
Answer Key (SOM 22nd June_2018)

1. (a)	16. (d)	31. (a)	46. (b)	61. (d)
2. (a)	17. (d)	32. (d)	47. (a)	62. (a)
3. (c)	18. (d)	33. (c)	48. (c)	63. (a)
4. (d)	19. (c)	34. (d)	49. (d)	64. (c)
5. (a)	20. (d)	35. (d)	50. (d)	65. (c)
6. (c)	21. (a)	36. (d)	51. (c)	66. (a)
7. (b)	22. (c)	37. (a)	52. (b)	67. (a)
8. (a)	23. (a)	38. (a)	53. (c)	68. (a)
9. (c)	24. (b)	39. (b)	54. (a)	69. (b)
10. (d)	25. (c)	40. (b)	55. (c)	70. (a)
11. (b)	26. (b)	41. (d)	56. (a)	71. (d)
12. (c)	27. (b)	42. (d)	57. (a)	72. (a)
13. (a)	28. (c)	43. (d)	58. (c)	73. (b)
14. (d)	29. (c)	44. (a)	59. (c)	74. (a)
15. (b)	30. (d)	45. (d)	60. (c)	75. (a)

SOM CLASS TEST-SOLUTIONS

1. (a)



$$\delta_B(\downarrow) = \frac{wL^4}{8EI}$$

$$\delta_B(\uparrow) = \frac{P\beta^3}{3EI}$$

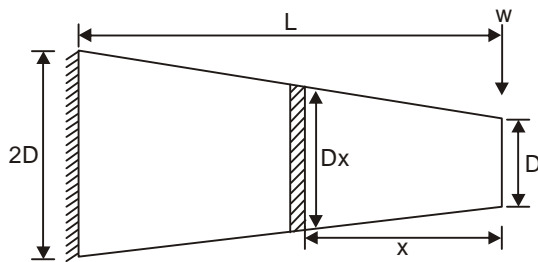
$$\delta_B(\downarrow) = \delta_B(\uparrow) \text{ (for zero vertical deflection at 'B')}$$

$$\Rightarrow \frac{wL^4}{8EI} = \frac{P \times \beta^3}{3EI}$$

$$\therefore \frac{800}{8} = \frac{P}{3}$$

$$\therefore P = 300 \text{ KN}$$

2. (a)



Assuming a cross section at distance x from free end.

$$D_x = D + \frac{2D-D}{L}x = D + \frac{Dx}{L}$$

$$\therefore I_x = \frac{\pi}{64} \left(D + \frac{Dx}{L} \right)^4 = \frac{\pi D^4}{64} \left(1 + \frac{x}{L} \right)^4$$

By unit method

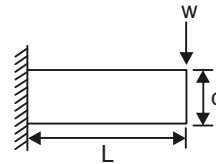
$$\Delta_{\text{free end}} = \int_0^L \frac{M \cdot m \, dx}{EI} = \int_0^L \frac{Wx \times x \, dx}{E \times \frac{\pi D^4}{64} \left(1 + \frac{x}{L} \right)^4} dx$$

$$\text{Let } 1 + \frac{x}{L} = t$$

$$\Rightarrow dx = L \, dt$$

$$x^2 = (t-1)^2 L^2$$

$$\begin{aligned} \therefore \Delta_{\text{free end}} &= \int_1^2 \frac{W \times 64 (t-1)^2}{\pi E D^4 t^4} dt \times L^3 \\ &= \frac{64WL^3}{\pi E D^4} \left[\int_1^2 \left(\frac{1}{t^2} + \frac{1}{t^4} - \frac{2}{t^3} \right) dt \right] \\ &= \frac{64WL^3}{\pi E D^4} \left[-\frac{1}{t} - \frac{1}{3t^3} + \frac{2}{2t^2} \right]_1^2 \\ &= \frac{64WL^3}{\pi E D^4} \left[-\frac{1}{2} - \frac{1}{24} + \frac{1}{4} + 1 + \frac{1}{3} - 1 \right] = \frac{64}{24} \frac{WL^3}{\pi E D^4} \\ \therefore \Delta_{\text{free end}} &= \frac{64}{24} \frac{WL^3}{\pi E D^4} \quad \dots (1) \end{aligned}$$



$$\Delta_{\text{free end}} = \frac{WL^3 \times 64}{3E \times \pi d^4} \quad \dots (2)$$

