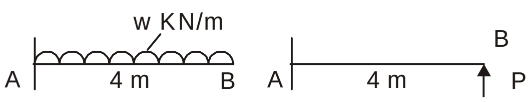
# Answer Key (SOM 11th June_2018)

| | | | | |
|---|---|---|---|---|
| 1. (a) | 16. (d) | 31. (a) | 46. (b) | 61. (d) |
| 2. (a) | 17. (d) | 32. (d) | 47. (a) | 62. (a) |
| 3. (c) | 18. (d) | 33. (c) | 48. (c) | 63. (a) |
| 4. (d) | 19. (c) | 34. (d) | 49. (d) | 64. (c) |
| 5. (a) | 20. (d) | 35. (d) | 50. (d) | 65. (c) |
| 6. (c) | 21. (a) | 36. (d) | 51. (c) | 66. (a) |
| 7. (b) | 22. (c) | 37. (a) | 52. (b) | 67. (a) |
| 8. (a) | 23. (a) | 38. (a) | 53. (c) | 68. (a) |
| 9. (c) | 24. (b) | 39. (b) | 54. (a) | 69. (b) |
| 10. (d) | 25. (c) | 40. (b) | 55. (c) | 70. (a) |
| 11. (b) | 26. (b) | 41. (d) | 56. (a) | 71. (d) |
| 12. (c) | 27. (b) | 42. (d) | 57. (a) | 72. (a) |
| 13. (a) | 28. (c) | 43. (d) | 58. (c) | 73. (b) |
| 14. (d) | 29. (c) | 44. (a) | 59. (c) | 74. (a) |
| 15. (b) | 30. (d) | 45. (d) | 60. (c) | 75. (a) |

# SOM CLASS TEST-SOLUTIONS

**1. (a)**




$$\delta_B(\downarrow) = \frac{wl^4}{8EI}$$

$$\delta_B(\uparrow) = \frac{Pl^3}{3EI}$$

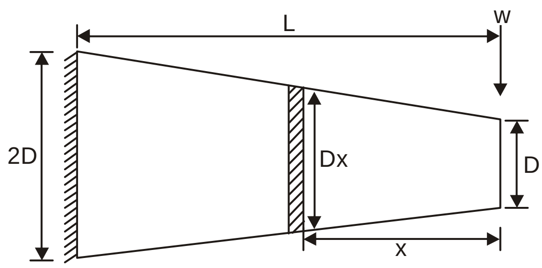
$$\delta_B(\downarrow) = \delta_B(\uparrow) \text{ (for zero vertical deflection at 'B')}$$

$$\Rightarrow \quad \frac{wl^4}{8EI} = \frac{P \times l^3}{3EI}$$

$$\therefore \quad \frac{800}{8} = \frac{P}{3}$$

$$\therefore \quad P = 300 \text{ KN}$$

**2. (a)**




Assuming a cross section at distance x from free end.

$$D_x = D + \frac{2D - D}{L}x = D + \frac{Dx}{L}$$

$$\therefore \ I_x = \frac{\pi}{64}\left(D + \frac{Dx}{L}\right)^4 = \frac{\pi D^4}{64}\left(1 + \frac{x}{L}\right)^4$$

By unit method

$$\Delta_{\text{free end}} = \int_0^L \frac{M.m\,dx}{EI} = \int_0^L \frac{Wx \times x\,dx}{E \times \frac{\pi D^4}{64}\left(1 + \frac{x}{L}\right)^4}\,dx$$

Let $1 + \frac{x}{L} = t$

$\Rightarrow$ dx = L dt

$x^2 = (t - 1)^2 L^2$

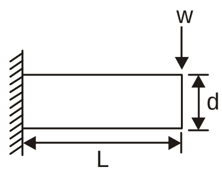
$$\therefore \quad \Delta_{\text{free end}} = \int_1^2 \frac{W \times 64}{\pi E D^4}\frac{(t-1)^2}{t^4}\,dt \times L^3$$

$$= \frac{64WL^3}{\pi ED^4}\left[\int_1^2 \left\{\frac{1}{t^2} + \frac{1}{t^4} - \frac{2}{t^3}\right\}dt\right]$$

$$= \frac{64WL^3}{\pi ED^4}\left[-\frac{1}{t} - \frac{1}{3t^3} + \frac{2}{2t^2}\right]_1^2$$

$$= \frac{64WL^3}{\pi ED^4}\left[-\frac{1}{2} - \frac{1}{24} + \frac{1}{4} + 1 + \frac{1}{3} - 1\right] = \frac{64}{24}\frac{WL^3}{\pi ED^4}$$

$$\therefore \quad \Delta_{\text{free end}} = \frac{64}{24}\frac{WL^3}{\pi ED^4} \qquad \text{... (1)}$$




$$\Delta_{\text{free end}} = \frac{WL^3 \times 64}{3E \times \pi d^4} \qquad \text{... (2)}$$

