

Solution (RCC)

4th March, 2019

Sol. 1(a) Transfer of axial force at base of column

In order to provide a plain concrete block footing, full force transfer must be possible at the column base, without the need for reinforcement at the interface.

⇒

$$P_u \leq \bar{v}_{br} \text{ (limiting bearing resistance)}$$

$$P_u = 1.5 \times 330 \text{ kN} = 495 \text{ kN}$$

$$\bar{v}_{br, \max} = 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$A_1 = A_2 = 300 \times 300 \text{ mm}^2$$

$$\bar{v}_{br} = 0.45 \times 20 \times \sqrt{\frac{300^2}{300^2}} = 9 \text{ MPa}$$

$$F_{br} = 9 \times 300^2 = 810 \text{ kN} > P_u = 495 \text{ kN}$$

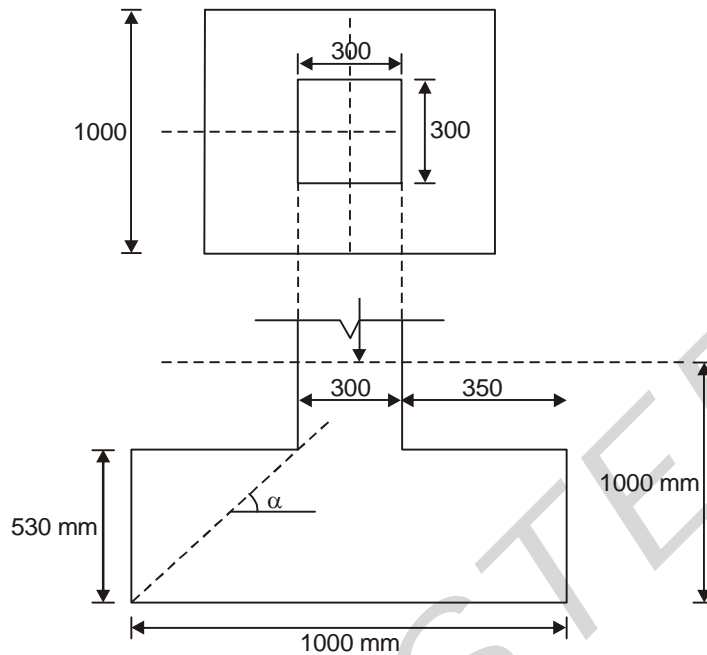
Full transfer of force is possible without the need for reinforcement

Size of footing

Assuming the weight of footing and backfill to comprise 10% of axial load,

$$\text{Base area required} = \frac{330 \times 1.1}{360} = 1.01 \text{ m}^2$$

Provide 1 m × 1 m footing



Thickness of footing

$$D = \left(\frac{1000 - 300}{2} \right) \tan \alpha = 350 \tan \alpha$$

$$\tan \alpha \geq 0.9 \sqrt{\frac{100q_{\max}}{f_{ck}} + 1}$$

$$\tan \alpha \geq 0.9 \sqrt{\frac{100 \times 360 \times 10^3}{10^6 \times 20} + 1}$$

$$\tan \alpha \geq 1.51$$

$$D = 350 \times 1.51 = 528.5 \text{ mm} \approx 530 \text{ mm}$$

Hence provide a concrete block $1000 \times 1000 \times 530 \text{ mm}$

(ii) Minimum reinforcement to provide 'tie action' and to account for temperature and shrinkage effects.

$$(A_{st})_{\min} = \frac{0.12}{100} BD = \frac{0.12}{100} \times 1000 \times 530 = 636 \text{ mm}^2$$

Provide 6-12 mm ϕ bars ($A_{st} = 678 \text{ mm}^2$) both ways with a clear cover of 75 mm

$$\text{Spacing} < 5d \text{ or } 300 \text{ mm Spacing} = \frac{1000 - 2 \times 75 - 2 \times \frac{12}{2}}{(6 - 1)} = 167.60 \text{ mm}$$

(iii) Check for gross base pressure

Assuming

$$\gamma_{\text{concrete}} = 24 \text{ kN/m}^3$$

$$\gamma_{\text{soil}} = 18 \text{ kN/m}^3$$

Actual gross soil pressure

$$q_{\text{max}} = \frac{330}{1 \times 1} + 24 \times 0.53 + 18 \times 0.47$$

$$q_{\text{max}} = 330 + 12.7 + 8.5$$

$$= 351.2 \text{ kN/m}^2$$

$$< q_a = 360 \text{ kN/m}^2 \quad \text{Hence, safe}$$

If the specified safe bearing capacity is based on net soil pressure,

Actual net soil pressure

$$q_{\text{net}} = \frac{330}{1 \times 1} + (24 - 18) \times 0.53$$

$$= 333.2 \text{ kN/m}^2$$

$$< q_a = 360 \text{ kN/m}^2 \text{ hence safe}$$

Sol.1.(b) Let us design a balanced doubly reinforced section.

$$\text{Factored moment } M_u = 1.5 \times 160$$

$$= 240 \text{ kNm}$$

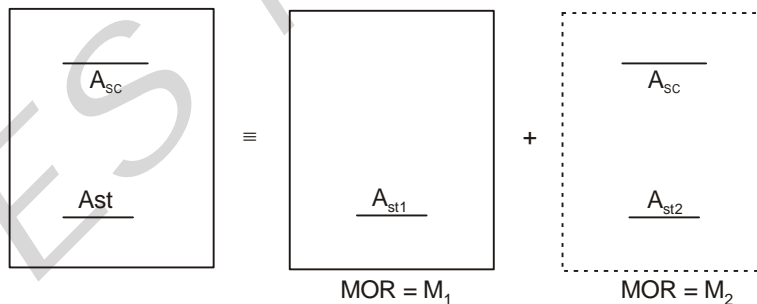
$$M_{u, \text{lim}} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 20 \times 250 \times (500)^2$$

$$= 172.5 \text{ kNm}$$

$$\therefore M_u > M_{u, \text{lim}}$$

Doubly reinforced section is needed.



$$\text{Determination of } A_{st1} = A_{st, \text{lim}}$$

$$x_u = x_{u, \text{lim}}$$

$$= 0.48 d$$

$$= 0.48 \times 500$$

$$= 240 \text{ mm}$$

$$C = T$$

$$\begin{aligned} 0.36f_{ck} bx_{u,lim} &= 0.87 f_y A_{st,lim} \\ 0.36 \times 20 \times 250 \times 240 &= 0.87 \times 415 \times A_{st,lim} \\ A_{st,lim} &= 1196.5 \text{ mm}^2 \\ M_2 &= M_u - M_1 \\ &= 240 - 172.5 \\ &= 67.5 \text{ kNm} \end{aligned}$$

Determination of A_{st2} ,

$$\begin{aligned} M_2 &= 0.87 f_y A_{st2} (d - d') \\ 67.5 \times 10^6 &= 0.87 \times 415 \times A_{st2} (500 - 50) \\ A_{st2} &= 415 \text{ mm}^2 \end{aligned}$$

For A_{SC}

$C = T$ in auxillary section.

$$A_{SC} (f_{SC} - 0.45f_{ck}) = 0.87 f_y A_{st2}$$

For f_{SC} , $\frac{d'}{d} = \frac{50}{500} = 0.10$

Corresponding to $\frac{d'}{d} = 0.10$ and Fe 415 grade steel.

$$f_{SC} = 352 \text{ N/mm}^2$$

$$\begin{aligned} A_{SC} &= \frac{0.87 f_y A_{st2}}{f_{SC} - 0.45 f_{ck}} \\ &= \frac{0.87 \times 415 \times 415}{352 - 0.45 \times 20} \end{aligned}$$

$$= 436 \text{ mm}^2$$

$$\begin{aligned} A_{st} &= A_{st,lim} + A_{st2} \\ &= 1196.5 + 415 \\ &= 1611.5 \text{ mm}^2 \end{aligned}$$

$$A_{SC} = 436 \text{ mm}^2$$

Sol. 1(c)

$$\begin{aligned} \text{Area of steel provided } A_{st} &= 3 \times \frac{\pi}{4} \times (20)^2 \\ &= 942 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Percentage of steel provided, } P_t &= \frac{A_{st}}{b_w d} \times 100 \\ &= \frac{942}{275 \times 470} \\ &= 0.73\% \end{aligned}$$

$$\begin{aligned} \text{Nominal shear stress } \tau_v &= \frac{V_u}{b_w d} \\ &= \frac{190 \times 1000}{275 \times 470} \\ &= 1.47 \text{ N/mm}^2 \end{aligned}$$

$$\tau_{c, \max} \text{ for M20 concrete} = 2.80 \text{ N/mm}^2 \quad (\tau_{c, \max} \cong 0.631 \sqrt{f_{ck}})$$

Design shear stress τ_c for 0.73% steel = 0.55 N/mm²

here, $\tau_v - \tau_c > 0.4 \text{ MPa}$, \Rightarrow shear reinforcement will be govern by $\tau_v - \tau_c$.

$$\Rightarrow 0.87 f_y A_{sv} \cdot \frac{d}{s_v} \geq (\tau_v - \tau_c) bd$$

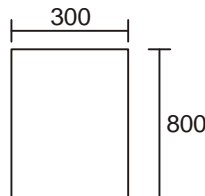
considering 2 legged 8mm dia. $A_{sv} = 100 \text{ mm}^2$

Spacing of stirrups should be least of following :

- (i) 142.71 mm
- (ii) 300 mm
- (iii) $0.75d = 0.75 \times 470 = 352.5 \text{ mm}$

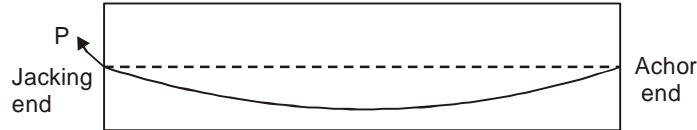
Provide 2-legged 8 mm dia stirrups @ 140 mm c/c

Sol. 1.(d)



$$\text{Equation of parabolic profile } y = \frac{4hx(l-x)}{l^2}$$

$$\text{Slope } \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$



$$\theta = \text{slope at end} = \frac{4h}{l}$$

Jacking at only one end

$$\alpha = \text{change in slope} = 2\theta = \frac{8h}{l}, \quad h = \text{eccentricity at mid span.}$$

$$x = L = 10 \text{ m}$$

(a) (ii)

$$\Delta P = \text{Loss due to friction} = P_0(Kx + \mu\alpha)$$

$$P_0 - P_0(kx + \mu\theta) = 250$$

$$\Rightarrow P_0 = \frac{250}{1 - kx - \mu\theta} = \frac{250}{1 - 0.003 \times 10 - 0.55 \times \frac{8 \times 250}{10000}} = 290.70 \text{ kN}$$

$$\Rightarrow \text{Friction loss} = 290.70 - 250 = 40.70 \text{ kN.}$$

$$(ii) \quad \text{Loss due to Anchorage slip} = \frac{\Delta l}{l} \times E_s$$

$$= \frac{3}{10000} \times 210$$

$$= 0.063 \text{ kN/mm}^2$$

$$= 63 \text{ N/mm}^2$$

$$(iii) \quad \text{Loss due to shrinkage strain} = \epsilon_s E_s$$

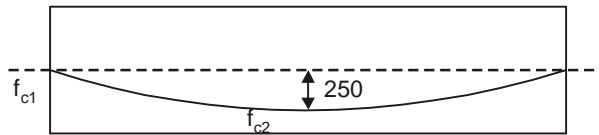
$$\epsilon_s = \frac{2 \times 10^{-4}}{\log(t+2)}$$

$$\text{Loss} = \frac{2 \times 10^{-4}}{\log(8+2)} \times 210 \text{ kN/mm}^2$$

$$= 0.042 \text{ kN/mm}^2$$

$$= 42 \text{ N/mm}^2$$

(iv) Loss due to creep



Stress in concrete, at ends

$$f_{c1} = \frac{P}{A} = \frac{218.5 \times 10^3}{300 \times 800} = 0.91 \text{ N/mm}^2$$

(Force remaining after friction and anchorage slip loss = $250 - 63 \times 500 \times 10^{-3} = 218.5 \text{ kN}$)

$$\begin{aligned} \text{at center, } f_{c2} &= \frac{P}{A} + \frac{Pe^2}{I} \\ &= \frac{218.5 \times 10^3}{300 \times 800} + \frac{218.5 \times 10^3 \times 250 \times 250}{300 \times \frac{(800)^3}{12}} \\ &= 1.98 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Avg. stress } f_{\text{cavg}} &= f_{c1} + \frac{2}{3}(f_{c2} - f_{c1}) \\ &= 0.91 + \frac{2}{3}(1.98 - 0.91) \\ &= 1.62 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Loss due to creep} &= m \phi f_{\text{cavg}} \\ &= 6 \times 2.2 \times 1.62 \quad (\text{Assuming } \phi \cong 2.2) \\ &= 21.38 \text{ N/mm}^2 \end{aligned}$$