For (1) and (2) to be equal

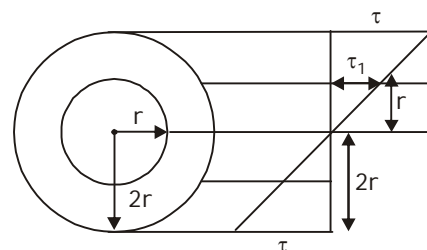
$$\frac{64}{24} \frac{WL^3}{\pi E D^4} = \frac{WL^3 \times 64}{3E \pi d^4}$$

$$\Rightarrow d^4 = \frac{24}{3} D^4$$

$$\text{or } d = 1.682D$$

3. (c)

4. (d)

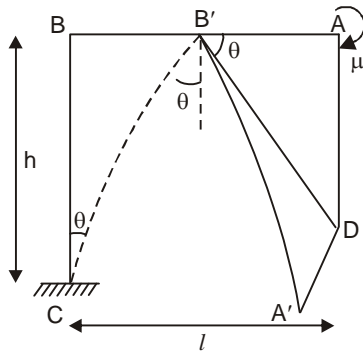


Shear stress in the inner tube

$$\tau_1 = \frac{\tau \times r}{2r}$$

$$\tau_1 = 0.5\tau$$

5. (a)



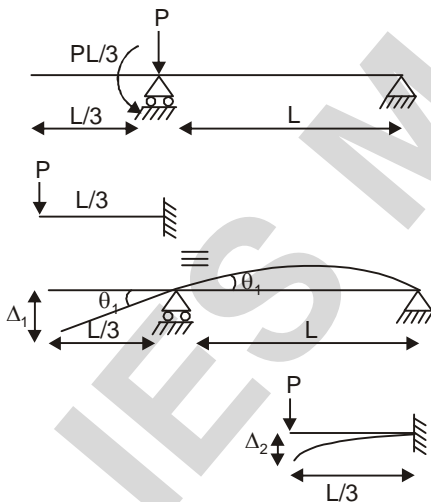
$$\theta = \frac{\mu h}{EI}, \quad AD = \theta \times l = \frac{\mu h l}{EI}$$

$$DA' = \frac{\mu l^2}{2EI}$$

Due to very small deflection, DC' may be taken as vertical deflection

$$\therefore (D_A)_{\text{vertical}} = \frac{\mu l}{EI} + \frac{\mu l^2}{2EI} = \frac{\mu l}{EI} \left[h + \frac{l}{2} \right]$$

6. (c)

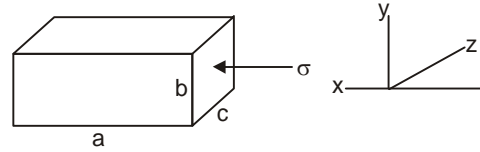


$$\theta_1 = \frac{ML}{3EI} = \frac{PL}{3} \times \frac{L}{3EI}$$

$$\therefore \Delta_1 = \theta_1 \times \frac{L}{3} = \frac{PL}{3} \times \frac{L}{3EI} \times \frac{L}{3} = \frac{PL^3}{27EI}$$

$$\Delta_2 = \frac{P \times \left(\frac{L}{3}\right)^3}{3EI} = \frac{PL^3}{81EI}$$

$$\therefore \Delta_{\text{Total}} = \Delta_1 + \Delta_2 = \frac{PL^3}{27EI} + \frac{PL^3}{81EI} = \frac{4PL^3}{81EI}$$



Final length in x, y, z direction is

$$a - ea, \quad b + e_{\mu}b \quad \text{and} \quad c + \mu ec$$

$$\therefore \text{Final volume} = (a - ea)(b + e_{\mu}b)(c + e_{\mu}c)$$

$$= abc(1 - e)(1 + e_{\mu})(1 + e_{\mu})$$

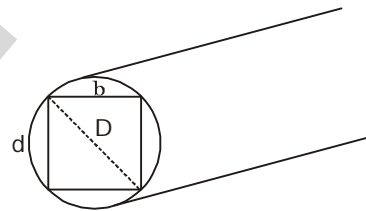
$$= (1 - e)(1 + e_{\mu})^2 V \quad \text{where } V = abc$$