For (1) and (2) to be equal

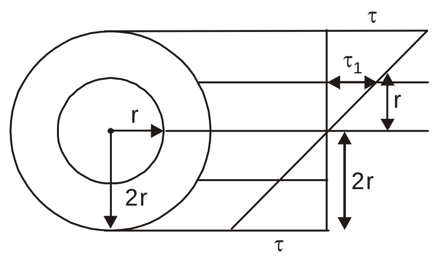
$$\frac{64}{24}\frac{WL^3}{\pi ED^4} = \frac{WL^3 \times 64}{3E\pi d^4}$$

$$\Rightarrow d^4 = \frac{24}{3}D^4$$

or d = 1.682D

**3. (c)**

**4. (d)**

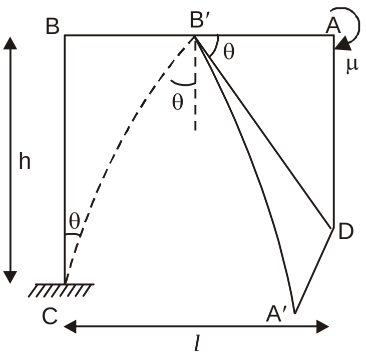



Shear stress in the inner tube

$$\tau_1 = \frac{\tau \times r}{2r}$$

$$\tau_1 = 0.5\tau$$

**5. (a)**



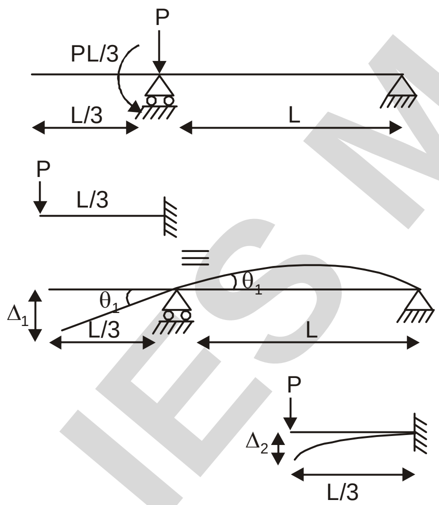

$$\theta = \frac{\mu h}{EI},\quad AD = \theta \times l = \frac{\mu h l}{EI}$$

$$DA' = \frac{\mu l^2}{2EI}$$

Due to very small deflection, DC′ may be taken as vertical deflection

$$\therefore\ (D_A)_{vertical} = \frac{\mu l}{EI} + \frac{\mu l^2}{2EI} = \frac{\mu l}{EI}\left[h + \frac{l}{2}\right]$$

**6. (c)**

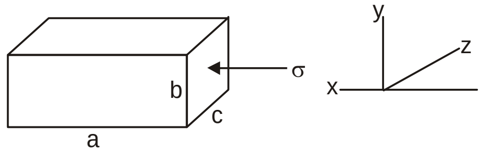



$$\theta_1 = \frac{ML}{3EI} = \frac{PL}{3} \times \frac{L}{3EI}$$

$$\therefore\ \Delta_1 = \theta_1 \times \frac{L}{3} = \frac{PL}{3} \times \frac{L}{3EI} \times \frac{L}{3} = \frac{PL^3}{27EI}$$

$$\Delta_2 = \frac{P \times \left(\frac{L}{3}\right)^3}{3EI} = \frac{PL^3}{81EI}$$

$$\therefore\ \Delta_{Total} = \Delta_1 + \Delta_2 = \frac{PL^3}{27EI} + \frac{PL^3}{81EI} = \frac{4PL^3}{81EI}$$