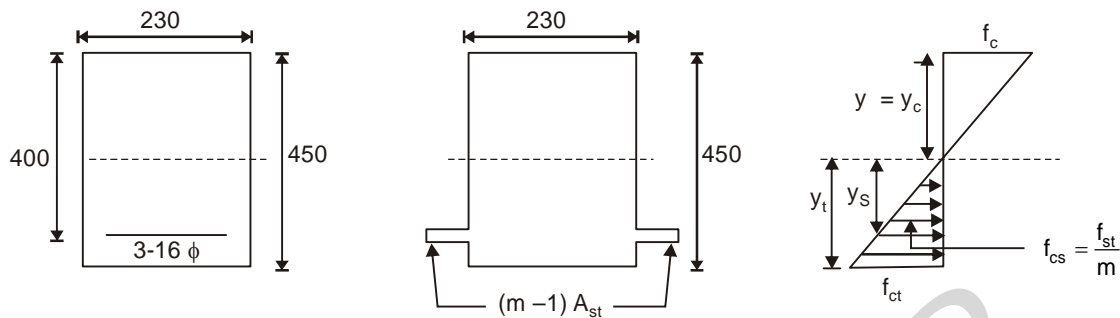
$$\text{(v) relaxation loss } = 290.7 \times \frac{4}{100} = 11.628 \text{ kN}$$

$$\begin{aligned} \text{Total Loss} &= 40.70 + 63 \times 500 \times 10^{-3} + 42 \times 500 \times 10^{-3} \\ &\quad + 21.38 \times 500 \times 10^{-3} + 11.629 \\ &= 115.52 \text{ kN} \end{aligned}$$

(b) Jacking force required at left end P'

$$\begin{aligned} P' &= P_0 + \text{Loss due to friction} \\ &= 250 + 40.7 \\ &= 290.70 \text{ kN} \end{aligned}$$

Sol. 2(a)



For M20 concrete

$$\text{Modular ratio } m = 13.33 \left(\frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33 \right)$$

$$f_{cr} = 0.7\sqrt{20} = 3.13 \text{ MPa}$$

$$\text{Area of tension steel} = 3 \times \frac{\pi}{4} \times 16^2 = 603.18 \text{ mm}^2$$

Depth of neutral axis

$$\bar{y} = \frac{230 \times 450 \times 225 + (13.33 - 1) \times 603.18 \times 400}{230 \times 45.0 + 12.33 \times 603.18}$$

$$\bar{y} = 236.73 \text{ mm}$$

$$y_c = 236.73 \text{ mm}$$

$$y_t = 213.27 \text{ mm}$$

$$\begin{aligned} \text{Distance of reinforcing steel from N.A} &= 213.27 - 50 \\ &= 163.27 \text{ mm} \end{aligned}$$

MOI of transformed section

$$I_T = \frac{by_c^3}{3} + \frac{by_t^3}{3} + (m-1) A_{st} (y_s)^2$$

$$= 1.959 \times 10^9 \text{ mm}^4$$

$$\text{Cracking moment } M_{cr} = f_{cr} \frac{I_T}{y_t}$$

$$= 3.13 \times \frac{1.959 \times 10^9}{213.27}$$

$$= 28.75 \text{ kNm}$$

$$M = 28.75 \text{ kN-m}$$

$$M = 20 \text{ kN-m}$$

(Here $M_{\text{applied}} < M_{\text{cracking}}$)

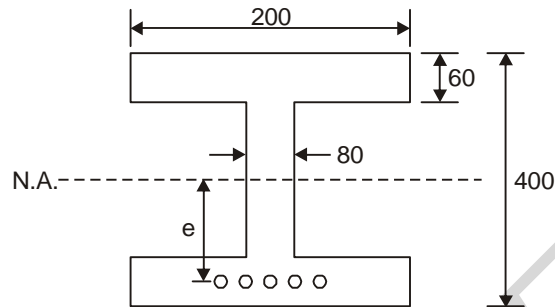
$$f_c = \frac{M y_c}{I_T} = 20 \times 10^6 \times \frac{236.73}{1.959 \times 10^9}$$

$$= 2.417 \text{ N/mm}^2$$

$$f_{ct} = M \frac{y_t}{I_t} = 20 \times 10^6 \times \frac{213.27}{1.959 \times 10^9}$$

$$= 2.177 \text{ N/mm}^2$$

Sol. 2(b)



$$\text{Area of the I-section} = 200 \times 400 - 280 \times 120$$

$$= 46400 \text{ mm}^2$$

$$\text{Density of concrete} = 24 \text{ kN/m}^3$$

$$\text{Dead load} = 24 \times 46400 \times 10^{-6} \text{ kN/m}$$

$$= 1.11 \text{ kN/m}$$

$$\text{Live Load} = 4 \text{ kN/m}$$

$$\text{Total Load} = 5.11 \text{ kN/m}$$

Moment due to dead load and live load at center of span

$$M = \frac{(W_d + W_l) l^2}{8}$$

$$= \frac{5.11 \times 8^2}{8}$$

$$= 40.88 \text{ kN-m}$$

$$\text{Prestressing Force, } P = 235 \text{ kN}$$

$$\text{Resultant stress at soffit, } \sigma = \frac{P}{A} + \frac{Pe}{z} - \frac{M}{z}$$

$$Z = \frac{I}{Y}$$

$$I = \frac{200 \times 400^3}{12} - \frac{120 \times 280^3}{12}$$

$$= 8.47 \times 10^8 \text{ mm}^4$$

$$Y = 200 \text{ mm at soffit}$$

$$z = 4.235 \times 10^6 \text{ mm}^3$$

$$\sigma = \frac{235 \times 10^3}{46400} + \frac{235 \times 10^3 e}{4.235 \times 10^6} - \frac{40.88 \times 10^3 \times 10^3}{4.234 \times 10^6} = 0$$

$$e = 82.7 \text{ mm}$$

Sol. 2(c) Column load = 600 kN
 Assume footing load = 10% of column load
 = 60 kN
 Total load = 600 + 60
 = 660 kN

$$\begin{aligned} \text{Plan area required} &= \frac{\text{Total load}}{\text{Safe bearing capacity}} \\ &= \frac{660 \times 10^3}{120 \times 10^3} \text{ m}^2 \\ &= 5.5 \text{ m}^2 \end{aligned}$$

$$\text{Let the ratio of } \frac{L}{B} = \frac{3}{2}$$

$$\frac{2}{3}L \times L = 5.5$$

$$L = 2.9 \text{ m}$$

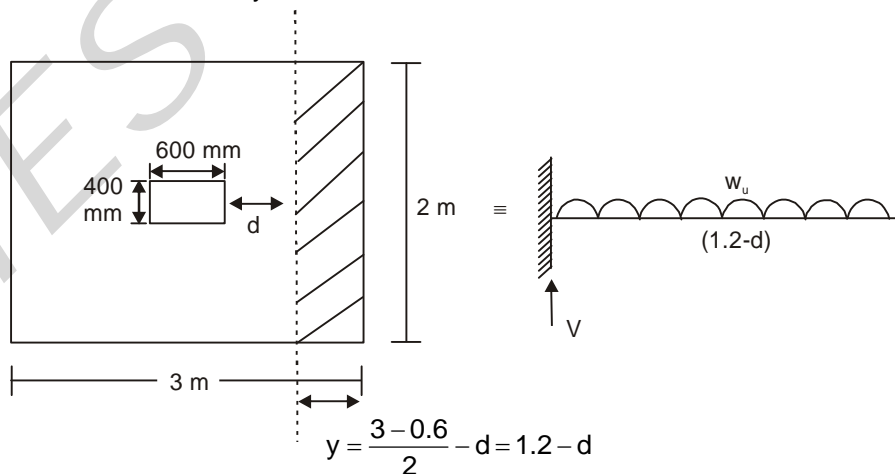
$$B = \frac{2}{3} \times 2.9 = 1.93$$

Provide a rectangular footing of 2 m × 3 m

$$\text{Net upward pressure, } w = \frac{600}{2 \times 3} = 100 \text{ kN/m}^2$$

$$\begin{aligned} \text{Factored net upward pressure } w_u &= 1.5 \times 100 \text{ kN/m}^2 \\ &= 150 \text{ kN/m}^2 \end{aligned}$$

(i) Depth calculation for one way shear



Critical section for one way shear is at a distance 'd' from the face of column.

$$V_u = w_u(1.2 - d) \times 2$$

$$\tau_v = \frac{w_u(1.2-d) \times 2}{2 \times d}$$

Assume 0.3% steel

$$\tau_c = 0.384 \text{ N/mm}^2$$

$$k_s = 1$$

$$\tau_v < K\tau_c$$

$$\frac{w_u(1.2-d)}{d} < 0.384 \text{ N/mm}^2$$

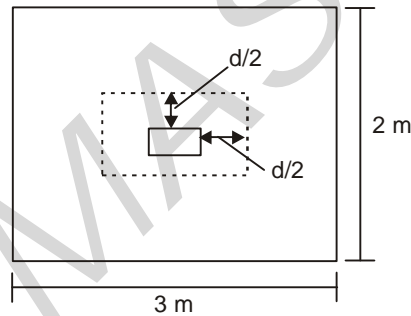
$$150 \times 10^3 \times \frac{(1.2-d)}{d} < 0.384 \times 10^6$$

$$1.2-d < 2.56 d$$

$$3.56 d > 1.2$$

$$d > 0.337 \text{ m}$$

(ii) For two way shear



Critical section for two-way shear is at a periphery at a distance ' $\frac{d}{2}$ ', from face of column.

$$\text{Punching force } \tau_p = F_p$$

$$F_p = w_u[2 \times 3 - (0.6+d)(0.4+d)]$$

$$\text{Punching stress } \tau_p = \frac{F_p}{\text{perimeter} \times d}$$

$$= \frac{150 \times 10^3 [6 - (0.6+d)(0.4+d)]}{2[0.4+d+0.6+d] \times d}$$

$$= \frac{75 \times 10^3 \times [6 - (0.24 + d + d^2)]}{(1+2d)d}$$

$$\text{Permissible stress} = K_s \tau_c$$

$$K_s = \beta + 0.5 \geq 1$$

$$\beta = \frac{0.4}{0.6}$$

$$K_s = 1$$

$$\begin{aligned}\tau_c &= 0.25\sqrt{f_{ck}} \\ &= 0.25\sqrt{20} \\ &= 1.118 \text{ N/mm}^2\end{aligned}$$

$$\tau_p < K_s \tau_c$$

$$\frac{75 \times 10^3 \times (5.76 - d - d^2)}{(1 + 2d)d} < 1.118 \times 10^6$$

$$5.76 - d - d^2 < 14.9(d + 2d^2)$$

$$30.8d^2 + 15.9d - 5.76 > 0$$

$$d > 0.24 \text{ m}$$

Assume clear cover of 50 mm & 12 mm bar

$$\begin{aligned}D &= 337 + 50 + \frac{12}{2} \\ &= 393 \text{ mm}\end{aligned}$$

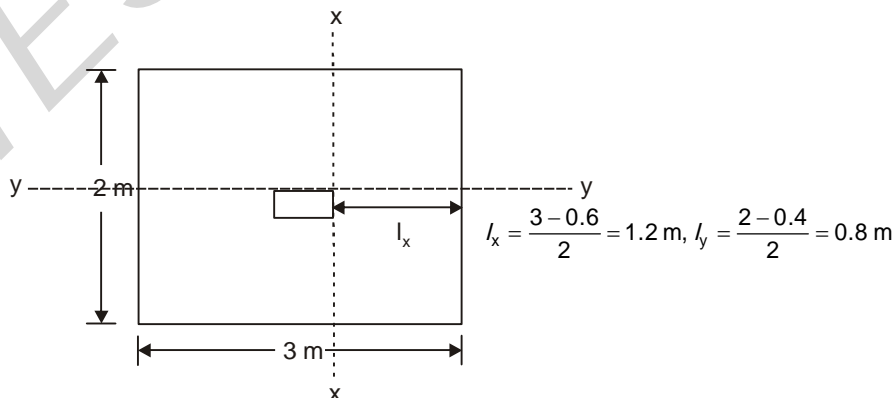
Take

$$D = 425 \text{ mm}$$

$$d = 425 - 50 - \frac{12}{2}$$

$$= 369 \text{ mm}$$

B.M. calculation



BM calculation

For long direction

$$M_{ux} = \frac{150 \times (1.2)^2}{2} = 108 \text{ kN-m}$$

For short direction

$$M_{uy} = \frac{150 \times 0.8^2}{2} = 48 \text{ kN-m}$$

∴ for under reinforced section

$$\begin{aligned} M_{ux} &< M_{u,lim} \\ 108 \times 10^6 &< 0.36 f_{ck} x_{u,lim} b (d - 0.42 x_{u,lim}) \\ 108 \times 10^6 &< 0.36 \times 20 \times 2000 \times 0.48 d (d - 0.42 \times 0.48 d) \\ d &> 139.89 \text{ mm, Hence OK} \end{aligned}$$

Area of steel

For long direction, effective depth, $d_x = 369 \text{ mm}$

$$\begin{aligned} M_{ux} &= 0.87 f_y A_{stx} \left[d_x - \frac{A_{stx} f_y}{f_{ck} b} \right] \\ 108 \times 10^6 &= 0.87 \times 415 A_{stx} \left[369 - \frac{A_{stx} \times 415}{20 \times 2000} \right] \end{aligned}$$

$$A_{stx} = 830.01 \text{ mm}^2$$

Provide 12 mm bar

$$\text{No. of bar} = \frac{830}{\frac{\pi}{4} \times 12^2} = 7.33 \approx 8$$

$$\% \text{ of steel} = \frac{8 \times \frac{\pi}{4} \times 12^2}{2000 \times 369} = 0.12\% < 0.3\%$$

So, Provide % of steel = 0.3%

$$A_{stx} = \frac{0.3}{100} \times 2000 \times 369 = 2214 \text{ mm}^2$$

Provide 20 bar of 12 mm dia.

