

Class Test Solution (FM) 09-06-2019

Answer key

1. (a)	16. (c)	31. (c)	46. (c)	61. (b)
2. (c)	17. (c)	32. (a)	47. (a)	62. (a)
3. (a)	18. (c)	33. (b)	48. (b)	63. (c)
4. (c)	19. (b)	34. (d)	49. (c)	64. (a)
5. (b)	20. (d)	35. (d)	50. (c)	65. (d)
6. (c)	21. (c)	36. (c)	51. (b)	66. (a)
7. (b)	22. (a)	37. (b)	52. (d)	67. (d)
8. (c)	23. (c)	38. (c)	53. (a)	68. (d)
9. (b)	24. (a)	39. (d)	54. (c)	69. (a)
10. (c)	25. (d)	40. (d)	55. (a)	70. (a)
11. (c)	26. (d)	41. (b)	56. (c)	71. (c)
12. (c)	27. (b)	42. (d)	57. (d)	72. (b)
13. (b)	28. (a)	43. (a)	58. (*)	73. (a)
14. (b)	29. (b)	44. (c)	59. (c)	74. (b)
15. (c)	30. (d)	45. (b)	60. (c)	75. (b)



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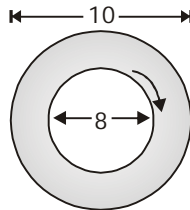
Institute for Engineers (IES/GATE/PSUs)

Regd. office : F-126, (Upper Basement), Katwaria Sarai, New Delhi-110016 • Phone : 011-41013406

Mob. : 8010009955, 9711853908 • E-mail: ies_master@yahoo.co.in, info@iesmaster.org

CLASS TEST-2 SOLUTIONS [FM]

1. (a)

Torque = force \times perpendicular distance \Rightarrow (shear stress \times Area) \times perpendicular distance $\Rightarrow \left(\mu \frac{du}{dy} \times \text{curved surface area} \right) \times \text{Radius}$ $\mu = 75 \text{ centi poise} = 0.75 \text{ poise}$ $\Rightarrow 0.075 \frac{\text{NS}}{\text{m}^2}$ $du = (R\omega - 0) \Rightarrow R \times \frac{2\pi N}{60} \Rightarrow \frac{\pi DN}{60}$ $= \frac{\pi \times 0.08 \times 1000}{60} = 4.188 \text{ m/sec}$ $dy = \frac{10 - 8}{2} = 1 \text{ cm} = 0.01 \text{ m}$ Curved surface area = $\pi D l = \pi \times 0.08 \times 0.15$
 $= 0.0377 \text{ m}^2$ Torque = $0.075 \times \frac{4.188}{0.01} \times 0.0377 \times \frac{0.08}{2}$ $\Rightarrow 0.04736 \text{ Nm}$

Power required to turn the shaft

 $P = T\omega$ $= 0.04736 \times \frac{2\pi \times 1000}{60}$ $= 4.96 \text{ watt.}$

2. (c) Conservation of mass

 $\frac{4}{3} \pi (8)^3 \times \rho = 8 \times \frac{4}{3} \pi r^3 \times \rho$ $r^3 = \frac{(8)^3}{8} \Rightarrow r = \frac{8}{2} \Rightarrow 4 \text{ mm}$ Initial surface area = $4\pi R^2$ $= 4\pi(8)^2$ final surface area = $8 \times 4\pi r^2$ $= 8 \times 4\pi(4)^2$ Increase in Area = $4\pi(8(4)^2 - 8^2)$ $= 4\pi \times 8(16 - 8)$ $= 4\pi \times 64$ $= 256\pi$ Work done = Surface tension \times Increase in area $\Rightarrow 256 \pi \sigma.$

3. (a)

4. (c)