Final length in x, y, z direction is

$a - ea$, $b + e\mu b$ and $c + \mu ec$

$$\therefore\ \text{Final volume} = (a - ea)(b + e\mu b)(c + e\mu c)$$

$$= abc(1 - e)(1 + e\mu)(1 + e\mu)$$

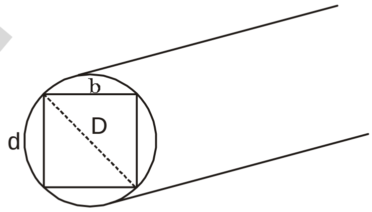
$$= (1 - e)(1 + e\mu)^2\ V \quad \text{where } V = abc$$

**7. (b)**

**8. (a)**

**9. (c)**

**10. (d)**




Let rectangular section is (b × d)

where $b^2 + d^2 = D^2$

For the beam to be strongest in the bending, Z should be maximum.

$$Z = \frac{bd^2}{6}$$

$$Z = \frac{b\left(D^2 - b^2\right)}{6}$$

For strongest section

$$\frac{dz}{db} = 0 = \frac{1}{6}\left[D^2 - 3b^2\right]$$

$$D^2 = 3b^2$$

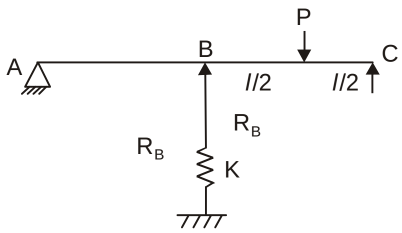
$$b = \frac{1}{\sqrt{3}}D$$

$$\because\ d^2 = D^2 - \frac{D^2}{3} = \frac{2}{3}D^2$$

$$\Rightarrow\ d = \sqrt{\frac{2}{3}}D$$

$$\frac{d}{b} = \sqrt{2}$$

**11. (b)**



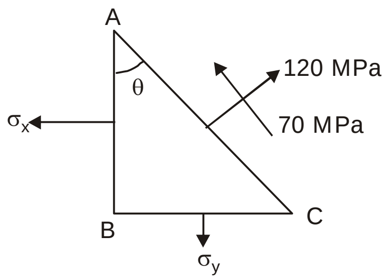

$$R_B + R_C = P$$

$$R_B = R_C = \frac{P}{2}$$

$$\text{Force in spring} = \frac{P}{2}$$

$$\text{Displacement at B} = \frac{P}{2K} \quad \text{(Downward)}$$

**12. (c)**




$$120 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\Rightarrow 120 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}(2\cos^2\theta - 1)$$

$$= \sigma_x(0.5 + 0.5(2\cos^2\theta - 1)) + \sigma_y(0.5 - 0.5(2\cos^2\theta - 1))$$

$$\cos\theta = \frac{AB}{AC} = \frac{4}{5}$$

$$\Rightarrow 120 = \sigma_x\left(0.5 + 0.5\left(2\times\left(\frac{4}{5}\right)^2 - 1\right)\right) + \sigma_y\left(0.5 - 0.5\left(2\times\left(\frac{4}{5}\right)^2 - 1\right)\right)$$

$$\Rightarrow 120 = \frac{16}{25}\sigma_x + \frac{9}{25}\sigma_y \quad ....(1)$$

$$\tau_{xy} = 70$$

$$= \frac{-(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

$$= \frac{-(\sigma_x - \sigma_y)}{2} 2\sin\theta\cos\theta$$

$$= \frac{-(\sigma_x - \sigma_y)}{2}\times 2\times\frac{3}{5}\times\frac{4}{5} = -(\sigma_x - \sigma_y)\times\frac{12}{25} \quad ..(2)$$

From (1) and (2)