Development length required: $L_d = 47\phi$

$$= 47 \times 12 = 564 \text{ mm}$$

Development length available = $1200 - 50 = 1150 \text{ mm} > 564 \text{ mm OK}$

For short direction, effective depth $d_y = 369 - 12$

$$= 357 \text{ mm}$$

$$M_{uy} = 0.87 f_y A_{sty} \left[d_y - \frac{A_{sty} f_y}{b f_{ck}} \right]$$

$$48 \times 10^6 = 0.87 \times 415 A_{sty} \left[357 - \frac{A_{sty} \times 415}{3000 \times 20} \right]$$

$$A_{sty} = 375.12 \text{ mm}^2$$

$$p_t = \frac{375.12 \times 100}{3000 \times 357} = 0.035\% < 0.3\%$$

∴ Provide % of steel = 0.3%

$$A_{sty} = \frac{0.3}{100} \times 3000 \times 357 = 3213 \text{ mm}^2$$

Provide 29 bar of 12 mm dia.

A_{st} to be provided within a control band, $B = 2000 \text{ mm}$ is

$$3213 \times \frac{2}{\beta+1} = 3213 \times \frac{2}{\frac{3}{2}+1} = 2570.4 \text{ mm}^2$$

Using 12ϕ , number required = 23 bar

Provide 23 nos. 12ϕ bars at uniform spacing within the central band of width 2000 mm and 3 nos.

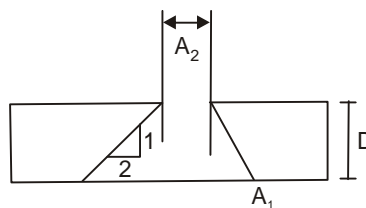
12ϕ bars each in the two outer segments.

The spacings are within limits ($3d$, 300 mm)

required development length = $47 \times 12 = 564 \text{ mm}$

development length available = $800 - 50 = 750 \text{ mm} > 564 \text{ mm}$ OK

Transfer of load,



$$\sigma_{br} = 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$A_1 = 2 \times 3 = 6 \text{ m}^2$$

$$A_2 = 0.6 \times 0.4 = 0.24$$

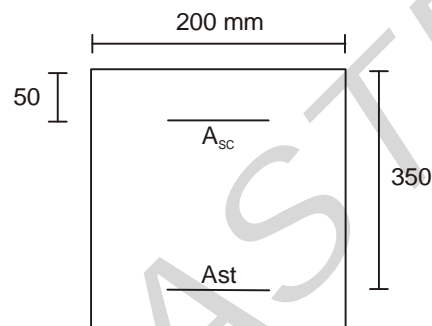
$$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{6}{0.24}} = 5 > 2.0$$

$$\therefore \sqrt{\frac{A_1}{A_2}} = 2$$

$$\sigma_{br} = 0.45 \times 20 \times 2 = 18 \text{ N/mm}^2$$

$$\begin{aligned} \text{Actual bearing stress} &= \frac{1.5 \times 600 \times 10^3}{600 \times 400} \\ &= 3.75 \text{ N/mm}^2 < \sigma_{br} \end{aligned}$$

Sol. 3 (a)



To find the N.A.

$$\text{Total compression} = \text{Total tension}$$

$$0.36f_{ck}bx_u + (f_{sc} - 0.45f_{ck})A_{sc} = 0.87f_yA_{st}$$

Let's take $f_{sc} = 0.87f_y$ as grade is Fe250

is more than 0.00109 (yield strain for mild steel)

$$0.36 \times 20 \times 200 \times x_u + (0.87 \times 250 - 0.45 \times 20) \times 1245 = 0.87 \times 250 \times 1600$$

$$x_u = 61.40 \text{ mm}$$

$$x_{u,lim} = 0.53 d$$

$$= 0.53 \times 350$$

$$= 185.5 \text{ mm}$$

$$x_u < x_{u,lim} \text{ under reinforced}$$

Limiting moment of resistance

$$M_u = 0.36f_{ck}bx_u(d - 0.42x_u) + (f_{sc} - 0.45f_{ck})A_{sc}(d - d')$$

$$= 0.36 \times 20 \times 200 \times 61.40 \times (350 - 0.42 \times 61.40)$$

$$+ (0.87 \times 250 - 0.45 \times 20) \times 1245 \times (350 - 50)$$

$$M_u = 106.54 \text{ kNm}$$

Sol. 3(b) Mean of the results,

$$f_m = \frac{37.8 + 42.5 + 39.8 + 40.7}{4}$$

$$= 40.2 \text{ N/mm}^2$$

For M30 to M60 concrete,

$$\text{Standard deviation, } S = 5 \text{ N/mm}^2$$

From acceptable criteria,

$$f_m \geq f_{ck} + 0.825 S \text{ or } f_{ck} + 4$$

$$\geq f_{ck} + 0.825 \times 5 \text{ or } f_{ck} + 4$$

$$f_m \geq f_{ck} + 4.125$$

$$f_{ck} \leq f_m - 4.125$$

$$\leq 40.2 - 4.125$$

$$\leq 36.075$$

ie., $f_{ck} = 36.075$

i.e., $f_{ck} = 36 \text{ N/mm}^2$

for individual results

$$f_{\min} \geq f_{ck} - 4$$

$$\geq 36 - 4$$

$$\geq 32$$

$$f_{\min} = 37.8 > 32 \text{ N/mm}^2$$

The results are complied for $f_{ck} = 36 \text{ N/mm}^2$

Sol. 3(c)

(i) Equilibrium of Torsion

It is torsion introduced as a result of maintaining equilibrium of structure. This type of torsion occurs mainly in statically determinate structures. It is determined only by equation of equilibrium and is independent of torsional or flexural stiffness of members.

e.g. A cantilever or a beam carrying an eccentric out of plane, transverse load or a beam curved in plane under vertical load and a helical stair.

Compatibility Torsion :

It is the torsion induced in a member due to compatibility of rotations at the joint of interconnected members.

e.g. A spandrel beam connected to a cross-beam, arch, slab or a shell; interconnected bridge girders; horizontal grids.

As per IS 456 : 2000

Equivalent shear,
$$V_e = V_a + \frac{1.6 \times T_u}{b}$$

where V_a = shear, T_u = torsional moment, b = width of beam

and Equivalent moment,
$$M_e = M_a + T_u \left(\frac{1 + \frac{D}{b}}{1.7} \right)$$

- (ii) Equilibrium torsion has to be transferred to supports by torsional resistance of members and cannot be ignored. It's neglectation will result in the vibration of equilibrium leading to disastrous collapse.

Consideration of compatibility type of torsion can be optional in design. Neglecting the torsional stiffness of spandrel beam, if it exists could only underestimate the actual strength of the structure. At the same time, it will lead to simplicity of analysis.

Torsional reinforcement is not calculated separately that required for bending and shear. Instead of that total longitudinal reinforcement is determined for a factious bending moment which is a function of actual bending moment and torsion. Similarly web reinforcement is determined for a fictitious shear which is a function of actual shear and torsion.

- (iii) In the reinforcement concrete beams to resist the torsion, two types of reinforcements are provided.

(a) Longitudinal Reinforcement :

Longitudinal reinforcement is provided to resist the equivalent bending moment M_e , given by,

$$M_e = M + M_t$$

where
$$M_t = T_u \left(\frac{1 + \frac{D}{b}}{1.7} \right)$$

And if $M_t > M_u$, we need to provide tensile reinforcement on the flexural compression side

$$M_t - M_u = 0.87 f_y A_{st1} (d - d')$$

(b) Transverse Reinforcement :

Transverse reinforcement is provided in the beams to resist the equivalent shear given by,

$$V_e = V_a + 1.6 T_u / b$$

where T_u is the torsional moment

Sol. 3. (d)

Analysis of end section :

$$\text{Stress at top} = \frac{P}{A} - \frac{Pe}{z}$$

$$\text{Stress at bottom} = \frac{P}{A} + \frac{Pe}{z}$$

$$Z = \frac{bd^2}{6} = \frac{400 \times 600^2}{6} = 24 \times 10^6 \text{ mm}^3$$

$$\frac{P}{A} = \frac{1000 \times 10^3}{400 \times 600} = 4.16 \text{ N/mm}^2$$

$$\frac{Pe}{z} = \frac{1000 \times 10^3 \times 50}{24 \times 10^6} = 2.08 \text{ N/mm}^2$$

$$\text{Stress at top} = 4.16 - 2.08 = 2.08 \text{ N/mm}^2$$

$$\text{Stress at bottom} = 4.16 + 2.08 = 6.24 \text{ N/mm}^2$$

Analysis of Mid Section :

$$\text{External downward load} = 160 \text{ kN}$$

$$\text{Upward point load provided by cable} = 2P \sin \theta$$

$$\text{Since } \theta \text{ is small, } \sin \theta = \tan \theta = \frac{50}{3000} = \frac{1}{60}$$

$$\text{Upward point load} = 2 \times 1000 \times \frac{1}{60} = 33.33 \text{ kN}$$

$$\begin{aligned} \text{Net downward load} &= 160 - 33.33 \text{ kN} \\ &= 126.67 \text{ kN} \end{aligned}$$

B.M. due to Net downwards load at mid-section = M

$$\begin{aligned} M &= \frac{P_{\text{net}} L}{4} = 126.67 \times \frac{6}{4} \\ &= 190 \text{ kN-m} \end{aligned}$$

$$\text{B.M due to dead load } M_d = \frac{W l^2}{8} = \frac{6 \times 6^2}{8} = 27 \text{ kNm} \quad (W = 0.6 \times 0.4 \times 1 \times 25 = 6 \text{ kN/m})$$

$$\text{Stress at top} = \frac{P}{A} - \frac{Pe}{z} + \frac{M + M_d}{z}$$

$$\text{Stress at bottom} = \frac{P}{A} + \frac{Pe}{z} - \frac{M + M_d}{z}$$

$$\frac{P}{A} = \frac{1000 \times 10^3}{400 \times 600} = 4.17 \text{ N/mm}^2$$

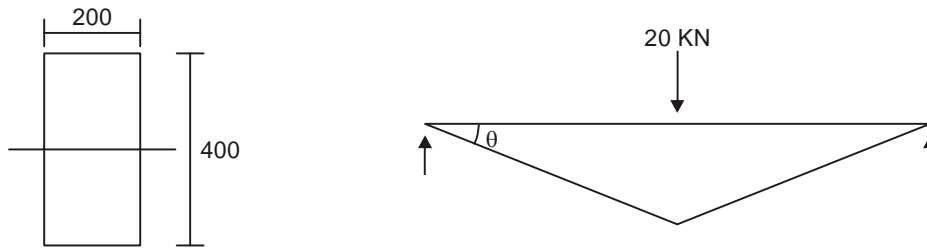
$$\frac{Pe}{z} = \frac{1000 \times 10^3 \times 50}{24 \times 10^6} = 2.08 \text{ N/mm}^2$$

$$\frac{M + M_d}{z} = \frac{(190 + 27) \times 10^6}{24 \times 10^6} = 9.04 \text{ N/mm}^2$$

$$\begin{aligned} \text{Stress at top} &= 4.16 - 2.08 + 9.04 \\ &= 11.12 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{Stress at bottom} &= 4.16 + 2.08 - 9.04 \\ &= -2.8 \text{ MPa} \end{aligned}$$