 $Q_1 + Q_2 = Q_3$ $\Rightarrow [(\pi)(0.03)^2 / 4](6) + [(\pi)(0.03)^2 / 4](10) = Q_3$ $Q_3 = 0.01131 \text{ m}^3/\text{s}$ $v_3 = Q_3 / A_3 = 0.01131 / [(\pi)(0.04)^2 / 4]$ $= 9.00 \text{ m/s}$ $\rho_{\text{alcohol}} A_1 v_1 + \rho_{\text{H}_2\text{O}} A_2 v_2 = \rho_{\text{mixture}} A_3 v_3$ $[(0.80)(998)][(\pi)(0.03)^2 / 4](6) + 998 [(\pi)(0.03)^2 / 4](10)$ $= (\rho_{\text{mixture}}) [(\pi)(0.04)^2 / 4](9.00)$ $\rho_{\text{mixture}} = 923 \text{ kg/m}^3$

5. (b)

6. (c) The bulk modulus of elasticity of fluid is defined as

$$K = -\frac{dp}{\frac{dv}{v}} \quad \dots (1)$$

 $\therefore v \propto \frac{1}{\rho} \text{ or } v\rho = \text{constant}$ where, $v = \text{volume}$ $\rho = \text{density}$

Differentiating both sides, we get

$$vdp + \rho dv = 0$$

$$\therefore \frac{dv}{v} = -\frac{dp}{\rho} \quad \dots (ii)$$

\therefore From equation (i) and (ii)

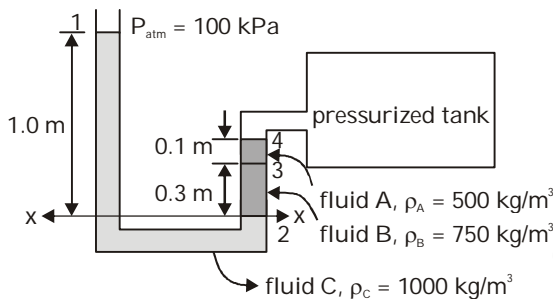
$$K = \frac{dp}{\left(\frac{dp}{\rho}\right)} = \frac{200 \times 10^4}{\left(\frac{0.1}{100}\right)}$$

$$= 200 \times 10^7 \text{ N/m}^2$$

$$= 2 \times 10^9 \text{ N/m}^2$$

$$= 2G \text{ N/m}^2$$

7. (b)



balancing pressure at section x-x

$$P_{atm} + \rho_C gh_1 = P + \rho_A gh_3 + \rho_B gh_2$$

$$\Rightarrow 100 \times 1000 + 1000 \times 9.81 \times 1$$

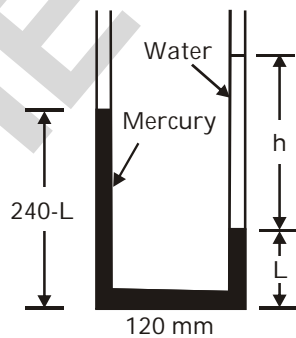
$$= P + 500 \times 9.81 \times 0.1 + 750 \times 9.81 \times 0.3$$

$$P = 107112.25 \text{ Pa}$$

$$= 107.11 \text{ kPa}$$

8. (c) After the water is poured, the orientation of the liquids will be shown in Figure below;

$$h = (12.0 \times 10^3 \text{ mm}^3) / \pi(5 \text{ mm})^2 = 152.8 \text{ mm}$$



$$\rho_w \times h + 13.6\rho_w \times L = 13.6\rho_w \times (240 - L)$$

$$\Rightarrow L = 114.38$$

$$\therefore \text{Height difference} = L + h - (240 - L)$$

$$= 141.2 \text{ mm}$$

9. (b)
10. (c)

$$h = x \left(\frac{G_{mercury}}{G_{sea \text{ water}}} - 1 \right) \text{ for a U-tube manometer}$$

$$= 0.17 \left[\frac{13.6}{1.026} - 1 \right] = 2.083 \text{ m of sea water}$$

$$\therefore \frac{P}{\gamma_{sea \text{ water}}} = \frac{V^2}{2g}$$

$$\frac{h \times \gamma_{sea \text{ water}}}{\gamma_{sea \text{ water}}} = \frac{V^2}{2g}$$

$$V = \sqrt{2gh}$$

$$V = \sqrt{2 \times 9.81 \times 2.083} = 6.39 \text{ m/sec}$$

11. (c)
12. (c)