$$\sigma_x = 67.5 \text{ MPa}$$

$$\sigma_y = 213.33 \text{ MPa}$$

**13. (a)**

T = 16 kN-M

M = 20 kN-M

as per Tresca theory

$$\tau_{max} \le \frac{f_y}{2}$$

$$\frac{16}{\pi d^3}\sqrt{m^2 + T^2} \le \frac{f_y}{2}$$

$$D^3 = \frac{16\times 2}{\pi\times f_y}\times\sqrt{M^2 + T^2}$$

D = 101.45 mmm

D = 102 mm

**14. (d)**

$$\frac{t_c}{t_s} = \frac{2 - \mu_c}{1 - \mu_h}$$

$$\frac{t_c}{t_s} = \frac{2 - 0.35}{1 - 0.25}$$

$$\frac{t_c}{t_s} = 2.2$$

**15. (b)**

$$\sigma_h = \frac{F}{A} - \sigma_l$$

$$\Rightarrow \quad \frac{P_r}{t} = \frac{F}{2\pi rt} - \frac{P_r}{2t}$$

$$\Rightarrow \quad F = 3\pi r^2 p$$

**16. (d)**

$$\text{Strain energy stored, } v = \frac{LT^2}{2GI_p}$$

$$v_1 = \frac{T^2\times 2L\times 32}{2G\times\pi D^4}$$

$$v_2 = \frac{T^2 L\times 32}{2\times G\times\pi\times 16D^4}$$

$$\frac{v_1}{v_2} = 32$$

**17. (d)**

twist angle at C

$$\theta_c = \frac{T_{AC} \times L_{AC}}{GI_P}$$

$$T_{AC} = \frac{T \times \frac{2L}{3}}{L} = \frac{2T}{3}$$

$$\therefore \quad \theta_c = \frac{\frac{2}{3}T \times \frac{L}{3}}{GI_P}$$

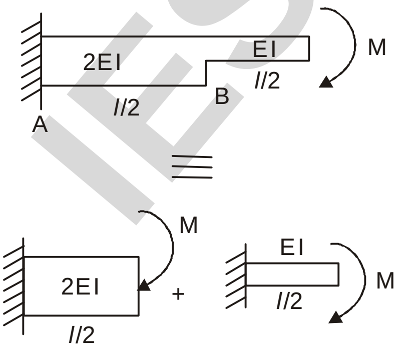
$$\Rightarrow \quad \theta_c = \frac{2TL}{9GI_P}$$

**18. (d)**

- Effective length of column is the distance between adjacent points of contraflexure or point of zero bending moment.
- Rankine theory of failure in applicable for both short and long column.
- $P_{CR} = \frac{\pi^2 EI}{L^2}$

  for all grade of steel, E is same. So various grades of steel will fail at same buckling load.

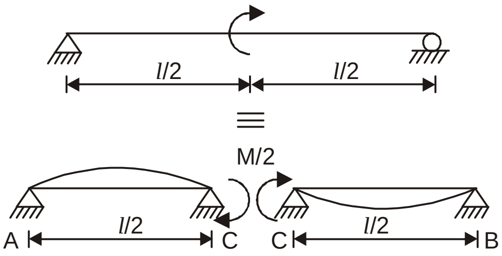
**19. (c)**




$$\Delta_C = \frac{M\left(\frac{l}{2}\right)^2}{2EI} + \frac{M\left(\frac{l}{2}\right)^2}{2 \times 2EI} + \frac{M\left(\frac{l}{2}\right)}{2EI} \times \frac{l}{2}$$

$$= \frac{Ml^2}{8EI} + \frac{Ml^2}{16EI} + \frac{Ml^2}{2 \times 4EI} = \frac{(2+1+2)Ml^2}{16EI} = \frac{5}{16}\frac{Ml^2}{EI}$$

**20. (d)**

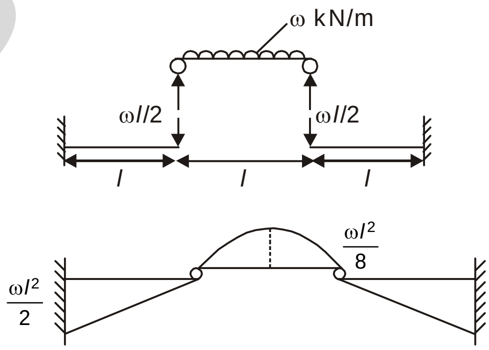



$$\theta_A = \frac{\frac{M}{2} \times \frac{l}{2}}{6EI} = \frac{Ml}{24EI}$$

$$\theta_C = \frac{\frac{M}{2} \times \frac{l}{2}}{3EI} = \frac{Ml}{12EI}$$

$$\theta_B = \frac{\frac{M}{2} \times \frac{l}{2}}{6EI} = \frac{Ml}{24EI}$$

**21. (a)**

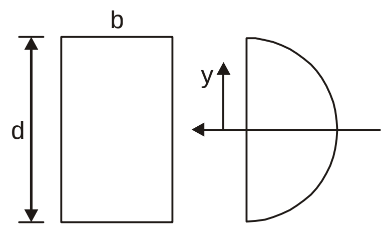



**22. (c)**

Theory of simple bending is only applicable to sections of beam in which plane of loading is axis of symmetry. △ and T have symmetry about loading axis (vertical axis) so theory of simple bending is applicable only to these sections.