Sol. 4(a)



$$I = \frac{bd^3}{12} = \frac{200 \times (400)^3}{12} = 10.67 \times 10^8 \text{ mm}^4$$

Initial force in tendons, $P = 1000 \times 120 \text{ N} = 120 \text{ kN}$

Rotation due to prestress, $\theta_1 = \frac{PeL}{2E_c I}$

$$\begin{aligned} \theta_1 &= \frac{120 \times 60 \times 6000}{2 \times 30 \times 10.67 \times 10^8} \\ &= 6.74 \times 10^{-4} \text{ radians} \end{aligned}$$

Rotation due to load, $\theta_2 = \frac{QL^2}{16E_c I}$

$$\begin{aligned} &= \frac{20 \times (6000)^2}{16 \times 30 \times 10.67 \times 10^8} \\ &= 14.05 \times 10^{-4} \text{ radians} \end{aligned}$$

\therefore Net radiation, $\theta = 7.31 \times 10^{-4} \text{ radians}$

Net Elongation of Tendon = $2e\theta$

$$\begin{aligned} &= 2 \times 60 \times 7.31 \times 10^{-4} \\ &= 0.08772 \text{ mm} \end{aligned}$$

Increase in stress in tendon = $\frac{0.08772}{6000} \times E_s$

$$\begin{aligned} &= \frac{0.08772}{6000} \times 210 \times 10^3 \text{ N/mm}^2 \\ &= 3.07 \text{ N/mm}^2 \end{aligned}$$

% increase in stress = $\frac{3.07}{1000} \times 100$

$$= 0.307 \%$$

Sol. 4 (b)

$$\begin{aligned} \text{Factored load } w_u &= 1.5w \\ &= 1.5 \times 74 \\ &= 111 \text{ KN/m} \end{aligned}$$

$$\text{Max. shear } V_{u, \max} = \frac{w_u L}{2} = \frac{111 \times 6}{2} = 333 \text{ KN}$$

Critical section for shear occurs at a distance of 'd' from the face of support i.e. a distance of $\frac{400}{2} + 600 = 800 \text{ mm}$ from the center of supports.

$$\begin{aligned} \therefore \text{Design shear force, } V_{uD} &= 333 - 111 \times 0.8 \\ &= 244.2 \text{ KN} \end{aligned}$$

$$\begin{aligned} \text{Nominal shear stress } \tau_v &= \frac{V_{uD}}{bd} \\ &= \frac{244.2 \times 1000}{300 \times 600} = 1.36 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Steel at support} &= (5 - 2) \times \frac{\pi}{4} \times (25)^2 \\ &= 1472.6 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Percentage of steel at support, } P_t &= \frac{1472.6}{300 \times 600} \times 1000 \\ &= 0.818\% \end{aligned}$$

$$\tau_c \text{ for } 0.818\% \text{ steel and M20 concrete} = 0.556 \text{ N/mm}^2$$

$$\tau_{c, \max} = 2.8 \text{ N/mm}^2 \text{ for M20 concrete}$$

Shear resistance to be provided for V_{us1}

$$\begin{aligned} V_{us1} &= V_u - \tau_c bd \\ &= 244.2 - 0.556 \times \frac{300 \times 600}{1000} \\ &= 144.12 \text{ kN} \end{aligned}$$

Shear resistance provided by bend up bar V_{us2}

$$\begin{aligned} V_{us2} &= 0.87 f_y A_{sv} \sin \alpha \\ &= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times (25)^2 \times \sin 45^\circ \\ &= 250.63 \text{ KN} \end{aligned}$$

Max. shear resistance to be assigned to the bent-up bar is half of shear resistance provided by shear reinforcement i.e. $0.5 \times 144.12 = 72.06 \text{ kN}$.

Resistance should provided by vertical stirrups

$$V_{us3} = V_{us1} - V_{us2}$$

$$= 72.06 \text{ kN}$$

Spacing of 2-legged 8 mm of stirrups S_v

$$S_v = \frac{0.87f_y A_{sv} d}{V_{us3}}$$

$$= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times (8)^2 \times 600}{72.06 \times 1000}$$

$$= 302.22 \text{ mm}$$

Spacing corresponding to minimum reinforcement $S_{v, \max}$

$$S_{v, \max} = \frac{0.87f_y A_{sv}}{0.4b}$$

$$= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times (8)^2}{0.4 \times 300}$$

$$= 302.5 \text{ mm}$$

Spacing of the stirrups should be least of the following :

- (i) 302.2 mm
- (ii) 302.5 mm
- (iii) 300 mm
- (iv) $0.75d = 0.75 \times 600 = 450 \text{ mm}$

Provide 2-legged 8 mm dia. stirrups @ 300 mm c/c

Sol. 4(c) Given,

$$b = 350 \text{ mm}$$

$$D = 750 \text{ mm}$$

$$f_{ck} = 25 \text{ MPa}$$

$$f_y = 415 \text{ MPa}$$

$$T_u = 140 \text{ kN-m}$$

$$M_u = 200 \text{ kN-m}$$

$$V_u = 110 \text{ kN}$$

Minimum required cover to stirrups is 20 mm. Assuming 50 mm effective cover all around

$$d = 700 \text{ mm}$$

Design of longitudinal reinforcement

⇒ Effective bending moment due to torsion

$$M_t = \frac{T_u}{1.7} \left(1 + \frac{D}{b} \right) = \frac{140}{1.7} \left(1 + \frac{750}{350} \right) = 258.82 \text{ kN-m}$$

⇒ Equivalent bending moments for design

$$M_e = M_t + M_u = 259 + 200 = 459 \text{ kN-m}$$

$$M_t > M_u$$

⇒ Provide tensile reinforcement in the flexural compression side.

Area of steel

$$MR_{bal} = Qbd^2$$

$$Q = 0.138f_{ck}$$

$$= 0.138 \times 25 = 3.45$$

$$\Rightarrow MR_{bal} = 3.45 \times 350 \times 700^2 \times 10^{-6} \text{ kN-m} = 591.675 \text{ kN-m}$$

$$M_e < MR_{bal}$$

⇒ We need singly reinforced section

$$A_{st} = \frac{M_e}{0.87f_y(d - 0.42x_u)}$$

$$C = T$$

$$0.36f_{ck}bx_u = 0.87f_yA_{st}$$

$$x_u = \frac{0.87 \times 415 \times A_{st}}{0.36 \times 25 \times 350} = 0.1146A_{st} \quad \dots (1)$$

$$A_{st} = \frac{459 \times 10^6}{0.87 \times 415 \times (700 - 0.42 \times 0.1146A_{st})}$$

$$A_{st} = 2127.29 \text{ mm}^2$$

$$\text{No. of 25 mm } \phi \text{ bars} = \frac{2127.29}{0.785 \times 25^2} \approx 5$$

Use 5 nos. of 25 mm bars

$$M_T > M_u$$

⇒ In this case, compression reinforcement is calculated for a moment M_{ez}

$$M_{ez} = M_T - M_u = 259 - 200 = 59 \text{ kN-m}$$

$$A'_{st} = \frac{M_{ez}}{0.87f_y(d - d')} = \frac{59 \times 10^6}{0.87 \times 415 \times (700 - d')}$$

$$A'_{st} = 251.40 \text{ mm}^2$$

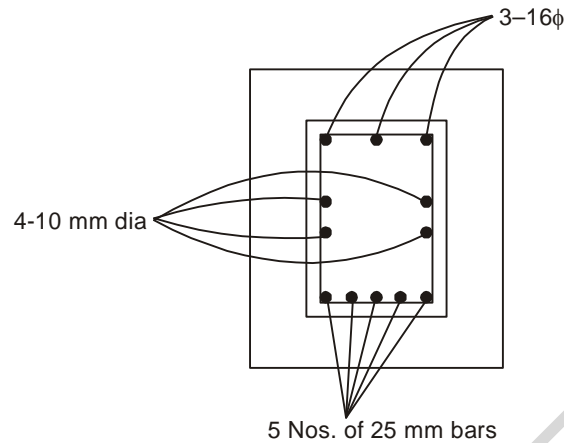
Provide, 2 – 16 ϕ at top

Side face reinforcement

$$A_{st} = \frac{0.1}{100} \times 350 \times 750 = 262.5 \text{ mm}^2$$

$$\text{No. of 10 mm } \phi \text{ bars} = \frac{262.5}{\frac{\pi}{4} \times 10^2} \approx 4 \text{ bars}$$

⇒ Provide 2 nos. of bars on each side. The spacing between longitudinal bars will be less than 300 mm



Equivalent shear

$$V_c = V_u + \frac{1.6T_u}{b} = 110 + 1.6 \times \frac{140}{0.35} = 750 \text{ kN}$$

Nominal shear stress

$$\tau_v = \frac{V_e}{bd} = \frac{750 \times 10^3}{350 \times 700} = 3.06 \text{ N/mm}^2 < \tau_{c1 \text{ max}}$$

for $p_t = 1\%$

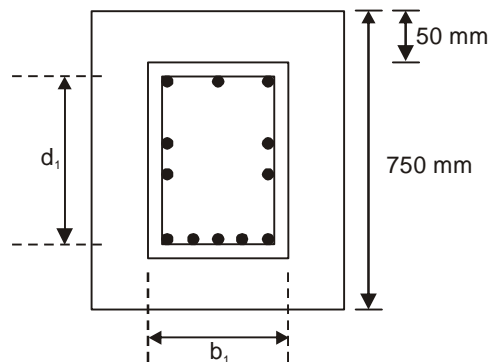
$$\tau_c = 0.64 \text{ N/mm}^2$$

Transverse reinforcement (shear reinforcement)

$$S_v = \frac{A_{sv}(0.87 f_y) d_1}{\frac{T}{b_1} + \frac{V_u}{2.5}}$$

Let us take 2-legged 10 mm stirrups 50 mm effective cover

$$A_{sv} = 2 \times \frac{\pi}{4} \times 10^2 = 157 \text{ mm}^2$$



$$b_1 = 350 - 50 - 50 = 250 \text{ mm}$$

$$d_1 = 750 - 50 - 50 = 650 \text{ mm}$$

$$S_v = \frac{157 \times 0.87 \times 415 \times 650}{\frac{140 \times 10^6}{250} + \frac{110 \times 10^3}{2.5}} = 61.0 \text{ mm}$$

⇒ Provide transverse reinforcement of 2 legged 12 mm dia

$$S_v = 61 \times \frac{226}{157} = 87.8 \text{ mm}$$

$$S_v = \frac{0.87 f_y A_{sv}}{(\tau_{ve} - \tau_c) b}$$

$$= \frac{0.87 \times 415 \times 226}{(3.06 - 0.64) \times 350}$$

$$= 96.34 \text{ mm}$$

Check minimum spacing

$$(1) \quad x_1 = b_1 + 25 + 10 = 250 + 25 + 10 = 285 \text{ mm}$$

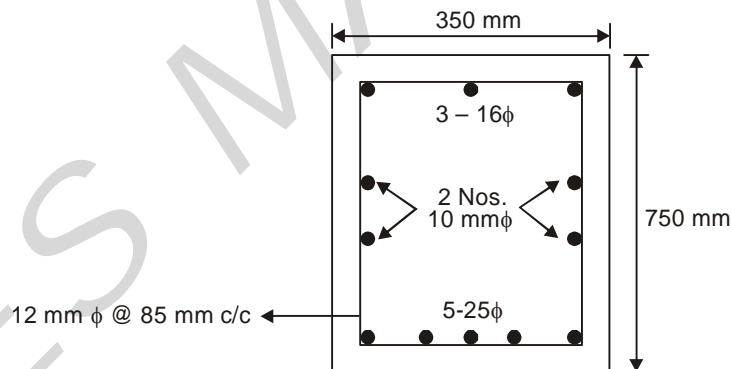
$$(2) \quad \frac{x_1 + y_1}{4}, \quad y_1 = d_1 + \frac{16}{2} + \frac{25}{2} + 12 = 650 + 8 + 12.5 + 12 = 682.5 \text{ mm}$$

$$\frac{x_1 + y_1}{4} = 241.875 \text{ mm}$$

$$(3) \quad 300 \text{ mm}$$

Provide 12 mm 2 legged stirrups at 85 mm c/c

Detailing



Sol. 5(a) As per verticle deflection criterion for SS slab.

$$\frac{\text{Span}}{d} = 20$$

$$d = \frac{3000}{20} = 150 \text{ mm}$$

(Assuming $k_1 = 1.0$)

Effective span in shorter direction. $l_{\text{eff } x}$ is lesser of

$$(i) \quad l_{0x} + d = 3000 + 150 = 3150 \text{ mm}$$

$$(ii) \quad l_{0x} + w = 3000 + 300 = 3300 \text{ mm}$$

$$l_x = 3150 \text{ mm}$$

$$l_{\text{eff } y} \text{ is lesser of (i) } l_{0y} + d = 7000 + 150 = 7150 \text{ mm}$$

$$\text{(ii) } l_{0y} + w = 7000 + 300 = 7300 \text{ mm}$$

$$\frac{l_y}{l_x} = \frac{7150}{3150} = 2.26 > 2, \text{ Design as a one way slab.}$$