7. (b)

8. (a)

9. (c)

10. (d)



Let rectangular section is $(b \times d)$

$$\text{where } b^2 + d^2 = D^2$$

For the beam to be strongest in the bending, Z should be maximum.

$$Z = \frac{bd^2}{6}$$

$$Z = \frac{b(D^2 - b^2)}{6}$$

For strongest section

$$\frac{dz}{db} = 0 = \frac{1}{6} [D^2 - 3b^2]$$

$$D^2 = 3b^2$$

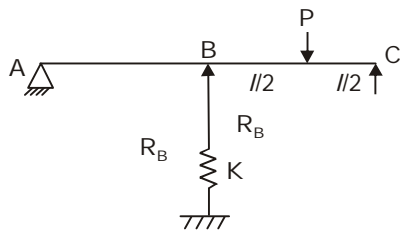
$$b = \frac{1}{\sqrt{3}} D$$

$$\therefore d^2 = D^2 - \frac{D^2}{3} = \frac{2}{3} D^2$$

$$\Rightarrow d = \sqrt{\frac{2}{3}} D$$

$$\frac{d}{b} = \sqrt{2}$$

11. (b)



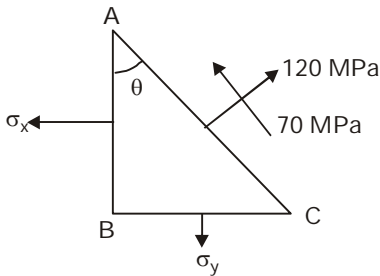
$$R_B + R_C = P$$

$$R_B = R_C = \frac{P}{2}$$

$$\text{Force in spring} = \frac{P}{2}$$

$$\text{Displacement at B} = \frac{P}{2K} \quad (\text{Downward})$$

12. (c)



$$120 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$\Rightarrow 120 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} (2\cos^2 \theta - 1)$$

$$= \sigma_x (0.5 + 0.5(2\cos^2 \theta - 1)) + \sigma_y (0.5 - 0.5(2\cos^2 \theta - 1))$$

$$\cos \theta = \frac{AB}{AC} = \frac{4}{5}$$

$$\Rightarrow 120 = \sigma_x \left(0.5 + 0.5 \left(2 \times \left(\frac{4}{5} \right)^2 - 1 \right) \right) + \sigma_y \left(0.5 - 0.5 \left(2 \times \left(\frac{4}{5} \right)^2 - 1 \right) \right)$$

$$\Rightarrow 120 = \frac{16}{25} \sigma_x + \frac{9}{25} \sigma_y \quad \dots(1)$$

$$\tau_{xy} = 70$$

$$= \frac{-(\sigma_x - \sigma_y)}{2} \sin 2\theta$$

$$= \frac{-(\sigma_x - \sigma_y)}{2} 2 \sin \theta \cos \theta$$

$$= \frac{-(\sigma_x - \sigma_y)}{2} \times 2 \times \frac{3}{5} \times \frac{4}{5} = -(\sigma_x - \sigma_y) \times \frac{12}{25} \dots(2)$$

From (1) and (2)

$$\sigma_x = 67.5 \text{ MPa}$$

$$\sigma_y = 213.33 \text{ MPa}$$

13. (a)

$$T = 16 \text{ kN-M}$$

$$M = 20 \text{ kN-M}$$

as per Tresca theory

$$\tau_{\max} \leq \frac{f_y}{2}$$

$$\frac{16}{\pi d^3} \sqrt{M^2 + T^2} \leq \frac{f_y}{2}$$

$$D^3 = \frac{16 \times 2}{\pi \times f_y} \times \sqrt{M^2 + T^2}$$

$$D = 101.45 \text{ mm}$$

$$D = 102 \text{ mm}$$

14. (d)

$$\frac{t_c}{t_s} = \frac{2 - \mu_c}{1 - \mu_h}$$

$$\frac{t_c}{t_s} = \frac{2 - 0.35}{1 - 0.25}$$

$$\frac{t_c}{t_s} = 2.2$$

15. (b)