**23. (a)**

**24. (b)**



$$\tau = \frac{6V\left(\frac{d^2}{4} - y^2\right)}{bd^3}$$

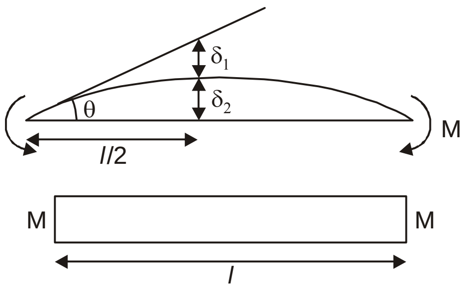
$$\tau_{avg} = \frac{V}{bd}$$

$$\tau = \tau_{avg}$$

$$\frac{6V\left(\frac{d^2}{4} - y^2\right)}{bd^3} = \frac{V}{bd}$$

$$y = \frac{d}{\sqrt{12}} = \frac{d}{2\sqrt{3}}$$

**25. (c)**




$$\theta = \frac{Ml}{2EI}$$

$$\delta_1 + \delta_2 = \frac{\theta l}{2} = \frac{Ml}{3EI} \times \frac{l}{2} = \frac{Ml^2}{4EI}$$

$$\delta_1 = \frac{ML}{2EI} \times \frac{l}{4} = \frac{Ml^2}{8EI}$$

$$\delta_2 = \frac{Ml^2}{4EI} - \frac{Ml^2}{8EI} = \frac{Ml^2}{8EI}$$

**26. (b)**

On the plane of maximum shear stress normal stress

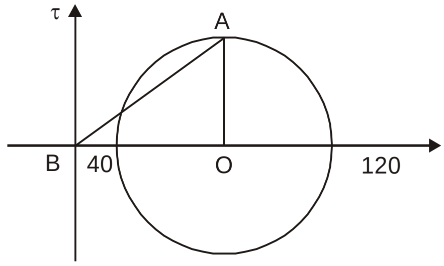
$$\sigma_{n_1} = \sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2}$$

$$= \frac{100 + 20}{2}$$

.

$$= 60 \text{ MPa}$$

**27. (b)**




$$\sqrt{OA + OB^2} = \sigma_R$$

$$OA = \frac{120 - 40}{2} = 40 \text{ MPa}$$

$$OB = \frac{120 + 40}{2} = 80 \text{ MPa}$$

$$\sigma = \sqrt{80^2 + 40^2}$$

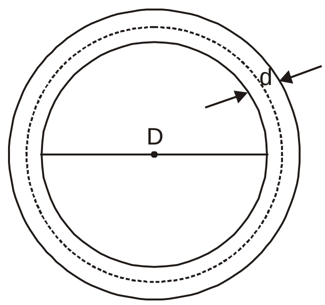
$$= 89.44 \text{ MPa}$$

$$\simeq 90 \text{ MPa}$$

**28. (c)**

- In non circular section apart from shearing stress, torsion also induces warping stress due to unsymmetrical distribution of shear stress.
- In rectangular section, due to torsion maximum shear stress induced is at the midpoint of the longest sides. This shear stress distribution is non liner from the centre.

**29. (c)**




$$\therefore \quad \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \quad \frac{\sigma}{d/2} = \frac{E}{\frac{D}{2} + \frac{d}{2}}$$

$$\Rightarrow \quad \sigma = E\left(\frac{d}{D+2}\right)$$

**30. (d)**

$$\varepsilon_1 = 80 \times 10^{-6}$$

$$\varepsilon_2 = 20 \times 10^{-6}$$

Maximum shear strain = $\gamma_{max}$

$$\frac{\gamma_{max}}{2} = \frac{(|\varepsilon_1 - \varepsilon_2|)}{2}$$

$$= 80 \times 10^{-6} - 20\times10^{-6}$$

$$\gamma_{max} = 60 \times 10^{-6}$$

**31. (a)**

**32. (d)**

$$\text{shear strain} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$= 6 \times 10^{-6} + 6 \times 10^{-6}$$

