SOLUTION

1. (c)

[SOLN]

If x(t) is real then a_k must be conjugate symmetric

$\Rightarrow \quad a_k = a^*_{-k}$; for all values of k

As $\quad a_k = j\left(\frac{1}{2}\right)^{|k|}$

$$a^*_{-k} = (-j)\left(\frac{1}{2}\right)^{|k|} \Rightarrow a_k \neq a^*_{(-k)}$$

since this is not true in this case so x(t) is not real.

If x(t) is even, then x(t) = x(–t)

Since this is true for this case, so x(t) is even.

$$a_k = +(a_{-k})$$

$$a_k = j\left(\frac{1}{2}\right)^{|k|}$$

$$a_{-k} = j\left(\frac{1}{2}\right)^{|(-k)|} = j\left(\frac{1}{2}\right)^{(k)} = a_k$$

2. (3)

[SOLN]

From the given figures it can be written as

$$x(t) = u(t+1) - u(t-1) \xleftrightarrow{LT} \frac{1}{s}e^{s} - \frac{1}{s}e^{-s} = x(s)$$

and $\quad y(t) = U(t) + U(t-2) - 2U(t-4)$

$$\xleftrightarrow{LT} \frac{1}{s} + \frac{e^{-2s}}{s} - \frac{2e^{-4s}}{s} = Y(s)$$

Given $\quad m(t) = x(t) * y(t) \quad$ ...(1)

Taking Laplace transform both side for eqn (1)

$$M(s) = X(s) \cdot Y(s)$$

Putting the value of x(s) and Y(s)

$$M(s) = \left(\frac{1}{s}e^{s} - \frac{1}{s}e^{-s}\right)\left(\frac{1}{s} + \frac{e^{-2s}}{s} - \frac{2e^{-4s}}{s}\right)$$

$$M(s) = \frac{1}{s^2}[e^{+s} - 3e^{-3s} - 2e^{-5s}]$$

Taking Inverse Laplace transform of M(s), we get m(t) as

m(t) = (t + 1) u(t + 1) – 3(t – 3) u(t – 3) – 2(t – 5) u(t – 5)

by putting t = 3.5, we get m(3.5) as

m(3.5) = (3.5 + 1) – 3(3.5 – 3) = 3.5 + 1 – 1.5 = 3

3. (12 to 13)

[SOLN]

We know

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{-j\omega t}\, d\omega$$

Put $\quad t = 0$

$$x(0)\cdot 2\pi = \int_{-\infty}^{\infty} X(\omega)\cdot d\omega$$

from given figure of x(t) it is clear that

$$x(t)|_{t=0} = 2$$

so $\displaystyle\int_{-\infty}^{\infty} X(\omega)\, d\omega = 4\pi = 12.56$

4. (d)

[SOLN]

H_1–It is causal because the output does not appear before the input.

H_2–It is non causal because the output appears at t = 0, one time unit before the delayed input at t = +1.

5. (6)

[SOLN]

Using Parseval's power theorem

$$P = \sum_{-\infty}^{\infty} C_n^2 = \frac{1}{T}\int_T [x(t)]^2\, dt$$

$$P = (-1)^2 + (-1)^2 + (2)^2$$

$$P = 6$$

6. (3 – 3.5)

[SOLN]

We know

If $\quad x(t) * h(t) = y(t)$

then $x(at) * h(at) = \frac{1}{a}y(at)$

Put $\quad a = 3$

$$x(3t) * h(3t) = \frac{1}{3}y(3t) = g(t) = A\,y(Bt),$$

on comparing

$$A = \frac{1}{3}$$

$$B = 3$$

$\Rightarrow \quad A + B = 3.33$

7. ( )

[SOLN]

The energy of the raised pulse is

$$E = \int_{-\infty}^{\infty} [x(t)]^2$$

Putting the value of x(t) from given in the question

$$\int_{-\pi/\omega}^{\pi/\omega} \frac{1}{4}(\cos(\omega t)+1)^2\, dt$$

$$\Rightarrow \frac{1}{2}\int_0^{\pi/\omega}\left[\cos^2(\omega t)+2\cos(\omega t)+1\right]dt$$

$$= \frac{1}{2}\int_0^{\pi/\omega}\left[\left(\frac{1+\cos(2\omega t)}{2}\right)+2\cos(\omega t)+1\right]dt$$

on solving the intergration and putting the upper and lower limits we set

$$E = \frac{1}{2}\cdot {}^{3}\!/_{2}\cdot {}^{\pi}\!/_{\omega}$$

$$E = \frac{3\pi}{4\omega}$$

8. (d)