$$\sigma_h = \frac{F}{A} - \sigma_l$$

$$\Rightarrow \frac{P_r}{t} = \frac{F}{2\pi r t} - \frac{P_r}{2t}$$

$$\Rightarrow F = 3\pi r^2 p$$

16. (d)

$$\text{Strain energy stored, } v = \frac{LT^2}{2GI_p}$$

$$V_1 = \frac{T^2 \times 2L \times 32}{2G \times \pi D^4}$$

$$V_2 = \frac{T^2 L \times 32}{2 \times G \times \pi \times 16D^4}$$

$$\frac{v_1}{v_2} = 32$$

17. (d)

twist angle at C

$$\theta_c = \frac{T_{AC} \times L_{AC}}{GI_P}$$

$$T_{AC} = \frac{T \times \frac{2L}{3}}{L} = \frac{2T}{3}$$

$$\therefore \theta_c = \frac{\frac{2}{3}T \times \frac{L}{3}}{GI_P}$$

$$\Rightarrow \theta_c = \frac{2TL}{9GI_P}$$

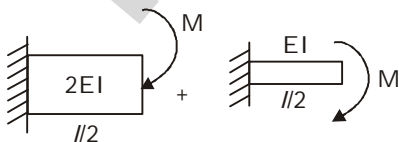
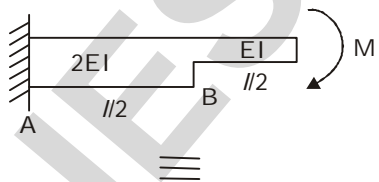
18. (d)

- Effective length of column is the distance between adjacent points of contraflexure or point of zero bending moment.
- Rankine theory of failure is applicable for both short and long column.

$$P_{CR} = \frac{\pi^2 EI}{L^2}$$

for all grade of steel, E is same. So various grades of steel will fail at same buckling load.

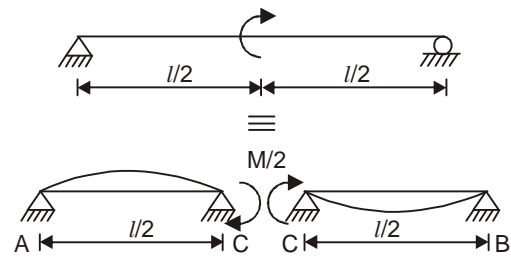
19. (c)



$$\Delta_C = \frac{M \left(\frac{l}{2}\right)^2}{2EI} + \frac{M \left(\frac{l}{2}\right)^2}{2 \times 2EI} + \frac{M \left(\frac{l}{2}\right)}{2EI} \times \frac{l}{2}$$

$$= \frac{Ml^2}{8EI} + \frac{Ml^2}{16EI} + \frac{Ml^2}{2 \times 4EI} = \frac{(2+1+2)Ml^2}{16EI} = \frac{5}{16} \frac{Ml^2}{EI}$$

20. (d)

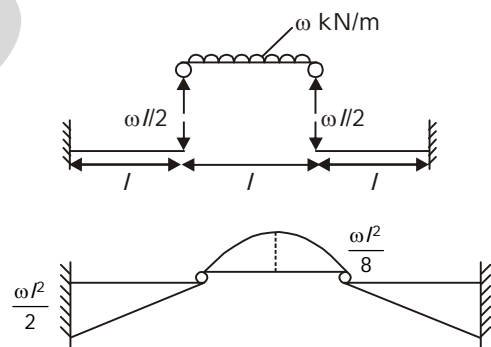


$$\theta_A = \frac{\frac{M}{2} \times \frac{l}{2}}{6EI} = \frac{Ml}{24EI}$$

$$\theta_C = \frac{\frac{M}{2} \times \frac{l}{2}}{3EI} = \frac{Ml}{12EI}$$

$$\theta_B = \frac{\frac{M}{2} \times \frac{l}{2}}{6EI} = \frac{Ml}{24EI}$$

21. (a)

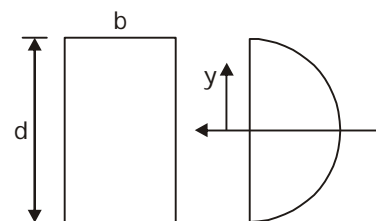


22. (c)

Theory of simple bending is only applicable to sections of beam in which plane of loading is axis of symmetry. Δ and T have symmetry about loading axis (vertical axis) so theory of simple bending is applicable only to these sections.