$$= 12 \times 10^{-6} \text{ unit}$$

**33. (c)**

$$\frac{\gamma_{yz}}{2} = 0.002$$

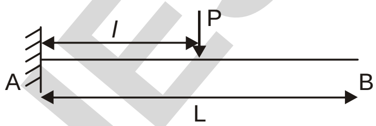
$$\gamma_{yz} = 0.004$$

Shear stress in yz plain = $\gamma_{yz} \times G$

$= 0.004 \times 100$ GPa

$= 400$ MPa

**34. (d)**




$$\Delta_B = \frac{Pl^3}{3EI} + \frac{Pl^2}{2EI} \times (L - l)$$

$$= \frac{Pl^2}{EI}\left(\frac{l}{3} + \frac{L-l}{2}\right)$$

$$= \frac{Pl^2}{2EI}\left(L - \frac{l}{3}\right)$$

**35. (d)**

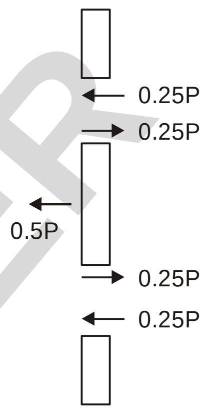
**36. (d)**

In traiangular section, maximum shear stress occurs at h/2 from vertex.

Therefore statement 1 is wrong.

In circular section, $\tau_{max} = \frac{4}{3}\tau_{avg}$

**37. (a)**

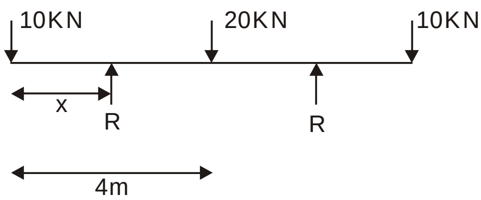



For carried by each pin = 0.5P

∴ Shear in each pin = 0.25P

**38. (a)**

**39. (b)**




R = 20 KN

Max negative BM = 10 × x

Max positive BM = $-40 \times 4 + 20(4 - x)$

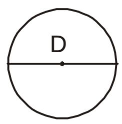
$\Rightarrow \quad 10 \times x = -10 \times 4 + 20(4 - x)$

$\Rightarrow \quad 10x = -40 + 80 - 20x$

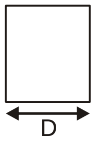
$\Rightarrow \quad 30x = 40 \Rightarrow x = 1.33\,m$

**40. (b)**

**41. (d)**




$$Z_C = \frac{\pi}{32}D^3$$



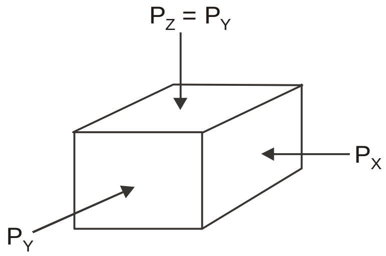

$$Z_S = \frac{D^3}{6}$$

$\because$ $$M = \sigma z$$

for same $\sigma$, $$\frac{M_c}{M_s} = \frac{Z_C}{Z_S} = \frac{\frac{\pi}{32}\times D^3}{\frac{D^3}{6}}$$

$$\frac{Z_C}{Z_S} = \frac{3\pi}{16}$$

**42. (d)**




$\because$ strain in lateral direction = 0

$$\Rightarrow \frac{P_y}{E} - \frac{\mu P_x}{E} - \frac{\mu P_y}{E} = 0$$

$$\Rightarrow P_y(1-\mu) = \mu P_x$$

or $$P_y = \frac{\mu P_x}{1-\mu}$$

**43. (d)**

**44. (a)**

**45. (d)**

$$F_1 + F_2 = 1000 \text{ Kg}$$

Taking moment about wire 2.