$$(2) 1800 \text{ millibars} = 1800 \times 10^{-3} \times 10^5 \text{ Pa}$$

$$= 1.8 \times 10^5 \text{ Pa}$$

$$(4) 150 \text{ kPa} = 150 \times 10^3 \text{ Pa} = 1.5 \times 10^5 \text{ Pa}$$

$$(1) 20 \text{ m of water} = 9810 \times 20 = 196200 \text{ Pa}$$

$$= 1.96 \times 10^5 \text{ Pa}$$

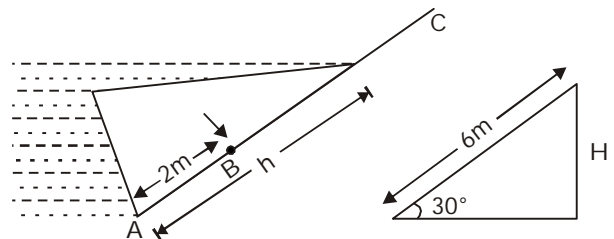
$$(3) 1240 \text{ mm of mercury}$$

$$= 13.6 \times 9810 \times 1.24 = 165,436 \text{ Pa}$$

$$= 1.65 \times 10^5 \text{ Pa}$$

13. (b) When C.G of force triangle will pass through B then flash board will be on verge of tipping

$$\text{so } \frac{h}{3} = 2.$$



$$h = 6\text{m}$$

$$\sin 30^\circ = \frac{H}{6}$$

$$H = 3\text{m}.$$

14. (b) Measuring distances from the water surface

Case (i)

$$C.G \rightarrow \frac{2h}{3}$$

$$C.P \rightarrow \frac{3h}{4}$$

$$\text{Distance moved by C.G} = \frac{2h}{3} - \frac{h}{3} = \frac{h}{3}$$

$$\text{Distance moved by C.P.} = \frac{3h}{4} - \frac{h}{2} = \frac{h}{4}$$

$$\text{Ratio} = \frac{h/3}{h/4} = \frac{4}{3} = 1.33.$$

15. (c)

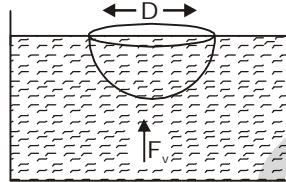
Vertical force

= wt of water contained in the hemisphere

$$\Rightarrow \frac{1}{2} \times \frac{4}{3} \pi R^3 \rho g$$

$$\Rightarrow \frac{2}{3} \pi \frac{D^3}{8} \rho g$$

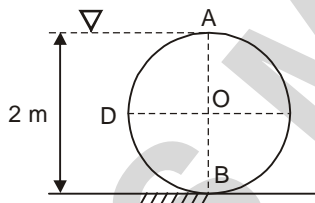
$$\Rightarrow \frac{1}{12} \rho \pi D^3 g$$



16. (c)

17. (c)

18. (c)



The vertical force acting on the cylindrical gates,
= weight of water displaced by section ADB

$$\begin{aligned} &= \rho g V = \rho g \times A \times L = \rho g \times \frac{1}{2} \times \frac{\pi}{4} \cdot 2^2 \times 1 \\ &= \frac{\pi \rho g}{2} = \frac{\pi \times 1000 \times 9.81}{2} = 15409.5 \text{ N.} \end{aligned}$$

19. (b) For viscous force to come in picture, the fluid must be motion i.e. velocity gradient,

$$\therefore \tau = \mu \frac{du}{dy}$$

For stationary fluid

$$\frac{du}{dy} = 0$$

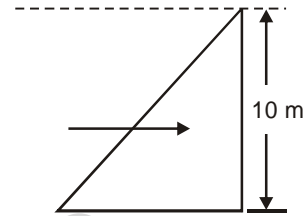
$$\therefore \tau = 0$$

20. (d)

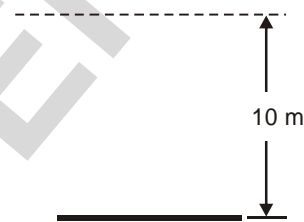
21. (c)

22. (a)

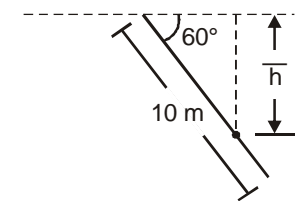
$$\therefore \text{Total force} = \gamma A \bar{h}$$



$$\begin{aligned} \text{total force} &= 10 \times (10 \times 1) \times 5 \\ &= 500 \text{ kN} \end{aligned}$$