23. (a)

24. (b)



$$\tau = \frac{6V \left(\frac{d^2}{4} - y^2 \right)}{bd^3}$$

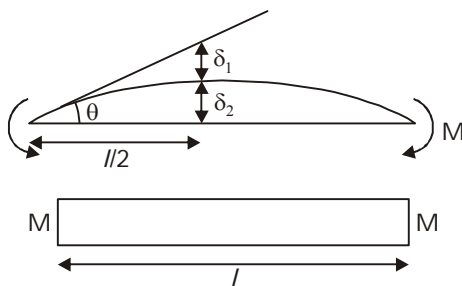
$$\tau_{avg} = \frac{V}{bd}$$

$$\tau = \tau_{avg}$$

$$\frac{6V \left(\frac{d^2}{4} - y^2 \right)}{bd^3} = \frac{V}{bd}$$

$$y = \frac{d}{\sqrt{12}} = \frac{d}{2\sqrt{3}}$$

25. (c)



$$\theta = \frac{Ml}{2EI}$$

$$\delta_1 + \delta_2 = \frac{\theta l}{2} = \frac{Ml}{3EI} \times \frac{l}{2} = \frac{Ml^2}{4EI}$$

$$\delta_1 = \frac{Ml}{2EI} \times \frac{l}{4} = \frac{Ml^2}{8EI}$$

$$\delta_2 = \frac{Ml^2}{4EI} - \frac{Ml^2}{8EI} = \frac{Ml^2}{8EI}$$

26. (b)

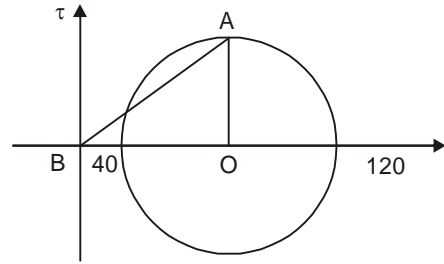
On the plane of maximum shear stress normal stress

$$\sigma_{n1} = \sigma_{n2} = \frac{\sigma_1 + \sigma_2}{2}$$

$$= \frac{100 + 20}{2}$$

$$= 60 \text{ MPa}$$

27. (b)



$$\sqrt{OA + OB^2} = \sigma_R$$

$$OA = \frac{120 - 40}{2} = 40 \text{ MPa}$$

$$OB = \frac{120 + 40}{2} = 80 \text{ MPa}$$

$$\sigma = \sqrt{80^2 + 40^2}$$

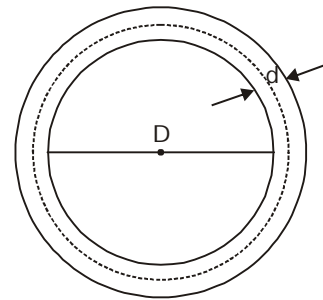
$$= 89.44 \text{ MPa}$$

$$\approx 90 \text{ MPa}$$

28. (c)

- In non circular section apart from shearing stress, torsion also induces warping stress due to unsymmetrical distribution of shear stress.
- In rectangular section, due to torsion maximum shear stress induced is at the midpoint of the longest sides. This shear stress distribution is non liner from the centre.

29. (c)



$$\therefore \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \frac{\sigma}{d/2} = \frac{E}{\frac{D}{2} + \frac{d}{2}}$$

$$\Rightarrow \sigma = E \left(\frac{d}{D+2} \right)$$

30. (d)

$$\varepsilon_1 = 80 \times 10^{-6}$$

$$\varepsilon_2 = 20 \times 10^{-6}$$

Maximum shear strain = γ_{\max}

$$\frac{\gamma_{\max}}{2} = \frac{(|\varepsilon_1 - \varepsilon_2|)}{2}$$

$$= 80 \times 10^{-6} - 20 \times 10^{-6}$$

$$\gamma_{\max} = 60 \times 10^{-6}$$

31. (a)