Assume clear cover of 20 mm and 10 mm bar

$$D = 150 + 20 + 5 = 175 \text{ mm}$$

$$\text{Dead load of slab} = 25 \times 175 = 4.375 \text{ kN/m}$$

$$\text{live load} = 2 \text{ kN/m}^2$$

$$\text{Weight of lime} = 20 \times 0.075 \times 1 = 1.5 \text{ kN/m}$$

$$\text{Total load} = 7.88 \text{ kN/m}$$

$$W_u = 1.5 \times 7.88 = 11.82 \text{ kN/m}$$

$$M_u = \frac{W_u l^2}{8} = 11.82 \times \frac{(3.15)^2}{8} = 14.66 \text{ kNm}$$

$$V_u = W_u \frac{l}{2} = 11.82 \times \frac{3.15}{2} = 18.61 \text{ kN}$$

Check for d (for under reinforced section)

$$M_u \leq 0.36 f_{ck} x_{u, \text{lim}} b (d - 0.42 x_{u, \text{lim}})$$

$$14.66 \times 10^6 \leq 0.36 \times 20 \times 0.48 d \times 1000 (d - 0.42 \times 0.48 d)$$

$$d \geq 72.89 \text{ mm}$$

∴ Hence, OK

Area of steel, A_{st}

$$M_u = 0.87 f_y A_{st} \left[d - \left(\frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b} \right) \right]$$

$$14.66 \times 10^6 = 0.87 \times 415 \times A_{st} \left[150 - \frac{0.42 \times 0.87 \times 415 A_{st}}{0.36 \times 20 \times 1000} \right]$$

$$14.66 \times 10^6 - 54157.5 A_{st} + 7.6 A_{st}^2 = 0$$

$$A_{st} = 282 \text{ mm}^2$$

$$\begin{aligned} \text{Spacing of 8 mm } \phi \text{ bar} &= \frac{1000 \times \frac{\pi}{4} \times (8)^2}{282} \\ &= 178 \text{ mm} \end{aligned}$$

Max. spacing is

(i) $3d = 3 \times 150 = 450 \text{ mm}$

(ii) 300 mm

Provide $8 \text{ mm } \phi @ 175 \text{ mm c/c}$

$$\begin{aligned} \text{Distribution bar} &= 0.12\% \text{ of BD} \\ &= \frac{0.12}{100} \times 1000 \times 175 \end{aligned}$$

$$= 210 \text{ mm}^2$$

$$\begin{aligned} \text{Spacing of 8 mm } \phi \text{ bar} &= \frac{1000 \times \frac{\pi}{4} \times (8)^2}{210} \\ &= 240 \text{ mm} \end{aligned}$$

Max. spacing is lesser of

(i) $5d = 5 \times 150 = 750 \text{ mm}$

(ii) 300 mm

Provide $8 \text{ mm } \phi @ 225 \text{ mm c/c}$

$$\begin{aligned} \text{Area of main steel provided} &= \frac{1000}{175} \times \frac{\pi}{4} \times (8)^2 \\ &= 288 \text{ mm}^2 \end{aligned}$$

Check for shear,

$$V_u = 18.61 \text{ kN}$$

V_u at distance 'd' from face of support

$$\begin{aligned} V_u \text{ at } d &= (18.61 - 11.82 \times 0.225) \quad (\text{Considering compression confinement}) \\ &= 15.96 \text{ kN} \end{aligned}$$

$$\tau_v = \frac{V_u}{bd} = \frac{15.96 \times 1000}{1000 \times 150}$$

$$= 0.11 \text{ N/mm}^2$$

$$\tau_v \neq 0.5 \tau_{c \text{ max}}$$

$$\tau_{c \text{ max}} \text{ for M20} = 2.8 \text{ N/mm}^2$$

$$0.5 \times 2.8 = 1.4 \text{ N/mm}^2$$

$$\tau_v < 0.5 \tau_{c \text{ max}} \quad \text{OK}$$

$$p_t = \frac{A_s}{bd} \times 100$$

50% bar are bent at a distance 0.1 L

$$A_{st} = 50\% \text{ of } A_{st}$$

$$p_t = \frac{288 \times 100}{1000 \times 150} = 0.192\%$$

$$\tau_c = 0.31 \text{ N/mm}^2$$

$$\tau_v < k\tau_c$$

$$\begin{aligned} k &= 1.6 - 0.002 D \\ &= 1.6 - 0.002 \times 175 \\ &= 1.25 \end{aligned}$$

$$k\tau_c = 1.25 \times 0.31 = 0.3875 \text{ MPa}$$

$$\tau_v < k(\tau_c) \text{ ok}$$

Check for development length L_d for safe in Bond

$$l_d < \frac{1.3M_1}{V} + l_0$$

$$l_d = \frac{0.87f_y\phi}{4\tau_{bd}}$$

$$= \frac{0.87 \times 415 \times 8}{4 \times 1.6 \times 1.2} = 376 \text{ mm}$$

Let curtail half the bar at the support

$$M_1 = 0.87f_y \frac{A_{st}}{2} (d - 0.42x_u)$$

$$x_u = \frac{0.87 \times 415 \times \left(\frac{1000}{350} \times \frac{\pi}{4} \times 8^2 \right)}{0.36 \times 20 \times 1000} = 7.2 \text{ mm}$$

$$M_1 = 7.64 \text{ kNm}$$

$$\frac{M_1}{V} = \frac{7.64 \times 10^3}{18.61} = 410.53 \text{ mm}$$

l_0 is maximum of d or 12ϕ

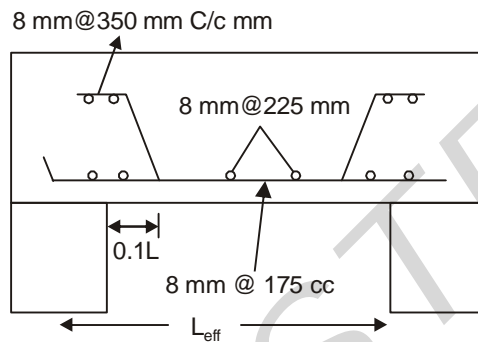
$$12\phi = 96 \text{ mm}$$

$$d = 150 \text{ mm}$$

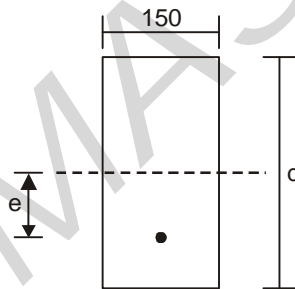
$$l_0 = 150 \text{ mm}$$

$$\frac{1.3M}{V} + l_0 = 1.3 \times (410.53) + 150 = 683.69 \text{ mm}$$

$$l_d < \left(1.3 \frac{M_1}{V} + l_0 \right) \text{ OR safe in bond. Also no need to provide 'l}_0\text{'}$$



Sol. 5(b)



$$M_d = 15 \times 10^6 \text{ N-mm}$$

$$M_L = 40 \times 10^6 \text{ N-mm}$$

At transfer :

$$\text{At top} \quad - \left[\frac{P}{A} - \frac{Pe}{z} + \frac{M_d}{z} \right] \leq 1 \quad \dots(1)$$

$$\text{At bottom} \quad \frac{P}{A} + \frac{Pe}{z} - \frac{M_d}{z} \leq 17 \quad \dots(2)$$

At service :

$$\text{At bottom} : \quad - \left[\frac{\eta P}{A} + \frac{\eta Pe}{z} - \frac{M_d + M_L}{z} \right] \leq 1 \quad \dots(3)$$

$$\text{At top} \quad \frac{\eta P}{A} - \frac{\eta Pe}{z} + \frac{M_d + M_L}{z} \leq 14 \quad \dots(4)$$

From $\eta(1) + (4)$

$$\frac{-\eta M_d + M_d + M_L}{z} \leq \eta + 14$$

$$\frac{[40 + (1 - 0.85) \times 15] \times 10^6}{z} \leq 14.85$$

$$z \geq 2.845 \times 10^6$$

$$z = \frac{bd^2}{6} = \frac{150 \times d^2}{6}$$

$$150 \times \frac{d^2}{6} \geq 2.845 \times 10^6$$

$$d \geq 338 \text{ mm}$$

From $\eta(2) + (3)$

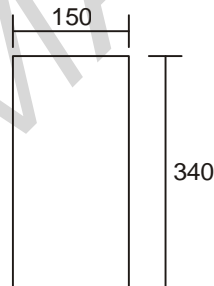
$$\frac{-\eta M_d + M_d + M_L}{z} \leq 17 \times 0.85 + 1$$

$$\frac{42.25 \times 10^6}{z} \leq 15.45$$

$$z \geq 2.73 \times 10^6$$

$$d \geq 330 \text{ mm}$$

Adopt, $d = 340 \text{ mm}$



$$Z = \frac{150 \times 340^2}{6} = 2.89 \times 10^6 \text{ mm}^3$$

To determine 'P'

$\eta(1) + (3)$

$$\frac{-2\eta P}{A} - \frac{\eta M_d}{z} + \frac{M_d + M_L}{z} \leq \eta + 1$$

$$\frac{2\eta P}{A} \geq 12.76$$

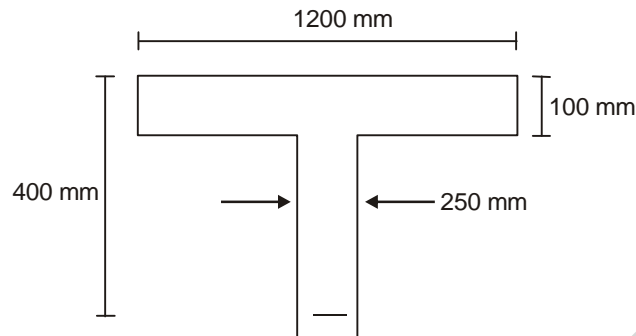
$$P \geq 383 \text{ kN}$$

Adopt, $P = 383 \text{ kN}$

$$e = 103.37 \text{ mm}$$

(using equation '1')

Sol. 5(c)



$$x_u = x_{u,lim} = 0.48 d = 0.48 \times 400 = 192 \text{ mm}$$

$$\frac{7}{3} D_f = 233.33 \text{ mm}$$

$$\Rightarrow D_f < x_{lim} < \frac{7}{3} D_f$$

Hence stress in the flange is not completing rectangular.

Equivalent stress block for uniform stress, y_f

$$\begin{aligned} y_f &= 0.15 x_{u,lim} + 0.65 D_f \\ &= 0.15 \times 192 + 0.65 \times 100 \\ &= 93.8 \text{ mm} \end{aligned}$$

Equating $C = T$

$$0.36 f_{ck} b_w x_{u,lim} + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 \times 192 + 0.45 \times 20 \times 950 \times 93.8 = 0.87 \times 415 \times A_{st}$$

$$A_{st} = 3178 \text{ mm}^2$$

$$\begin{aligned} M_{u,lim} &= 0.36 f_{ck} b_r x_{u,lim} (d - 0.42 x_{u,lim}) + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right) \\ &= 393.55 \text{ kNm} \end{aligned}$$

Sol. 5. (d)

For M 30 grade concrete

$$\sigma_{cbc} = 10 \text{ MPa}, \sigma_{cf} = 1.5 \text{ MPa}$$

$$\Rightarrow m = \frac{280}{3\sigma_{bcc}} = \frac{250}{3 \times 10} = 9.33$$

For Fe415 steel $\sigma_{st} = 130 \text{ MPa}$

$$\text{Area of tension steel} = \frac{T}{\sigma_{st}} = \frac{150 \times 10^3}{130} = 1153.85 \text{ mm}^2/\text{m}$$

For no crack

$$\sigma_{ct} \geq \frac{T}{bD + (m-1)A_{st}}$$

$$\Rightarrow 1.5 \geq \frac{150 \times 10^3}{1000 \times D + (9.33 - 1) \times 1153.85}$$

$$\Rightarrow D \geq 90.39 \text{ mm}$$

$$\text{Adopt } D = 100 \text{ mm}$$

Minimum steel = 0.35% of surface zone

$$= \frac{0.35}{100} \times 1000 \times \frac{100}{2} = 350 \text{ mm}^2$$

Provide 12 mm dia @ 90 mm c/c in one layer only.

$$A_{st} \text{ per meter width} = \frac{1000}{90} \times \frac{\pi}{4} \times (12)^2 = 1255 \text{ mm}^2$$

Provide 8 mm dia @ 125 mm c/c as temperature reinforcement.