[SOLN]

$\because \quad r[n] = n\,u[n]$

$$x[n] = 2^{n\,u[n]} = \begin{cases} 1 & ; \quad n < 0 \\ 2^n & ; \quad n \geq 0 \end{cases}$$

$\Rightarrow \quad x[n] = u[-n-1] + 2^n\,u[n]$

$\because \quad u[-n-1] \rightarrow ROC_1\ |z| < 0$

and $\quad 2^n u[n] \rightarrow ROC_2\ |z| > 2$

soROC of x[n] is $ROC_1 \cap ROC_2$

i.e. no common ROC exists, so z-transform of x[n] does not exist.

9. (3)

[SOLN]

The poles of z-transform obtained from characteristic eqn

$z_0 = +\frac{1}{2}j, \quad z_1 = -\frac{1}{2}j, \quad z_2 = -\frac{1}{2}, \quad z_3 = \frac{3}{4}$

Based on these pole locations, we may choose from the following regions of convergence

(i) $0 < |z| < \frac{1}{2}$

(ii) $\frac{1}{2} < |z| < \frac{3}{4}$

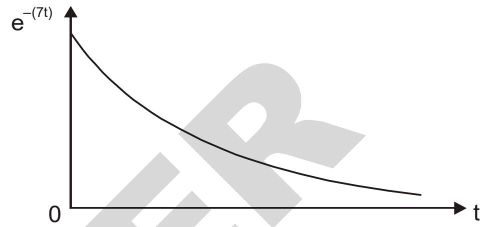
(iii) $|z| > \frac{3}{4}$

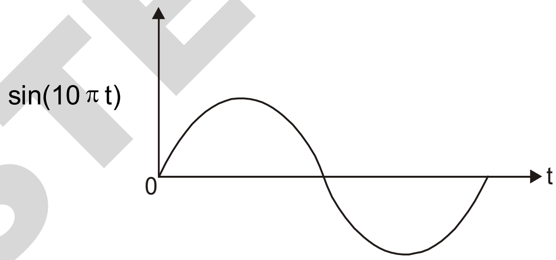
$\because$ Characteristic equation

$$\left(1+\frac{1}{4}z^{-2}\right)\left(1+\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2}\right) = 0$$

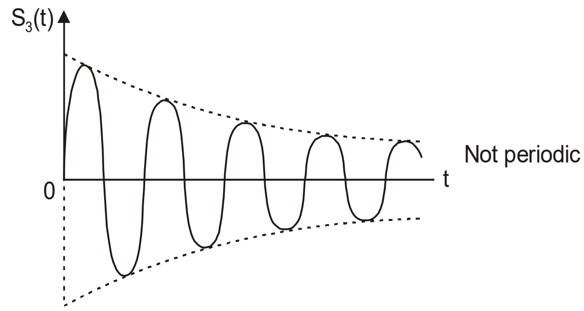
10. (c)

[SOLN]





$s_3(t) = \exp(-7t)\cdot\sin(10\pi t)$



$$S_1(t) = \cos 2t + \cos 3t + \cos 5t$$

$$\omega_1 = 2,\ \omega_2 = 3,\ \omega_3 = 5$$

$$T_1 = \frac{2\pi}{\omega_1},\ T_2 = \frac{2\pi}{\omega_2},\ T_3 = \frac{2\pi}{\omega_3}$$

$$T_1 = \frac{2\pi}{2},\ T_2 = \frac{2\pi}{3},\ T_3 = \frac{2\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{3}{2} \text{ rational}$$

Similarly, $\frac{T_2}{T_3}, \frac{T_3}{T_1}$ are also rational. So $S_1(t)$ is periodic

$S_2 = e^{(j8\pi t)}$ is also periodic with frequency $\omega_3 = 8\pi$ rad / sec

11. (1)

[SOLN]