32. (d)

$$\text{shear strain} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$= 6 \times 10^{-6} + 6 \times 10^{-6}$$

$$= 12 \times 10^{-6} \text{ unit}$$

33. (c)

$$\frac{\gamma_{yz}}{2} = 0.002$$

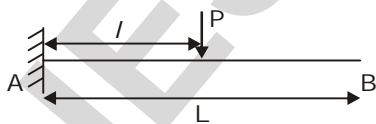
$$\gamma_{yz} = 0.004$$

$$\text{Shear stress in yz plain} = \gamma_{yz} \times G$$

$$= 0.004 \times 100 \text{ GPa}$$

$$= 400 \text{ MPa}$$

34. (d)



$$\Delta_B = \frac{Pl^3}{3EI} + \frac{Pl^2}{2EI} \times (L-l)$$

$$= \frac{Pl^2}{EI} \left(\frac{l}{3} + \frac{L-l}{2} \right)$$

$$= \frac{Pl^2}{2EI} \left(L - \frac{l}{3} \right)$$

35. (d)

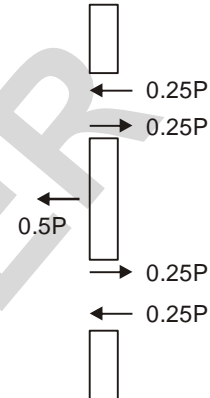
36. (d)

In triangular section, maximum shear stress occurs at $h/2$ from vertex.

Therefore statement 1 is wrong.

In circular section, $\tau_{\max} = \frac{4}{3} \tau_{\text{avg}}$

37. (a)

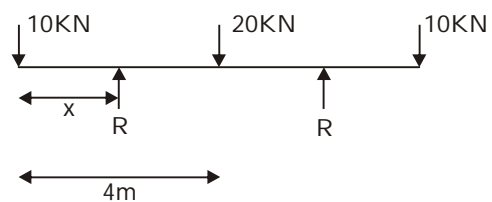


For carried by each pin = $0.5P$

\therefore Shear in each pin = $0.25P$

38. (a)

39. (b)



$$R = 20 \text{ kN}$$

$$\text{Max negative BM} = 10 \times x$$

$$\text{Max positive BM} = -40 \times 4 + 20(4-x)$$

$$\Rightarrow 10 \times x = -10 \times 4 + 20(4-x)$$

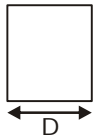
$$\Rightarrow 10x = -40 + 80 - 20x$$

$$\Rightarrow 30x = 40 \Rightarrow x = 1.33 \text{ m}$$

40. (b)

41. (d)

$$Z_c = \frac{\pi}{32} D^3$$



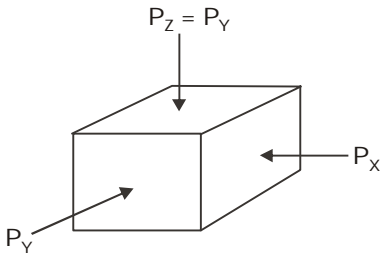
$$Z_s = \frac{D^3}{6}$$

$$\therefore M = \sigma z$$

$$\text{for same } \sigma, \frac{M_c}{M_s} = \frac{Z_c}{Z_s} = \frac{\frac{\pi}{32} \times D^3}{\frac{D^3}{6}}$$

$$\frac{Z_c}{Z_s} = \frac{3\pi}{16}$$

42. (d)



\therefore strain in lateral direction = 0

$$\Rightarrow \frac{P_y}{E} - \frac{\mu P_x}{E} - \frac{\mu P_y}{E} = 0$$

$$\Rightarrow P_y(1 - \mu) = \mu P_x$$

$$\text{or } P_y = \frac{\mu P_x}{1 - \mu}$$

43. (d)

44. (a)

45. (d)

$$F_1 + F_2 = 1000 \text{ Kg}$$

Taking moment about wire 2.

$$F_1 \times 40 = 1000 \times 20$$

$$\Rightarrow F_1 = 500 \text{ Kg}$$

$$F_2 = 500 \text{ Kg}$$

$$\frac{\Delta_1}{\Delta_2} = \frac{F_1}{A_1 E_1} \times \frac{A_2 E_2}{F_2}$$

$$= \frac{2}{4} \times \frac{2}{1} = 1$$

46. (b)