$$= \frac{1000}{125} \times \frac{\pi}{4} \times 8^2 = 400 \text{ mm}^2 > 350 \text{ mm}^2 \text{ O.K.}$$

Sol. 6 (a). Given data

$$l_{eff} = 1.2 \text{ m}, T = 300 \text{ mm}, R = 150 \text{ mm}$$

M 20, Fe415.

$$\text{Let us take a thickness} = \frac{l_{eff}}{10} = 120 \text{ mm}$$

For cantilever tread slab, thickness can be reduce at the free end upto 80 mm.

$$\Rightarrow \text{Average thickness} = \frac{120 + 80}{2} = 100 \text{ mm}$$

$$\text{Dead load} = 25 \times 0.1 \times 0.3 = 0.75 \text{ kN/m}$$

$$\text{Finishes} = 0.6 \times 0.3 = 0.18 \text{ kN/m}$$

$$\Rightarrow \text{Factored D.L.} = 1.5 \times (0.75 + 0.18) = 1.40 \text{ kN/m}$$

Live load: For residence L.L. = 3 kN/m² or 1.3 kN at free end.

$$\text{Case - 1: Factored imposed load} = 1.5 \times 3 \times 0.3 = 1.35 \text{ kN/m}$$

$$\text{Case - 2: Factored imposed load} = 1.5 \times 1.3 = 1.95 \text{ kN/m}$$

$$\text{B.M. due to dead load} = \frac{wl^2}{2} = \frac{1.40 \times 1.2^2}{2} = 1.01 \text{ kNm}$$

$$\text{B.M. due to imposed load} = \frac{1.35 \times l^2}{2} = 0.972 \text{ kNm}$$

$$\text{or} \quad = 1.95 \times 1.2 = 2.34 \text{ kNm (governing)}$$

$$\Rightarrow \text{Total B.M.} = 1.01 + 2.34 = 3.35 \text{ kNm}$$

Depth check

$$d \geq \sqrt{\frac{M}{0.138f_{ck}b}} = \sqrt{\frac{3.35 \times 10^6}{0.138 \times 20 \times 300}} = 63 \text{ mm} \ll 120 \text{ mm}$$

So, depth adopted is sufficient.

$$d = 120 - 20 - \frac{10}{2} = 95 \text{ mm}$$

(Assuming a clear cover of 20 mm and main dia as 10 mm)

$$\Rightarrow M = 0.87f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} b} \right]$$

$$\Rightarrow 3.35 \times 10^6 = 0.87 \times 415 \times A_{st} \left[95 - \frac{415 \times A_{st}}{20 \times 300} \right]$$

$$\Rightarrow A_{st} = 105.82 \text{ mm}^2$$

Provide 3-8 ϕ

$$\text{Area provided} = 3 \times 50 = 150 \text{ mm}^2$$

Spacing \succ 3d, 300 mm

Distribution R/F

$$A_{st, \min} = 0.0012 bD$$

$$= 0.0012 \times 1200 \times 120 = 172.8 \text{ mm}^2$$

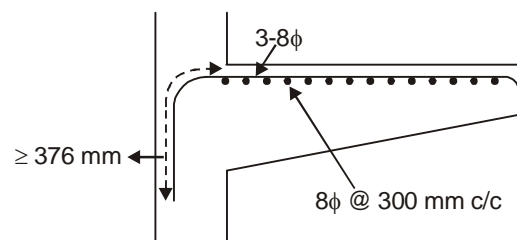
$$\text{No. of 8 mm bars} = \frac{172.8}{50} \cong 4 \text{ nos.}$$

Provide 8 mm dia @ 300 mm c/c.

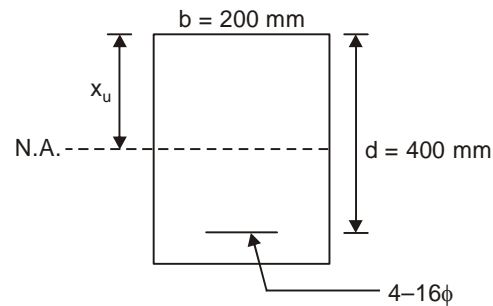
Spacing \succ 5d, 300 mm

Development length $L_d = 47\phi = 376 \text{ mm}$.

Staircase are generally not critical in shear, and hence not checked for shear.



Sol.6. (b)



$$\begin{aligned} \text{Area of steel} &= 4 \times \frac{\pi}{4} \times (16)^2 \\ &= 804.24 \text{ mm}^2 \end{aligned}$$

Let the neutral axis be at x_u from the top fibre.

Applying total compression = Total Tension

$$C = T$$

$$0.36f_{ck}bx_u = 0.87 \times 415 \times 804.24$$

$$x_u = 201.64 \text{ mm}$$

Limiting depth of N.A.

$$\begin{aligned} x_{u,lim} &= 0.48d \\ &= 0.48 \times 400 \\ &= 192 \text{ mm} \end{aligned}$$

As $x_u > x_{u,lim}$ over reinforced section. So take

$$\begin{aligned} x_u &= x_{u,lim} \\ &= 192 \text{ mm} \end{aligned}$$

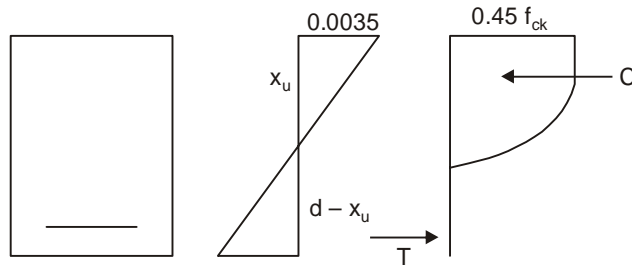
Moment of resistance = $M_{u,lim}$

$$\begin{aligned} M_{u,lim} &= 0.36f_{ck}bx_{u,lim}(d - 0.42x_{u,lim}) \\ &= 0.36 \times 20 \times 200 \times 192 \times (400 - 0.42 \times 192) \end{aligned}$$

$$M_{u,lim} = 88.29 \text{ kNm}$$

Since section is limiting section

$$\begin{aligned} \text{Stress in concrete} &= 0.45 f_{ck} \\ &= 0.45 \times 20 \\ &= 9 \text{ N/mm}^2 \end{aligned}$$



For stress in steel

$$A_{st} f_s = 0.36 f_{ck} b x_u \quad [x_u = x_{u, lim}]$$

$$f_s = \frac{0.36 \times 20 \times 200 \times 192}{804.24}$$

$$= 343.77 \text{ N/mm}^2$$

Sol. 6(c) Design as a short square column

$$\frac{l_{eff}}{b} \leq 12$$

$$\frac{2.75 \times 1000}{b} \leq 12$$

$$b \geq 229.1$$

so take $b = 300 \text{ mm}$

$$e_{min} \leq 0.05 \times 300$$

$$e_{min} \leq 15 \quad \text{Not OK. as } e_{min} = 20 \text{ mm}$$

Take $b = 425 \text{ mm}$

$$e_{min} \leq 0.05 \times 425$$

$$e_{min} \leq 21.25 \text{ mm}$$

e_{min} is more than:

$$(i) \quad \frac{L}{500} + \frac{b}{30} = \frac{2750}{500} + \frac{425}{30} = 19.66$$

$$(ii) \quad 20 \text{ mm}$$

$$e_{min} = 20 < 0.05 b$$

$$\therefore P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1.5 \times 1600 \times 10^3 = 0.4 \times 20 \times (180625 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$A_{sc} = 3536 \text{ mm}^2$$

Provide 8 – 25 ϕ diameter having area

$$8 \times \frac{\pi}{4} \times (25)^2 = 3928 \text{ mm}^2$$

Diameter of lateral ties is max. of

(i) $\frac{\phi_{max}}{4} = \frac{25}{4} = 6.25 \text{ mm}$

(ii) 6 mm

Provide 8 mm ϕ diameter

Spacing of ties, S_v should be minimum of

(i) Least lateral dimension = 425 mm

(ii) $16 \times \phi_{main} = 16 \times 25 = 400 \text{ mm}$

(iii) 300 mm

Provide 8 mm ϕ @ 300 mm c/c

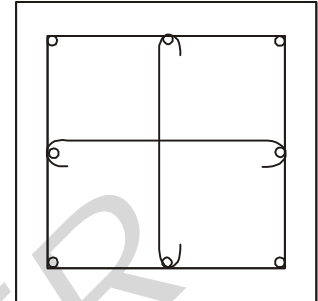
$$\text{c/c distance of bars} = \frac{425 - 2 \times 40 - 2 \times 8 - \frac{2 \times 25}{2}}{2}$$

$$= 152 \text{ mm} < 300 \text{ mm O.K.}$$

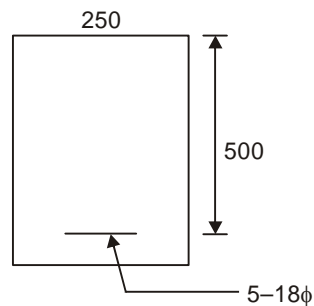
c/c distance is > 75

$$\text{c/c b/w corner bars} = 304 \text{ mm} < 48 \phi_t$$

\Rightarrow open ties can be provided.



Sol. 6(d)



$$\begin{aligned} \text{Nominal shear at supports, } \tau_v &= \frac{V_u}{bd} \\ &= \frac{105 \times 1000}{250 \times 500} \\ &= 0.84 \text{ N/mm}^2 \end{aligned}$$

$$\tau_{c, \max} \text{ for M20 concrete} = 2.8 \text{ N/mm}^2$$

$$\begin{aligned} \text{Percentage of steel, } P_t &= \frac{A_{st}}{bd} \times 100 \\ &= \frac{5 \times \frac{\pi}{4} \times (18)^2}{250 \times 500} \times 100 \\ &= 1.01\% \end{aligned}$$

τ_c for 1.01% steel and M20 concrete is

$$\tau_c = 0.62 \text{ N/mm}^2$$

As $\tau_c < \tau_v < \tau_{\max}$

Therefore, shear reinforcement is required for V_{us}

$$\begin{aligned} V_{us} &= V_u - \tau_c bd \\ &= 105 - 0.62 \times \frac{250 \times 500}{1000} \\ &= 27.5 \text{ kN} \end{aligned}$$

Providing 8mm 2-legged stirrups

$$\begin{aligned} A_{sv} &= 2 \times \frac{\pi}{4} \times (8)^2 \\ &= 100.53 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Spacing } S_v &= \frac{0.87 f_y A_{sv} d}{V_{us}} \\ &= \frac{0.87 \times 250 \times 100.53 \times 500}{27.5 \times 1000} \\ &= 397.55 \text{ mm} \end{aligned}$$

Maximum spacing $S_{v, \max}$ for minimum shear reinforcement criteria

$$\begin{aligned} S_{v, \max} &= \frac{0.87 f_y A_{sv}}{0.4b} \\ &= \frac{0.87 \times 250 \times 100.53}{0.4 \times 250} = 218.6 \text{ mm} \end{aligned}$$

Spacing should not exceed.