Given

$$y(t) = x(t) * h(t)$$

Taking Fourier transform both side we get

$$\Rightarrow \quad Y(j\omega) = X(j\omega) \cdot H(j\omega) \quad ...(1)$$

Taking Fourier transform of x(t), we get $X(j\omega)$ as,

$$X(j\omega) = \frac{1}{(2+j\omega)^2} \quad ...(2)$$

Taking Fourier transform of h(t), we get $H(j\omega)$ as,

$$H(j\omega) = \frac{1}{(4+j\omega)^2} \quad ...(3)$$

Putting the value of $H(j\omega)$ and $X(j\omega)$ in eqn (1) from eqn (3) and (2) we get

$$Y(j\omega) = \frac{\frac{1}{4}}{2+j\omega} + \frac{\frac{1}{4}}{(2+j\omega)^2} - \frac{\frac{1}{4}}{(4+j\omega)} + \frac{\frac{1}{4}}{(4+j\omega)^2}$$

(By using partial fraction)

Now taking inverse Fourier transform of $y(t)$, we get

$$y(t) = \frac{1}{4}\left[e^{-2t} + te^{-2t} - e^{-4t} + te^{-4t}\right]u(t)$$

on comparing with given eqn of $y(t)$, we get

$$\Rightarrow \quad A = B = C = D = \frac{1}{4}$$

$$\Rightarrow A + B + C + D = 1$$

12. (4)

[SOLN]

Poles location from characterstic eqn.

$(s + 2)(s + 3)(s^2 + s + 1) = 0$

then $\quad s = -2$

and $\quad s = -3$ and $s = -\frac{1}{2} + \frac{\sqrt{3}}{2} j$

and $\quad s = -\frac{1}{2} - \frac{\sqrt{3}}{2} j$

Based on the locations of these poles, we may choose the following regions of convergence

(a)$\text{Re}\{s\} > -\frac{1}{2}$ (b) $-2 < \text{Re}\{s\} < -\frac{1}{2}$

(ii)$-3 < \text{Re}\{s\} < -2$

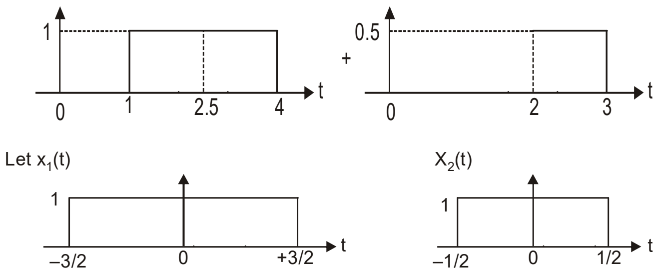
(d)$\text{Re}\{s\} < -3$

Hence total 4 signals are possible according to 4 different region of convergence.

13. (d)

[SOLN]

x(t) =



x(t) can be represented in term of $x_1(t)$ and $x_2(t)$ as

$$x(t) = x_1(t-2.5) + \frac{1}{2}x_2(t-2.5)$$

$$x_1(t-2.5) \rightleftharpoons 3.\text{Sa}\left(\frac{3\omega}{2}\right)e^{-2.5j\omega}$$

$$x_2(t-2.5) \rightleftharpoons \text{Sa}\left(\omega\frac{1}{2}\right)e^{-j2.5\omega}$$

$$x(t) \rightleftharpoons X(j\omega) = \left[3.\text{Sa}\left(\omega\frac{3}{2}\right) \cdot + \frac{1}{2}\text{Sa}\left(\frac{\omega}{2}\right)\right]e^{-j2.5\omega}$$

$$= \left[3\frac{\sin\left(\frac{3\omega}{2}\right)}{\frac{3\omega}{2}} + \frac{1}{2}\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}\right]e^{-2.5\omega j}$$

$$= \left[\frac{2\sin\left(\frac{3\omega}{2}\right)}{\omega} + \frac{\sin\left(\frac{\omega}{2}\right)}{\omega}\right]e^{-j2.5\omega}$$

$$= \left[\frac{2\sin\left(\frac{3\omega}{2}\right) + \sin\left(\frac{\omega}{2}\right)}{\omega}\right]e^{-j2.5\omega}$$

14. (c)

[SOLN]

given

$$X(j\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)$$

Taking inverse Fourier transform

$$x(t) = \frac{1}{2\pi} + \frac{1}{2\pi}e^{j\pi t} + \frac{1}{2\pi}e^{j5t}$$

Signal x(t) has two complex exponentials whose fundametnal frequencies are $\frac{2\pi}{5}$ red/sec.and 2 rad/sec.