47. (a)

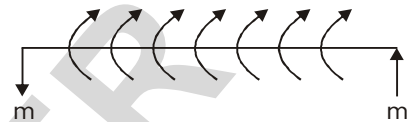
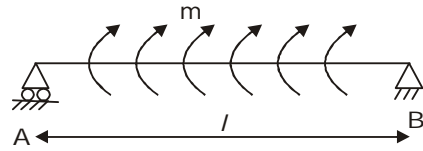
$$\delta_1 = \frac{P_1 L}{AE}, \delta_2 = \frac{P_2 L}{AE}$$

$$\delta = (P_1 + P_2) \frac{l}{AE}$$

$$\text{So, } \delta = \delta_1 + \delta_2$$

48. (c)

49. (d)



S.F. at any point.

$$(S.F.)_x = R_A = m$$

So S.F.D.



50. (d)

51. (c)

52. (b)

53. (c)

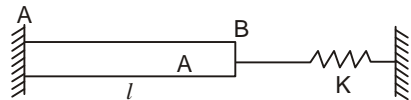
54. (a)

55. (c)

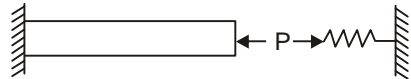
56. (a)

57. (a)

58. (c)



Free expansion of bar (without spring presence) = $l\alpha t$



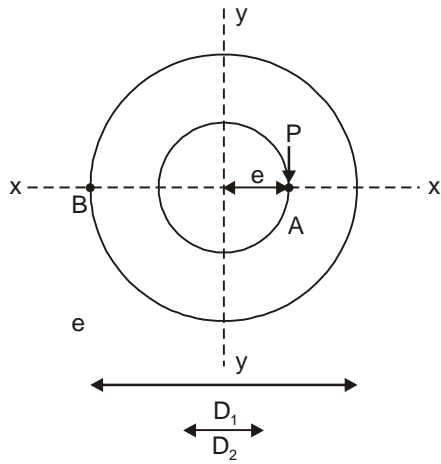
Compression of spring = $\frac{P}{K}$

$$\therefore l\alpha t - \frac{P}{K} = \frac{Pl}{AE} \Rightarrow P \left(\frac{1}{K} + \frac{l}{AE} \right) = l\alpha t$$

$$\Rightarrow \frac{P}{A} = \frac{\alpha t E}{1 + \frac{Kl}{AE}} = \sigma$$

59. (c)

60. (c)



Suppose load P is acting at point 'A' at eccentricity of 'e', then

$$\sigma_B = \frac{P}{A} - \frac{My}{I}$$

$$= \frac{P \times 4}{\pi(D_1^2 - D_2^2)} = \frac{Pe \times \frac{D_1}{2}}{\pi(D_1^4 - D_2^4)} \times 64$$

For no tension at point B,

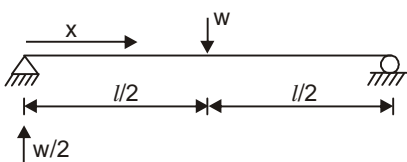
$$\sigma_B = 0$$

$$\Rightarrow \frac{P \times 4}{\pi(D_1^2 - D_2^2)} = \frac{Pe \times \frac{D_1}{2}}{\pi(D_1^4 - D_2^4)} \times 64$$

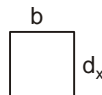
$$\Rightarrow e = \frac{D_1^4 - D_2^4}{8D_1(D_1^2 - D_2^2)}$$

$$\Rightarrow e = \frac{D_1^2 + D_2^2}{8D_1}$$

61. (d)



$$M = \sigma z$$



$$\Rightarrow \frac{w}{2} x = \frac{f \times b d_x^2}{6}$$

$$\Rightarrow d_x^2 = \frac{3wx}{fb}$$

$$\text{or } d_x = \sqrt{\frac{3wx}{fb}}$$

62. (a)