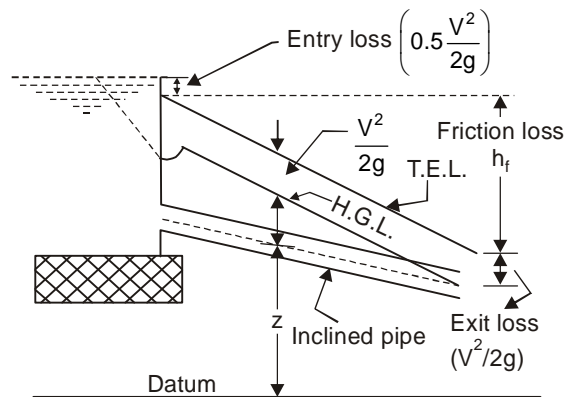
$$\text{total force} = 10 \times (10 \times 1) \times 10 = 1000 \text{ kN}$$



$$\bar{h} = \frac{10}{2} \sin 60^\circ = 4.33 \text{ m}$$

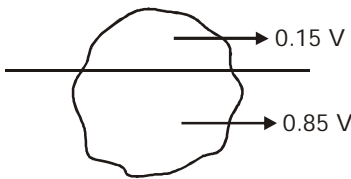
$$\begin{aligned} \therefore \text{total force} &= 10 \times (10 \times 1) \times 4.33 \\ &= 433 \text{ kN} \end{aligned}$$

23. (c)



The vertical distance between the datum and the hydraulic grade line is equal to the piezometric head $\left(\frac{p}{\gamma} + z\right)$. The energy grade line (or total energy line) will be parallel to the hydraulic grade line with a vertical distance between them equal to $\frac{V^2}{2g}$.

24. (a)



Let volume of ice berg = V
 Let specific weight of ice be w
 from Archimede's principle
 weight of ice berg = wt. of sea water displaced
 $V \times w = 0.85 V \times 10.5$

$$w = 8.925 \text{ kN/m}^3$$

25. (d)

26. (d) The period of rolling, is inversely proportional to metacentric height. So by increasing it, period of oscillation reduces. At the same time the frequency increases with increase in metacentric height. The period of oscillation-

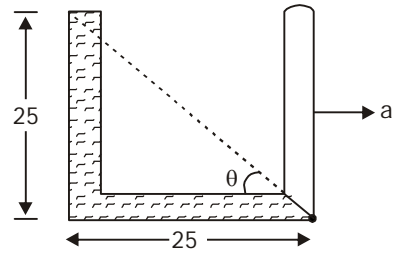
$$T = 2\pi \sqrt{\frac{k^2}{g \times GM}}$$

By adding load below centre of gravity, the centroid comes down and centre of buoyancy goes up. Due to this centre of buoyancy upward movement, metacentre in pitching comes down and height of metacentre in pitching reduces and frequency of oscillation in pitching reduces.

Every oscillation has its own metacenter.

$$\left[\begin{array}{l} (GM = BM - BG) \\ GM \downarrow = \frac{I}{V} \downarrow - BG \uparrow \end{array} \right]$$

27. (b) For atmosphere pressure at c,



$$\tan \theta = \frac{a}{g} = 1$$

$$a = g.$$

28. (a)

- The forced vortex motion is essentially a rotational motion because every fluid particle in such a motion is also found to rotate about its own axis as it moves along the curved path.
- The velocity of flow in a free vortex motion varies inversely with radial distance from the centre of vortex motion.
- Flow-field of a free vortex motion is everywhere irrotational except at the axis and therefore the free vortex motion is also called irrotational vortex motion.
- In free vortex, the streamlines are concentric circles