$$F_1 \times 40 = 1000 \times 20$$

$$\Rightarrow F_1 = 500 \text{ Kg}$$

$$F_2 = 500 \text{ Kg}$$

$$\frac{\Delta_1}{\Delta_2} = \frac{F_1}{A_1E_1} \times \frac{A_2E_2}{F_2}$$

$$= \frac{2}{4} \times \frac{2}{1} = 1$$

**46. (b)**

**47. (a)**

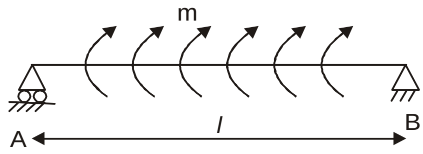
$$\delta_1 = \frac{P_1L}{AE}, \delta_2 = \frac{P_2L}{AE}$$

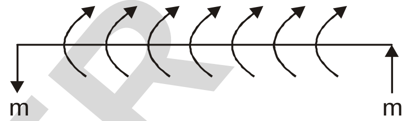
$$\delta = (P_1 + P_2)\frac{l}{AE}$$

So, $$\delta = \delta_1 + \delta_2$$

**48. (c)**

**49. (d)**




S.F. at any point.

$(S.F.)x = R_A = m$

So S.F.D.


**50. (d)**

**51. (c)**

**52. (b)**

**53. (c)**

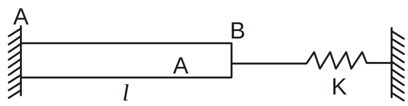
**54. (a)**

**55. (c)**

**56. (a)**

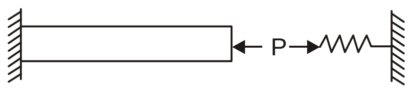
**57. (a)**

**58. (c)**




Free expansion of bar (without spring presence)

$= l\alpha t$



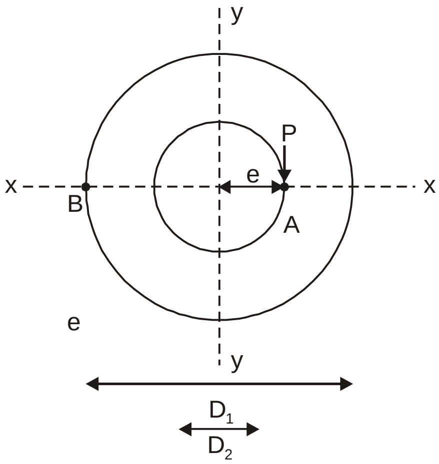

Compression of spring $= \frac{P}{K}$

$$\therefore \quad l\alpha t - \frac{P}{K} = \frac{Pl}{AE} \Rightarrow P\left(\frac{1}{K} + \frac{l}{AE}\right) = l\alpha t$$

$$\Rightarrow \frac{P}{A} = \frac{\alpha t E}{1 + \frac{AE}{Kl}} = \sigma$$

**59. (c)**

**60. (c)**




Suppose load P is acting at point 'A' at eccentricity of 'e', then

$$\sigma_B = \frac{P}{A} - \frac{My}{I}$$

$$= \frac{P \times 4}{\pi\left(D_1^2 - D_2^2\right)} = \frac{Pe \times \frac{D_1}{2}}{\frac{\pi\left(D_1^4 - D_2^4\right)}{64}}$$

For no tension at point B,

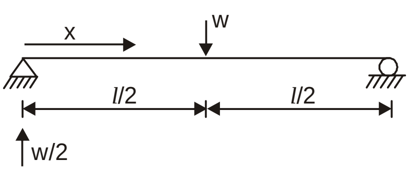
$$\sigma_B = 0$$

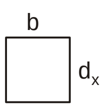
$$\Rightarrow \frac{P \times 4}{\pi\left(D_1^2 - D_2^2\right)} = \frac{Pe \times \frac{D_1}{2}}{\frac{\pi\left(D_1^4 - D_2^4\right)}{64}}$$