- (i) 397.55 mm
- (ii) 218.6 mm
- (iii) 300 mm
- (iv) $0.75d = 0.75 \times 500 = 375$ mm

Provide 8 mm 2-legged stirrups @ 200 mm c/c

Sol.7(a)

$$D = \frac{7}{10} = 0.7 \text{ m}$$

$$D = 700 \text{ mm}$$

Use clear cover 25 mm and 16 mm dia. bar

$$d = 700 - 25 - \frac{16}{2}$$

$$= 667 \text{ mm}$$

$$b = \frac{d}{2} = \frac{667}{2}$$

$$b = 333.5 \text{ mm} \approx 335 \text{ mm}$$

$$\text{Self weight} = 0.335 \times 0.7 \times 25 = 5.86 \text{ kN/m}$$

$$\text{Total weight} = 5.86 + 20 = 25.86 \text{ kN/m}$$

$$\text{Factor load} = 1.5 \times 25.86 = 38.79 \text{ kN/m}$$

$$M_u = \frac{w_u l^2}{8} = \frac{38.79 \times 7^2}{8}$$

$$M_u = 237.59 \text{ kN-m}$$

For under reinforced section

$$M_u < M_{u, \text{lim}}$$

$$237.59 \times 10^6 < 0.36 f_{ck} x_{u, \text{lim}} b (d - 0.42 x_{u, \text{lim}})$$

$$< 0.36 f_{ck} 0.48 d \left(\frac{d}{2}\right) (d - 0.42 \times 0.48 d)$$

$$d > 556.36 \text{ mm}$$

$$D = 556.36 \times 1.05 + 25 + \frac{16}{2}$$

$$D = 617.17 \text{ mm}$$

$$\text{provide, } D = 625 \text{ mm}$$

$$d = 625 - 25 - \frac{16}{2}$$

$$= 592 \text{ mm}$$

$$b = \frac{592}{2} = 296 \text{ mm}$$

Area of steel A_{st}

$$M_u = 0.87 f_y A_{st} \left[d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b} \right]$$

$$237.59 \times 10^6 = 0.87 \times 415 A_{st} \left[592 - \frac{0.42 \times 0.87 \times 415 A_{st}}{0.36 \times 20 \times 296} \right]$$

$$A_{st} = 1321.46 \text{ mm}^2$$

use 7 nos. of 16 mm bar

$$A_{st \text{ provided}} = 1407.43 \text{ mm}^2$$

$$A_{st, \text{ lim}} = \frac{0.36 f_{ck} b \times x_{u, \text{ lim}}}{0.87 f_y}$$

$$= \frac{0.36 \times 20 \times 296 \times 0.48 \times 592}{0.87 \times 415}$$

$$= 1677.33 \text{ mm}^2$$

$$A_{st \text{ provided}} < A_{st \text{ balanced}} \Rightarrow \text{UR section OK}$$

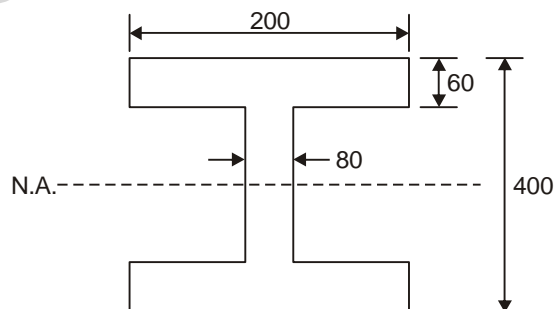
Check for $A_{st \text{ min}}$

$$\frac{0.85}{f_y} = \frac{A_{st \text{ min}}}{bd}$$

$$A_{st \text{ min}} = 358.9 \text{ mm}^2$$

$$A_{st \text{ provided}} > A_{st \text{ min}} \text{ (o.k.)}$$

Sol. 7(b)



$$\begin{aligned} \text{Area of the I-section, } A &= 200 \times 400 - 280 \times 120 \\ &= 46400 \text{ mm}^2 \end{aligned}$$

$$\text{Dead Load } W_d = 24 \times 46400 \times 10^{-6} \text{ KN/m}$$

$$= 1.11 \text{ KN/m}$$

$$\text{Live Load} = 4 \text{ KN/m}$$

$$\text{total Load} = 5.11 \text{ KN/m}$$

Moment due to dead load and live load at center of span

$$M = \frac{(W_d + W_L) l^2}{8}$$

$$M = \frac{5.11 \times 8^2}{8}$$

$$= 40.88 \text{ KN-m}$$

$$\text{Prestressing Force} = P$$

$$\text{Eccentricity, } e = 0$$

$$\text{Resultant stress at bottom, } \sigma = \frac{P}{A} + \frac{Pe}{z} - \frac{M}{z} = 0$$

$$Z = \frac{I}{Y}$$

$$I = \frac{200 \times 400^3}{12} - \frac{120 \times 280^3}{12}$$

$$= 8.47 \times 10^8 \text{ mm}^4$$

$$Y = 200 \text{ mm at soffit}$$

$$Z = 4.235 \times 10^6 \text{ mm}^3$$

$$\sigma = \frac{P}{46400} - \frac{40.88 \times 10^6}{4.235 \times 10^6} = 0$$

$$P = 447.9 \text{ KN}$$

Sol. 7 (c) Given data,

$$H = 4.35 \text{ m, } k_a = \frac{1}{3}, \gamma_s = 18 \text{ kN/m}^3$$

M20, Fe415

$$B.M_{\max} = \frac{1}{2} K_a \gamma_s H^2 \cdot \frac{H}{3} = \frac{1}{2} \times \frac{1}{3} \times 18 \times \frac{4.35^3}{3}$$

$$= 82.31 \text{ kNm (considering 1.90 m length of wall)}$$

$$\text{Factored B.M}_{\max} = 1.5 \times 82.31 = 123.47 \text{ kNm}$$

$$d \text{ required} = \sqrt{\frac{M}{0.138 f_{ck} b}} = \sqrt{\frac{123.47 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$= 211.51 \text{ mm}$$

$$d_{\text{adopt}} = 1.05 \times 211.51 = 222.08 \text{ mm} \cong 230 \text{ mm}$$

$$\text{Total depth } D = 230 + 50 + \frac{16}{2} = 288 \text{ mm}$$

Take $D = 300 \text{ mm}$

$$d = 300 - \left(50 + \frac{16}{2}\right) = 242 \text{ mm}$$

Cover = 50 mm, main dia = 16 mm (assume)

R/F in stem

$$M = 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} b} \right]$$

$$\Rightarrow 123.47 \times 10^6 = 0.87 \times 415 \times A_{st} \left[242 - \frac{f_y A_{st}}{20 \times 1000} \right]$$

$$\Rightarrow A_{st} = 1645.20 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000}{\left(\frac{1645.20}{201}\right)} = 122.17 \text{ mm}$$

$$\text{Spacing} \leq \min \begin{cases} 3d \\ 300 \text{ mm} \end{cases}$$

\Rightarrow Provide 16 mm dia. bar @ 120 mm c/c

$$\text{Distribution R/F} = \frac{0.12}{100} bD = \frac{0.12}{100} \times 1000 \times 300 = 360 \text{ mm}^2 < A_{st, \text{ provided}}$$

Using 8mm dia for distribution R/F, spacing

$$= \frac{1000}{\left(\frac{360}{50}\right)} = 138.88 \text{ mm}$$

$$\text{Spacing} \leq \min \begin{cases} 5d \\ 300 \text{ mm} \end{cases}$$

Provided 8ϕ @ 135 mm c/c.

$$\text{Development length} = 47\phi = 47 \times 16 = 752 \text{ mm}$$

Sol. 7(d)

(i) The clear spacing between beams

$$= \frac{14.5 - 0.3 \times 3}{4} = 3.4 \text{ m}$$

⇒ Each slab panel (with clear spans 3.4 m × 8 m has $\frac{L}{B} > 2$

⇒ It may be treated as one way (continuous) slab.

(ii) Effective depth = $\frac{\text{Span}}{26 \times \text{MF}}$
 MF = 1 (Assuming $k_1 = 1.0$)

$$d = \frac{3400}{26} = 130.76 \text{ mm}$$

Take clear cover = 20 mm and 10 mm ϕ bar

$$\text{Total depth} = 130.76 + 25 = 155.76 \text{ mm}$$

$$D = 155.76 \text{ mm}$$

$$\text{Adopt } D = 175 \text{ mm}$$

$$d = 175 - 25 \text{ mm} = 150 \text{ mm}$$

(ii) Loads

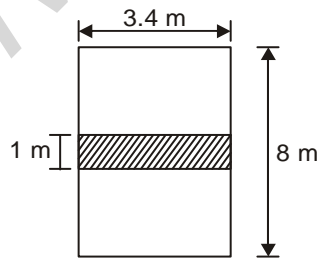
$$\text{Live load} = 4 \text{ kN/m}^2$$

$$\text{Dead load (other than self weight)} = 1.5 \text{ kN/m}^2$$

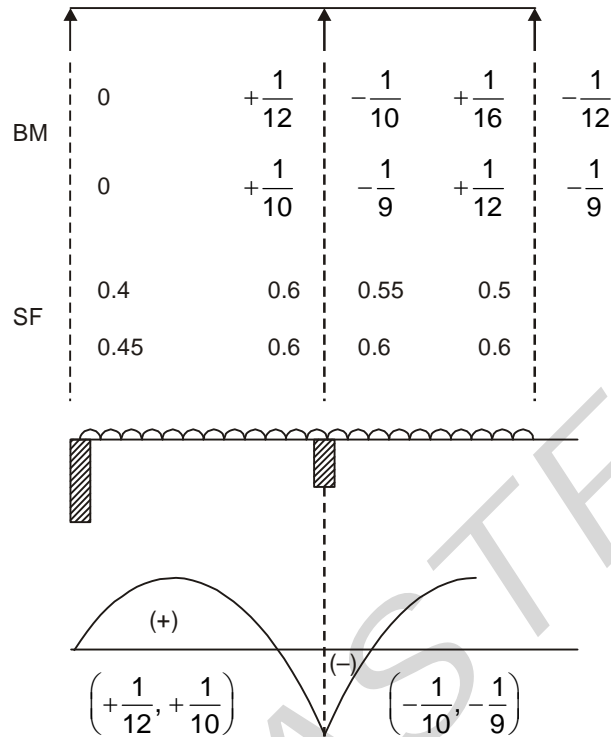
$$\text{(Total dead load)} w_d = 1.5 \times 1 + 4.375 = 5.875 \text{ kN/m}$$

$$\text{Factored load} = \begin{cases} w_u(\text{DL}) = 1.5 \times 5.875 = 8.812 \text{ kN} \\ w_u(\text{LL}) = 1.5 \times 4 = 6 \text{ kN/m}^2 \end{cases}$$

Plan of each panel



Let us design end panel



Effective length : As the beam width exceeds $\frac{\text{span}}{12}$

$$w = 300 \text{ mm}$$

$$\frac{\text{span}}{12} = 283.33 \text{ mm} \quad \left\{ w > \frac{\text{span}}{12} \right\}$$

For interior spans

$$l = 3400 \text{ mm (clear span)}$$

For end span

$$\begin{aligned}
 l &= 3400 + \frac{d}{2} = 3475 \text{ mm} \\
 &= 3400 + \frac{w}{2} = 3550 \text{ mm}
 \end{aligned}
 \left. \vphantom{\begin{aligned} l &= 3400 + \frac{d}{2} \\ &= 3400 + \frac{w}{2} \end{aligned}} \right\} \text{whichever is less}$$

$$l_{\text{eff}} = 3475 \text{ mm}$$

$$\begin{aligned}
 M_u^+ &= \frac{1}{12} w_d l^2 + \frac{1}{10} w_l l^2 \\
 &= \frac{1}{12} \times 8.812 \times 3.475^2 + \frac{1}{10} \times 6 \times 3.475^2 \\
 &= 16.11 \text{ kN-m}
 \end{aligned}$$

Max -ve BM

$$\begin{aligned}
 (M_u^-) &= -\frac{1}{10} w_d l^2 - \frac{1}{9} w_l l^2 \\
 &= -\frac{1}{10} \times 8.812 \times 3.475^2 - \frac{1}{9} \times 6 \times 3.475^2
 \end{aligned}$$

$$= -18.69 \text{ kN-m}$$

(iv) Depth calculation

$$Q = 0.138 f_{ck} \quad (\because \text{Fe415 steel})$$

$$= 0.138 \times 20 = 2.76$$

$$d = \sqrt{\frac{M}{QB}} = \sqrt{\frac{18.69 \times 10^6}{2.76 \times 1000}} = 82.3 \text{ mm} < 150 \text{ mm}$$

So, we have to use the effective depth as calculated from deflection point of view.