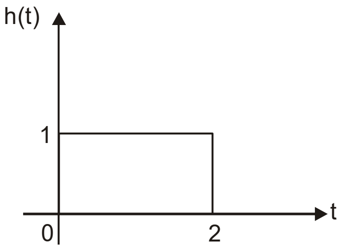
These two complex exponentials are not harmonically related. So signal x(t) is not periodic.

Consider

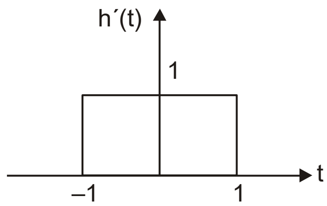
$$y(t) = x(t) * h(t)$$

$$Y(j\omega) = X(j\omega)H(j\omega)$$

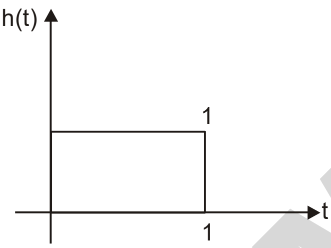
$$h(t) = u(t) - u(t-2)$$



Let h'(t)



h(t) = h'(t–1)



$$h'(t) \rightleftharpoons H'(j\omega) = 2Sa(\omega)$$

$$H(j\omega) = e^{-j\omega}2Sa(\omega)$$

$$= e^{-j\omega}\frac{2\sin\omega}{\omega}$$

$$Y(j\omega) = [\delta(\omega)+\delta(\omega-\pi)+\delta(\omega-5)]e^{-j\omega}\left(\frac{2\sin\omega}{\omega}\right)$$

Since when $\omega = k\pi$, then $H(j\omega) = 0$

if $k = 1$ then $\omega = \pi$, $H(j\omega) = 0$

$$Y(j\omega) = [\delta(\omega)+\delta(\omega-5)]$$

$$y(t) = \frac{1}{2\pi}+\frac{1}{2\pi}e^{j5t}$$

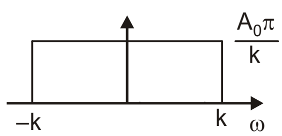
$\Rightarrow$ y(t) is a complex exponetial summed with a constant and y(t) is periodic

15. 0 (zero)

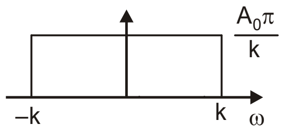
[SOLN]

Since we know

$$A_0 Sa(kt) \rightleftharpoons$$

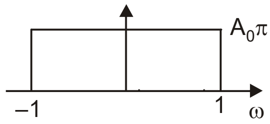


$$A_0\frac{\sin(kt)}{kt} \rightleftharpoons$$



k = 1

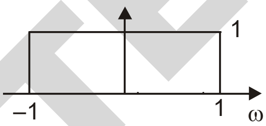
$$A_0\frac{\sin(t)}{t} \rightleftharpoons$$



$$A_0 = \frac{1}{\pi}$$

Let

$$y_1(t) = x_1(t) = \left(\frac{\sin t}{\pi t}\right) \rightleftharpoons$$

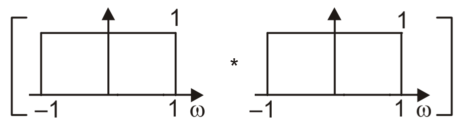


$$= X_1(j\omega) = Y_1(j\omega) \qquad \ldots(1)$$

$$x_1(t)\cdot y_1(t) \rightleftharpoons \frac{1}{2\pi}X_1(j\omega) * Y_1(j\omega)$$

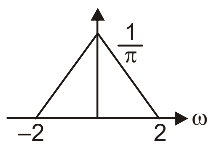
from eqn. (1)

$$\left(\frac{\sin t}{\pi t}\right)^2 \rightleftharpoons \frac{1}{2\pi}$$



Let

$$x'(t) = \left(\frac{\sin t}{\pi t}\right)^2 \rightleftharpoons$$



$$= x'(j\omega) \qquad \ldots(2)$$

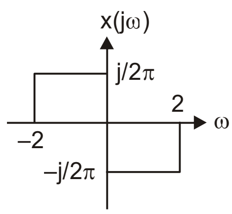
$$\because \quad x'(t) \rightleftharpoons X'(j\omega)$$

$$t\,x'(t) \rightleftharpoons j\frac{dX'(j\omega)}{d\omega}$$

from eqn. (2)

$$t\left(\frac{\sin t}{\pi t}\right)^2 \rightleftharpoons j\frac{d(X(j\omega))}{d\omega}$$

$$t\left(\frac{\sin t}{\pi t}\right)^2 \rightleftharpoons$$



$$X(j\omega) = \begin{cases} j/2\pi & -2 \le \omega < 0 \\ -j/2\pi & 0 < \omega \le 2 \\ 0 & \text{otherwise} \end{cases}$$