$$K = \frac{E}{\sigma_y}$$

$$\alpha \propto \frac{\sigma_y}{\pi^2 E}$$

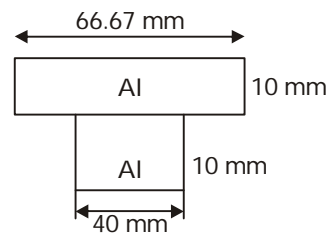
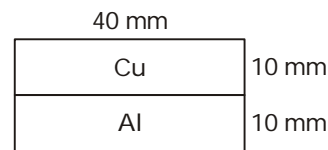
Rankine constant

$$\alpha = \frac{1}{\pi^2 \frac{E}{\sigma_y}}$$

$$\alpha \propto \frac{1}{K}$$

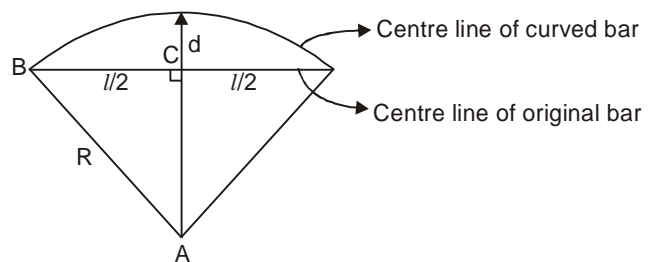
63. (a)

64. (c)



$$\bar{y} \text{ from top} = \frac{(66.67 \times 10 \times 5 + 40 \times 10 \times 15)}{66.67 \times 10 + 40 \times 10} = 8.75 \text{ mm}$$

65. (c)



$$R^2 = (R-d)^2 + \frac{l^2}{4}$$

$$\Rightarrow R^2 = R^2 + d^2 - 2Rd + \frac{l^2}{4}$$

$$\text{or } R = l \left(\frac{d}{2l} + \frac{l}{8d} \right)$$

$$\therefore d \ll l$$

$$\therefore R = \frac{l^2}{8d}$$

$$\therefore \frac{\sigma}{E} = \frac{y}{R} \Rightarrow \varepsilon = \frac{y}{R}$$

$$\therefore \varepsilon = \frac{\frac{t}{2}}{\frac{l^2}{8d}} = \frac{4td}{l^2}$$

66. (a)

67. (a)

68. (a)

$$\varepsilon_1 = 8 \times 10^{-4}$$

$$\varepsilon_2 = 4 \times 10^{-4}$$

$$\sigma_1 = \frac{E}{1-\mu^2} \times (\varepsilon_1 + \mu\varepsilon_2)$$

$$\sigma_1 = \frac{75000}{1-0.25} (8 \times 10^{-4} + 2 \times 10^{-4})$$

$$\sigma_1 = \frac{75000 \times 10 \times 10^{-4}}{0.75}$$

$$\sigma_1 = 100 \text{ KPa}$$

69. (b)

70. (a)

Deflection at B

$$6 \times 10^{-3} = \frac{w l^4}{8EI} - \frac{R_B L^3}{3EI}$$

$$6 \times 10^{-3} \times 625000 = \frac{100 \times 5^4}{8} - \frac{R_B \times 5^3}{3}$$

$$6 \times 625 = 125 \left[\frac{100 \times 5}{8} - \frac{R_B}{3} \right]$$

$$30 = \frac{500}{8} - \frac{R_B}{3}$$

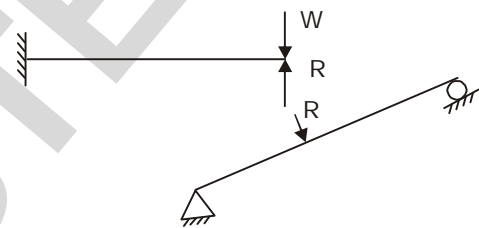
$$\frac{R_B}{3} = 32.5$$

$$R_B = 97.5 \text{ kN}$$

$$\therefore R_A = 5 \times 100 - 97.5$$

$$R_A = 402.5 \text{ kN}$$

71. (d)



$$\frac{(w-R)\ell^3}{3EI} = \frac{R\ell^3}{48EI}$$

$$16W - 16R = R$$

$$R = \frac{16}{17}W$$

72. (a)

73. (b)

74. (a)

75. (a)