$$\Rightarrow e = \frac{D_1^4 - D_2^4}{8D_1\left(D_1^2 - D_2^2\right)}$$

$$\Rightarrow e = \frac{D_1^2 + D_2^2}{8D_1}$$

**61. (d)**




$$M = \sigma z$$

$$\Rightarrow \frac{w}{2}x = \frac{f \times bd_x^2}{6}$$

$$\Rightarrow d_x^2 = \frac{3wx}{fb}$$

$$\text{or} \quad d_x = \sqrt{\frac{3wx}{fb}}$$

**62. (a)**

$$K = \frac{E}{\sigma_y}$$

$$\alpha \propto \frac{\sigma_y}{\pi^2 E}$$

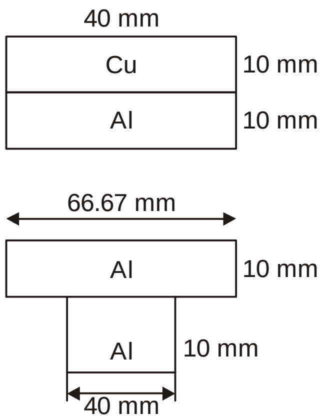
Rankine constant

$$\alpha = \frac{1}{\pi^2 \frac{E}{\sigma_y}}$$

$$\alpha \propto \frac{1}{K}$$

**63. (a)**

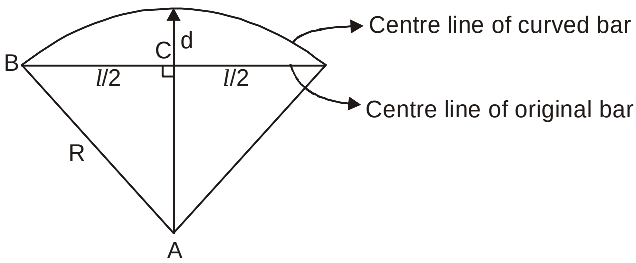
**64. (c)**




$$\bar{y} \text{ from top} = \frac{(66.67 \times 10 \times 5 + 40 \times 10 \times 15)}{66.67 \times 10 + 40 \times 10}$$

$$= 8.75 \text{ mm}$$

**65. (c)**



$$R^2 = (R-d)^2 + \frac{l^2}{4}$$

$$\Rightarrow R^2 = R^2 + d^2 - 2Rd + \frac{l^2}{4}$$

or $R = l\left(\frac{d}{2l} + \frac{l}{8d}\right)$

$\because$ d << $l$

$$\therefore R = \frac{l^2}{8d}$$

$$\because \frac{\sigma}{E} = \frac{y}{R} \Rightarrow \varepsilon = \frac{y}{R}$$

$$\therefore \varepsilon = \frac{\frac{t}{2}}{\frac{l^2}{8d}} = \frac{4td}{l^2}$$

**66. (a)**

**67. (a)**

**68. (a)**

$$\varepsilon_1 = 8 \times 10^{-4}$$

$$\varepsilon_2 = 4 \times 10^{-4}$$

$$\sigma_1 = \frac{E}{1-\mu^2} \times (\varepsilon_1 + \mu\varepsilon_2)$$

$$\sigma_1 = \frac{75000}{1-0.25}(8 \times 10^{-4} + 2 \times 10^{-4})$$

$$\sigma_1 = \frac{75000 \times 10 \times 10^{-4}}{0.75}$$

$$\sigma_1 = 100 \text{ KPa}$$

**69. (b)**

**70. (a)**

Deflection at B

$$6 \times 10^{-3} = \frac{wl^4}{8EI} - \frac{R_B L^3}{3EI}$$

$$6 \times 10^{-3} \times 625000 = \frac{100 \times 5^4}{8} - \frac{R_B \times 5^3}{3}$$

$$6 \times 625 = 125\left[\frac{100 \times 5}{8} - \frac{R_B}{3}\right]$$

$$30 = \frac{500}{8} - \frac{R_B}{3}$$

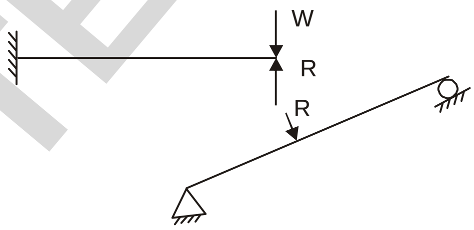
$$\frac{R_B}{3} = 32.5$$

$$\boxed{R_B = 97.5 \text{ kN}}$$

$$\boxed{\therefore R_A = 5 \times 100 - 97.5}$$

$$\boxed{R_A = 402.5 \text{ kN}}$$

**71. (d)**




$$\frac{(w-R)\ell^3}{3EI} = \frac{R\ell^3}{48EI}$$

$$16\text{ W} - 16\text{ R} = \text{R}$$

$$R = \frac{16}{17}W$$

**72. (a)**

**73. (b)**

**74. (a)**

**75. (a)**