16. (d)

[SOLN]

To exist a real α such that $e^{j\alpha\omega}X(j\omega)$ is real, we require $x(t+\alpha)$ be a real and even signal

$$x(t) = \frac{1}{2\pi}\int X(j\omega)e^{+j\omega t}d(\omega)$$

At t = 0

$$x(0) = \frac{1}{2\pi}\int X(j\omega)d\omega = 0$$

if signal x(t) is periodic then $X(j\omega)$ will be periodic

For $\text{Img}(X(j\omega)) = 0$, the signal must be real and even

17. (b)

[SOLN]

Let $x_1(t) = \cos(\pi t)[u(t+1) - u(t-1)]$ ...(1)

& $x_2(t) = u(t)$ ...(2)

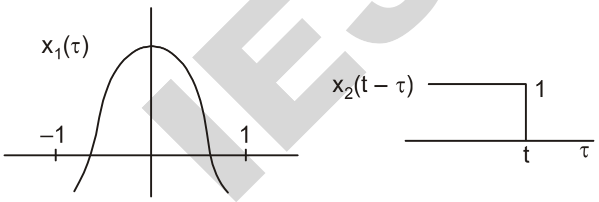
& $y(t) = x_1(t) * x_2(t)$

$$\Rightarrow \quad y(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau \quad ...(3)$$

Putting the values from eqn. (1) and eqn. (2)

$$x_1(\tau) = \cos(\pi\tau)[u(\tau+1) - u(\tau-1)] \quad ...(4)$$

$$x_2(t-\tau) = u(t-\tau) \quad ...(5)$$



for t < – 1 and t > + 1

y(t) = 0

for –1 < t < 1

So from eqn. (3)

$$y(t) = \int_{-1}^{t} \cos(\pi t)d\tau$$

$$= \frac{1}{\pi}\sin(\pi t)$$

$$y(t) = \begin{cases} \frac{1}{\pi}\sin(\pi t) & -1 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

18. (b)

[SOLN]

Given

$$x(t) = 1 + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + 2\left[\frac{e^{+j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] + \frac{e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}}{2} \quad ...(1)$$

Since we know

$$x(t) = \sum_{n=0}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(t) = C_0 + C_1 e^{j\omega_0 t} + C_{-1}e^{-j\omega_0 t} + C_2 e^{+2j\omega_0 t} + C_{-2}e^{-j2\omega_0 t} \quad ...(2)$$

on comparing eqn (1) and eqn. (2)

$$C_{-2} = \frac{1}{2}e^{-j\pi/4} \quad ...(3)$$

$$C_2 = \frac{1}{2}e^{j\pi/4} \quad ...(4)$$

$$\Rightarrow \quad C_2 + C_{-2} = \frac{1}{2}\left(e^{j\pi/4} + e^{-j\pi/4}\right)$$

$$= \frac{1}{2}[2\cos\pi/4]$$

$$= \cos\pi/4$$

$$= \frac{1}{\sqrt{2}}$$

19. (b)

[SOLN]

Taking fourier transform of given differential equation

$(j\omega)^2\, Y(j\omega) + 4(j\omega)\, Y(j\omega) + 3Y(j\omega) = (j\omega)\, X(j\omega) + 2\, X(j\omega)$

$\Rightarrow Y(j\omega)\, [(j\omega)^2 + 4(j\omega) + 3] = X(j\omega)[(j\omega) + 2]$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega + 2}{(j\omega)^2 + 4(j\omega) + 3}$$

The frequency response is

$$H(j\omega) = \frac{(j\omega) + 2}{(j\omega)^2 + 4(j\omega) + 3}$$

$$H(j\omega) = \frac{j\omega + 2}{(j\omega+1)(j\omega+3)} = \frac{A}{(j\omega+1)} + \frac{B}{(j\omega+3)}$$