Since, this thickness of slab is very less

So, use

$$d = 150 \text{ mm}$$

(v) Area of steel for max. +ve BM (A_{st}^+)

$$M_u^+ = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$C = T$$

$$0.36 f_{ck} B x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B}$$

$$= \frac{0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 1000} = 0.0501 A_{st}$$

$$16.11 \times 10^6 = 0.87 \times 415 A_{st} (150 - 0.4 \times 0.0501 A_{st})$$

$$A_{st}^+ = 310.33 \text{ mm}^2$$

$$(A_{st})_{\min} = \frac{0.12}{100} \times BD = \frac{0.12}{100} \times 1000 \times 175 = 210 \text{ mm}^2$$

$$A_{st}^+ > (A_{st})_{\min}$$

$$\text{Spacing of 10 mm dia bar} = \frac{\frac{\pi}{4} \times 10^2 \times 1000}{325.43} = 253.08 \text{ mm}$$

$$\text{Spacing} \times 3d \times 150 = 450 \text{ mm}$$

$$\times 300 \text{ mm}$$

So provide 10 mm bar @ 250 mm c/c

Area of steel for max -ve BM (A_{st}^-)

$$M_u^- = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$18.69 \times 10^6 = 0.87 \times 415 \times A_{st} (150 - 0.42 x_u)$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 A_{st}}{0.36 \times 20 \times 1000} = 0.0501 A_{st}$$

$$18.69 \times 10^6 = 0.87 \times 415 \times A_{st} (150 - 0.42 \times 0.0501 A_{st})$$

$$A_{st}^{-ve} = 363.66 \text{ mm}^2$$

$$A_{st}^- > (A_{st})_{\min}$$

$$\text{Spacing} = \frac{\frac{\pi}{4} \times 10^2}{363.66} \times 1000 = 215.97 \text{ mm}$$

Spacing ∇ (i) 450 mm (ii) 300 mm

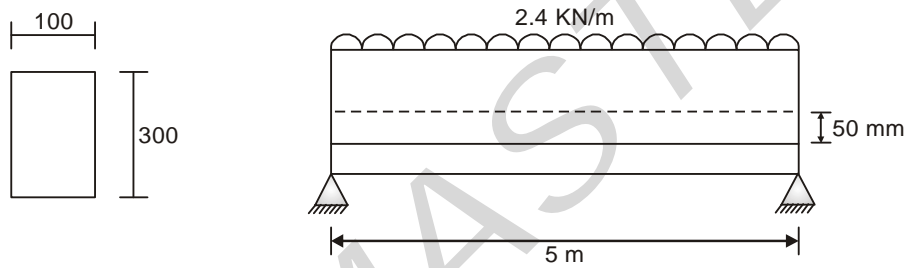
Provide 10 mm dia bars @ 210 mm C/C

$$\text{Area of distribution steel} = \frac{0.12BD}{100} = \frac{0.12 \times 1000 \times 175}{100} = 210 \text{ mm}^2$$

$$\text{Spacing of 8 mm bar} = \frac{\pi}{4} \times 8^2 \times 1000 = 239.23 \text{ mm}$$

Provide 8 mm @ 230 mm C/C

Sol.8(a)



$$\text{Reaction at each support, } R = 2.4 \times \frac{5}{2} = 6 \text{ kN}$$

$$\begin{aligned} \text{B.M. at quarter span, } M_1 &= R \times \frac{L}{4} - W \left(\frac{L}{4} \right)^2 \times \frac{1}{2} \\ &= 6 \times 1.25 - 2.4 \frac{(1.25)^2}{2} \\ &= 5.625 \text{ kN-m} \end{aligned}$$

$$\text{B.M. at mid-span} = \frac{WL^2}{8} = \frac{2.4 \times 5^2}{8} = 7.50 \text{ kN-m}$$

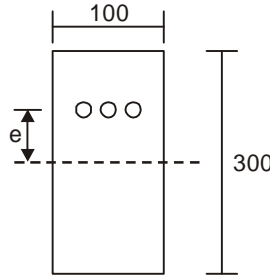
$$\text{Shift of C-line from p-line, } e' = \frac{M}{P}$$

$$\begin{aligned} \text{Shift at quarter span, } e'_1 &= \frac{5.625 \times 10^3}{75} \\ &= 75 \text{ mm} \end{aligned}$$

Eccentricity of C-line = $-50 + 75 = 25$ mm above centroidal axis

$$\text{Shift at mid-span, } e'_2 = \frac{7.5 \times 10^3}{75} = 100 \text{ mm}$$

Eccentricity of C-line = $100 - 50 = 50$ mm above centroidal axis



$$\text{Stress at top at Mid-span} = \frac{P}{A} + \frac{Pe}{z}$$

$$\text{Stress at bottom at mid-span} = \frac{P}{A} - \frac{Pe}{z}$$

$$\frac{P}{A} = \frac{75 \times 10^3}{100 \times (300)} = 2.5 \text{ N/mm}^2$$

$$\frac{Pe}{z} = \frac{75 \times 10^3 \times 50}{100 \times \frac{(300)^2}{6}} = 2.5 \text{ N/mm}^2$$

$$\sigma_{\text{top}} = 2.5 + 2.5 = 5 \text{ N/mm}^2$$

$$\sigma_{\text{bottom}} = 2.5 - 2.5 = 0 \text{ N/mm}^2$$

Sol. 8(b)

$$b_f = \frac{l_0}{\frac{l_0}{B} + 4} + b_w = \frac{6000}{\frac{6000}{1000} + 4} + 250 = 850 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$b_w = 250 \text{ mm}$$

$$d = 520 \text{ mm}$$

$$A_{st} = 6 \times \frac{\pi}{4} \times 28^2 = 3694.51 \text{ mm}^2$$

$$f_{ck} = 20 \text{ MPa}$$

$$f_y = 250 \text{ MPa}$$

Case I : Assuming

$$x_u < D_f$$

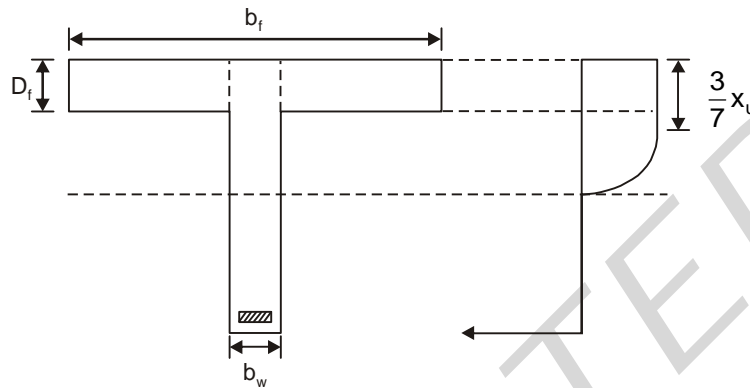
$$C = T$$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87f_y A_{st}}{0.36f_{ck} b_f} = \frac{0.87 \times 250 \times 3694.51}{0.36 \times 20 \times 850} = 131.30 \text{ mm}$$

⇒ Assumption is not correct as $x_u > 100 \text{ mm}$

Case II : Assuming $x > \frac{7}{3}D_f$



$$C = T$$

$$0.36f_{ck} b_w x_u + 0.45f_{ck} (B_f - b_w) D_f = 0.87f_y A_{st}$$

$$0.36 \times 20 \times 250 \times x_u + 0.45 \times 20(850 - 250) \times 100 = 0.87 \times 250 \times 3694.51$$

$$x_u = 146.42 \text{ mm}$$

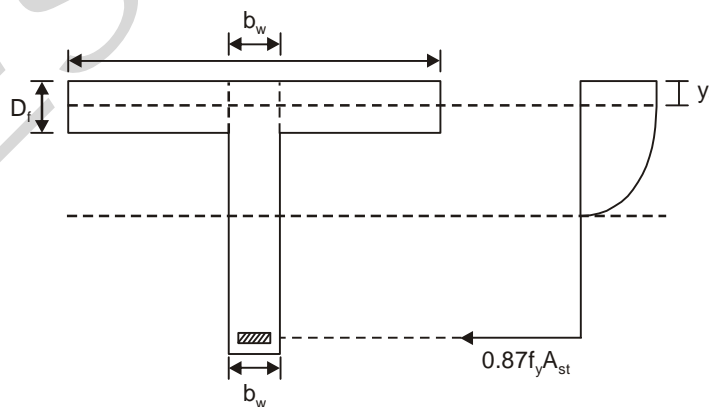
$$\frac{7}{3}D_f = 233.33 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

⇒ Assumption is not correct.

Case III:

$$D_f < x < \frac{7}{3}D_f$$



$$C = T$$

$$0.36f_{ck} b_w x_u + (b_f - b_w) y_f \times 0.45f_{ck} = 0.87f_y A_{st}$$

$$y_f = 0.15x_u + 0.65D_f$$

$$0.36 \times 20 \times 250 \times x_u + (850 - 250) \times (0.15x_u + 0.65 \times 100) \times 0.45 \times 20$$

$$= 0.87 \times 250 \times 3694.51$$

$$x_u = 173.39 \text{ mm}$$

$$y_f = 0.15 \times 173.39 + 0.65 \times 100$$

$$= 90.01 \text{ mm}$$

$$M_u = 0.36f_{ck}b_w x_u (d - 0.42x_u) + 0.45f_{ck}(B_f - b_w)y_f \left(d - \frac{y_f}{2} \right)$$

$$= 0.36 \times 20 \times 250 \times 173.23(520 - 0.42 \times 173.23)$$

$$+ 0.45 \times 20 \times (850 - 250) \times 90.01 \times \left(520 - \frac{90.01}{2} \right)$$

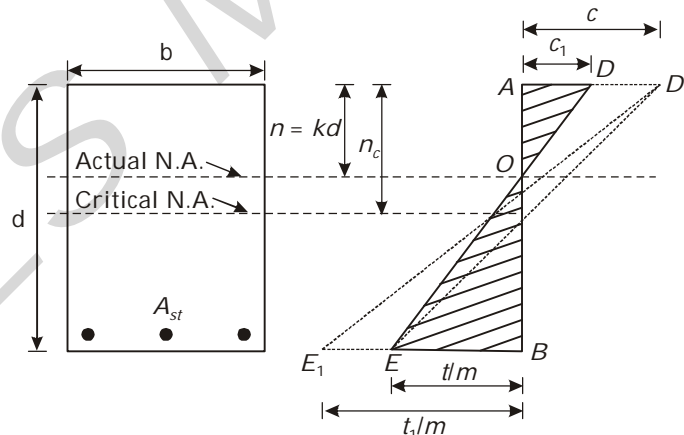
$$= 370.33 \text{ kN-m}$$

Sol. 8(c) Under-Reinforced and Over-Reinforced Section

(i) Under Reinforced Section: An under reinforced section is the one in which the percentage of steel provided is less than critical one and therefore full strength of concrete in compression is not developed. The actual neutral axis of such a section will fall above the critical neutral axis of a balanced section.

Figure below shows the under-reinforced section in which the actual N.A. is above the critical N.A. Let c be the permissible stress in concrete and t be the permissible stress in steel. If full permissible compressive stress c is permitted to be developed in concrete, the corresponding stress t_1 in steel will be such that t_1/m . Evidently, since $t_1 > t$, full compressive stress c cannot develop in concrete. Instead, a compressive stress c_1 is developed of such a magnitude that $t/m = \sigma_{st}m$ is obtained. Thus, in a under-reinforced concrete, the concrete is not fully stressed to its permissible value when stress in steel reaches its maximum value of $t = \sigma_{st}$. The moment of resistance of an under-reinforced section is, therefore, computed on the basis of the tensile force in steel:

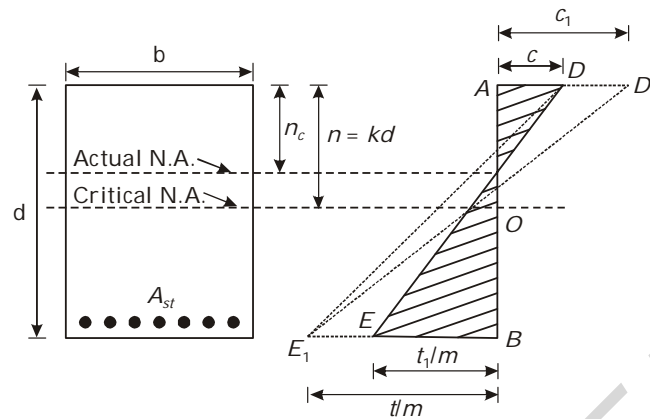
$$M_r = tA_{st} \cdot jd = \sigma_{st} \times A_{st} \cdot jd$$



(ii) Over-Reinforced Section: In an over-reinforced section, the reinforcement provided is more than critical one and therefore the actual N.A. of such a section falls below the critical N.A. of a balanced section.

Figure shows an over-reinforced section in which the actual N.A. falls below the critical one. If stress in steel is permitted to be equal to maximum permissible value t the corresponding stress in concrete will be equal to c_1 which is greater than permissible value c in concrete. Hence the stress distribution will be governed by line DOE (and not $D_1 O E_1$), giving rise to a stress t_1 in steel which is lesser than t . Thus, in an over-reinforced section, steel reinforcement is not fully stressed to its permissible value

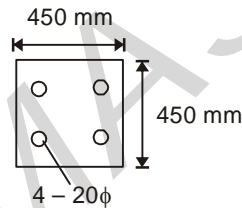
and the moment of resistance is determined on the basis of compressive force developed in concrete:



$$M_r = \frac{1}{2} c \cdot kd \cdot b \cdot jd = \frac{1}{2} c \cdot j \cdot k \cdot bd^2 = \frac{1}{2} \sigma_{cbc} \cdot j \cdot k \cdot bd^2$$

$$\text{or } M_r = \frac{1}{2} c \cdot n \cdot \left(d - \frac{n}{3}\right) b = \frac{1}{2} \sigma_{cbc} \cdot n \left(d - \frac{n}{3}\right) b$$

Sol. 8. (d)



$$A_{st} = \frac{4 \times \pi}{4} \times (20)^2 = 1256.6 \text{ mm}^2$$

Adopting a min eccentricity of 20 mm

$$0.05 D = 0.05 \times 450 = 22.5 \text{ mm}$$

$e_{\min} < 0.05 D$ hence ultimate load is given by

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$$

$$P_u = 0.4 \times 20 \times (450 \times 450 - 1256.6) + 0.67 \times 415 \times 1256.6$$

$$P_u = 1959.3 \text{ kN}$$