On solving value of A = 1/2 and $B = \frac{1}{2}$, we get

$$H(j\omega) = \frac{\frac{1}{2}}{j\omega + 1} + \frac{\frac{1}{2}}{j\omega + 3}$$

Taking inverse fourier transform of $H(j\omega)$

$$h(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)$$

20. (0.2 to 0.3)

[SOLN]

For given signal, it is clear that since T = 2

$$\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

Since we know

$$x[k] = \frac{1}{4}\int_0^1 x(t)e^{-jk\omega_0 t}\ dt$$

By putting the value of T and ω_0

$$= \frac{1}{2}\int_0^1 e^{-t}e^{-j(k\pi t)}dt \quad = \frac{1}{2}\int_0^1 e^{-(1+jk\pi)tdt}$$

$$\Rightarrow \qquad x[k] = \frac{1-e^{-1[1+k\pi j]}}{2(1+k\pi j)}$$

by putting

$$k = \frac{1}{j\pi}$$

$$x\left[\frac{1}{j\pi}\right] = \frac{1-e^{-1\left(1+\frac{\pi}{\pi}\right)}}{2(2)}$$

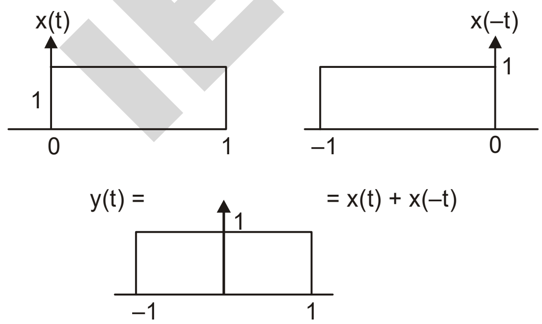
$$= \frac{1-e^{-(2)}}{4} = 0.2161$$

21. (c)

[SOLN]

$$x(t) = \text{rect}\left(t-\frac{1}{2}\right)$$

Given



Taking Fourier transform of y(t)

$$Y(j\omega) = 2Sa(\omega)$$

$$= 2\frac{\sin\omega}{\omega}$$

$$= 2\cdot\frac{2\sin(\omega/2)\cdot\cos(\omega/2)}{\omega}$$

$$Y(j\omega) = 2\frac{\sin(\omega/2)}{(\omega/2)}\cos(\omega/2) \qquad \ldots(1)$$

$$\because \qquad Sa(k) = \text{Sinc}(k/\pi)$$

$$Sa\left(\frac{\omega}{2}\right) = \text{Sinc}\left(\frac{\omega}{2\pi}\right) \qquad \ldots(2)$$

From eqn. (1) and (2)

$$\Rightarrow \qquad Y(j\omega) = 2\ Sa\left(\frac{\omega}{2}\right)\cdot\cos\left(\frac{\omega}{2}\right)$$

$$Y(j\omega) = 2\ \text{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)$$

22. (d)

[SOLN]

For $\text{Img}\{X(j\omega)\} = 0$, signal must be real and even

$$x(t) = \frac{1}{2\pi}\int X(j\omega)e^{+j\omega t}d\omega$$

At t = 0

$$x(0) = \frac{1}{2\pi}\int X(j\omega)d\omega = 0$$

$$x(t) = \frac{1}{2\pi}\int X(j\omega)e^{+j\omega t}d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}(j\omega)\int X(j\omega)\cdot e^{+j\omega t}d\omega$$

At t = 0

$$\left.\frac{dx(t)}{dt}\right|_{t=0} = \frac{1}{2\pi}\int \omega X(j\omega)d\omega = 0$$

derivative of $\left.\frac{dx(t)}{dt}\right|_{t=0} = 0$

$$x(t) = t^2e^{-|t|}$$

$$\frac{dx(t)}{dt} = 2t\ e^{-|t|} + t^2\ e^{-|t|}\ (-1)$$

$$\left.\frac{dx(t)}{dt}\right|_{t=0} = 0$$

For real $\{X(j\omega)\} = 0$, signal x(t) must be real and odd.

23. (6)

[SOLN]

Since the Fourier series coefficents repeat at every N, we have

$$a_1 = a_{15},\ a_2 = a_{16} \text{ and } a_3 = a_{17}$$

Further more, since the signal is real and odd, the Fourier series coefficients a_k will be purely imaginary and odd. Therefore, $a_0 = 0$

and

$$a_1 = -a_{-1}$$

$a_2 = -a_{-2}$

$a_3 = -a_{-3}$

Finally $a_{-1} = -j$

$a_{-2} = -2j$

$a_{-3} = -3j$

So, $a_0 + a_{-1} + a_{-2} + a_{-3} = -6j$

$\Rightarrow j(a_0 + a_{-1} + a_2 + a_{-3}) = j(-6j) = 6$

24. (c)

[SOLN]

For any signal to be real its Fourier series coefficient should be conjugate symmetric

$x_1(t) : a_{1k} = \cos(k\pi)$ ...(1)

$a_{1k}^* = \cos k\pi$

$a_{1(-k)}^* = \cos(-k\pi) = \cos k\pi$ ...(2)

So, $x_1(t)$ is real valued signal

Since $a_{1k} = a_{1(-k)}$ then $x_1(t)$ is even Signal

$x_2(t) : a_{2k} = j\sin\left(\frac{k\pi}{2}\right)$

$a_{2(-k)} = (-j)\sin\left(\frac{(-k)\pi}{2}\right)$

$= j\sin\left(\frac{k\pi}{2}\right)$

$a_{2(-k)}^* = a_{2k}$

$\Rightarrow x_2(t)$ is real valued signal

but $a_{2k} = -a_{2(-k)}$

$\Rightarrow x_2(t)$ is odd signal

25. (c)

[SOLN]

Taking z transform of given difference equation

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z) + \frac{1}{3}z^{-1}X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

when ROC $|z| < \frac{1}{2}$ does not include unit circle, so not stable

By using time shifting property

So,

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{3}\frac{z^{-1}}{1-\frac{1}{2}z^{-1}} \quad ...(1)$$

taking inverse z transform of eqn. (1) yields

$h[n] = h_1[n] + h_2[n]$

$$h_1[n] \xleftrightarrow{Z.T} \frac{1}{1-2z^{-1}}$$

So, $h_1[n] = -\left(\frac{1}{2}\right)^n u[n-1]$

and $h_2[n] \leftrightarrow \frac{1}{3}\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$

Since $H_2(z) = \frac{1}{3}z^{-1}H_1(z)$

$$\Rightarrow h_2[n] = \frac{1}{3}h_1[n-1] = \frac{1}{3}\left(-\frac{1}{2}\right)^n u[-n-1] \quad ...(3)$$

Put n = n – 1 in eqn. (3)

$$h_2[n] = \frac{1}{3}\left(-\frac{1}{2}\right)^{n-1} u[-n]$$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - \frac{1}{3}\left(\frac{1}{2}\right)^{n-1} u[-n]$$

$h[n] \neq 0$ for $n < 0$

So system is not causal

when ROC $|z| > \frac{1}{2}$

ROC include unit circle so stable

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - \frac{1}{3}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$h[n] = 0$ for $n < 0$ = system is causal and stable

26. (c)

[SOLN]

As initial condition y[–1] is given then use uniletral z-transform (U.Z.T)

$$y[n-n_0] \xleftrightarrow{U.ZT} z^{-n_0}Y(z) - y[-n_0]$$

So by using given eqn. (taking z transform)

$z^{-1}Y(z) - Y[-1] + 2Y(z) = X(z)$ ...(1)

as zero input response $x[n] = 0$ $\Rightarrow X(z) = 0$

So, eqn. (1) become as

$$Y(z) = \frac{y[-1]}{2+z^{-1}}$$

$$= \frac{2}{2+z^{-1}} = \frac{1}{1+(2z)^{-1}} \qquad \ldots(2)$$

Taking inverse uniletral z-transform of eqn. (2)

$$y[n] = -\left(-\frac{1}{2}\right)^n u[n]$$

27. (a)

[SOLN]

Two discrete time signal x[n] and y[n] are said to be orthogonal if

$\sum_{n=0}^{\infty} x[n]y^*[n] = 0$, where * denotes the complex conjugation…(1)

$Y_1(z) = z^{-1} + 4z^{-3} + 23z^{-5}$ (given) …(2)

Taking inverse z transform of given eqn (2), we get

$$y_1[n] = \delta(n-1) + 4\delta(n-3) + 23\delta(n-5) + ---$$

$$Y_2(z) = \sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n z^{-n} \text{ (given)} \qquad \ldots(3)$$

Taking z transform of given eqn. (3)

$y_2[n] = (2/3)^n u[n]$

$$Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n} \text{ (given)}$$

$$\Rightarrow \qquad y_3[n] = 2^{-|n|}$$

$$Y_4(z) = 2z^{-4} + 3z^{-2} + 1 \text{ (given)}$$

Taking inverse z transform, we get

$$y_4[n] = \delta(n) + 3\delta(n-2) + 2\delta(n-4)$$

$$X(z) = \sum_{n=0}^{\infty} \frac{3}{2+n} z^{2z}$$

$$X(z) = \frac{1}{2} + z^2 + \ \ldots..$$

Taking inverse z transform

$$x[n] = \frac{1}{2}\delta(n) + \delta(n+2) +$$

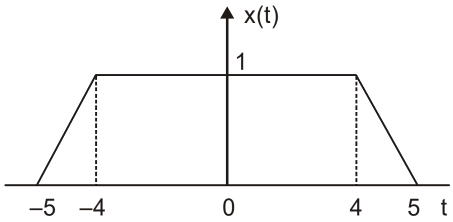
So only $Y_1[n]$ satisfy eqn. (1)

$$\int_0^{\infty}\left(\frac{1}{2}\delta(n) + \delta(n+2) + \ldots\right)(\delta(n-1) + 4\delta(n-3) + 23\delta(n-5) \ldots\ldots = 0$$

28. (8 – 9)

[SOLN]

Signal x(t) is as shown in figure below



the signal x(t) is even, its total energy is therefore

$$E = 2\int_0^5 x^2(t)\,dt$$

$$= 2\int_0^4 (1)^2\,dt + 2\int_4^5 (5-t)^2\,dt$$

$$= 2(t)_{t=0}^{t=4} + 2\left[-\frac{1}{3}(5-t)^3\right]_{t=4}^{5}$$

$= 8 + 2/3$

$= 26/3$

$= 8.666$

29. (a)

[SOLN]

R_1 : $y(t) = t^2x(t)$

linear but time variant

R_2 : $y(t) = t|x(t)|$

non linear and time variant

R_3 : $y(t) = |x(t)|$

nonlinear but time invarient

R_4 : $y(t) = x(t - 5)$

linear but time variant

30. (a)

[SOLN]

Given

$$y[n] = x[n] * h[n]$$

$$\Rightarrow \qquad y[n] = \sum_{k=-\infty}^{\infty} x[k]\ h[n-k] \qquad \ldots(1)$$

putting the x[n] and h[n] in eqn. (1), we get

$$= \sum_{k=-\infty}^{\infty} 2^k u[-k]\, u[n-k]$$

$\because$ $u[-k] = 1$

for $k \le 0$

& $u[n - k] = 1$

for $n - k \ge 0$

Since h[n] = u[n] h[n] will be non zero for $n \ge 0$

So range of k is k{–∞ to 0}

$$y[n] = \sum_{k=-\infty}^{0} 2^k$$

$$= 2^0 + 2^{-1} + 2^{-2} + 2^{-3} + ...$$

$$= 1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+---$$

$$= \frac{1}{1-0.5} = 2$$

31. (d)

[SOLN]

Given

$$h(t) = e^{\alpha t}u(t)+e^{\beta t}u(-t)$$

$\because e^{\alpha t}u(t)$ is right handed signal so for stability of $e^{\alpha t}u(t)$, $e^{\alpha t}u(t)$ should be absolutely integrable

$$\Rightarrow \left|\int_{-\infty}^{\infty} e^{\alpha t}u(t)dt\right| < \infty$$

$$\left|\int_{0}^{\infty} e^{\alpha t}dt\right| < \infty$$

and this is posible only when α is negative

Similarly

$e^{\beta t}u(-t)$ is left sided signal so for stability of $e^{\beta t}u(-t)$,

$e^{\beta t}u(-t)$ should be absolutely integrable, then

Similarly,

$$\left|\int_{-\infty}^{\infty} e^{\beta t}u(-t)dt\right| < \infty$$

$$\left|\int_{-\infty}^{0} e^{\beta t}dt\right| < \infty$$

and this is possible when β is positive.