29. (b)

$$P = \rho gh$$

$$100 = 1 \times 10 \times h$$

$$h = 10\text{m}$$

$$V = C_v \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 10}$$

$$= 10\sqrt{2} = 14.14\text{m/sec}$$

30. (d) For flow to be possible

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$$

$$a + d = 0$$

for irrotational flow $\frac{\delta v}{\delta x} = \frac{\delta u}{\delta y}$

$$c = b.$$

31. (c)

$$\text{Local acceleration, } a_L = \frac{\partial V}{\partial t} = 2 \times \left(1 - \frac{x}{2L}\right)^2$$

$$= 2 \times \left(1 - \frac{0.5}{2 \times 0.8}\right)^2 = 0.945 \text{ m/s}^2$$

$$\text{Convective acceleration, } a_c = V \times \frac{\partial V}{\partial x}$$

$$= 2t \left(1 - \frac{x}{2L}\right)^2 \times 4t \times \left(1 - \frac{x}{2L}\right) \times \left(-\frac{1}{2L}\right)$$

$$= -\frac{4t^2}{L} \left(1 - \frac{x}{2L}\right)^3 = -14.62 \text{ m/s}^2$$

32. (a)

The velocity potential, ϕ is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

The stream function ψ is defined as a scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

33. (b) $P_A = P_B + v^2/2g$

34. (d)

$$q = \left(\frac{1}{2} \times V_{\max} \times \frac{2}{3} b \times 1\right) - \left(\frac{1}{2} \times \frac{V_{\max}}{2} \times \frac{1}{3} b \times 1\right)$$

$$= \frac{b V_{\max}}{3} - \frac{b V_{\max}}{12}$$

$$\frac{q}{b} = \frac{V_{\max}}{4}$$

$$v_{\text{mean}} = 0.25 V_{\max} \\ = 0.25.$$

35. (d)

36. (c) **Uniform flow:** When the velocity of flow of fluid doesnot change, both in magnitude and direction, from point to point in the flowing fluid for any given instant of time, the flow is said to be uniform. In Mathematical form,

$$\frac{\partial V}{\partial S} = 0$$

Irrotational flow: A flow is said to be irrotational if the fluid particles while moving in the

direction of flow do not rotate about their mass centre. It may however be stated that a true irrotational flow exists only in the case of flow of an ideal fluid for which no tangential or shear stresses occur. But the flow of practical fluids, may also be assumed to be irrotational if the viscosity of the fluid has little significance.

37. (b) In a pitot tube, the velocity of the stream is given by

$$v = C\sqrt{2g\Delta h}$$

$$\Delta h = y \left(\frac{S_w}{S_{\text{air}}} - 1\right)$$

$$= 0.012 \left(\frac{1000}{1.2} - 1\right) \\ = 9.988 \text{ m}$$

$$V = \sqrt{2g \times 9.98} \\ = 14 \text{ m/sec}$$

38. (c)

Given definition is for coanda effect.

- When flow past a bluff body generates a vortex street in the wake region (this being the well-known Karman vortex street).
- The Bauschinger effect refers to a property of materials stress/strain characteristics change as a result of the microscopic stress distribution of the materials.
- The force exerted on a rapidly spinning cylinder or sphere moving through fluid in a direction at an angle to the axis of spin is called magnus effect.

39 (d)

$$u = 2x - 3y \text{ and}$$

$$v = -3x - 2y$$

$$\therefore u = -\frac{\partial \phi}{\partial x} \Rightarrow \frac{\partial \phi}{\partial x}$$

$$= -(2x - 3y)$$

$$= -2x + 3y$$

$$\phi = -x^2 + 3xy + f(y) \quad \dots(i)$$

again, $\frac{\partial \phi}{\partial y} = -v = 3x + 2y$

$$\therefore \phi = 3xy + y^2 + f(x) \quad \dots(ii)$$

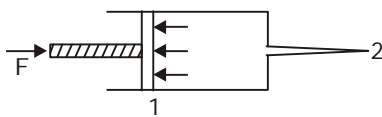
From eq (i) & (ii)

$$\phi = -x^2 + 3xy + y^2$$



40. (d) It is more convenient to express the momentum flux flowing through any cross-section in terms of the mean velocity of flow. But the actual momentum flux is always greater than that computed by using the mean velocity of flow. Hence in order to account for this difference in the values of the momentum flux due to the non-uniform velocity distribution at any cross-section a factor called momentum correction factor is introduced so that the momentum flux computed by using the mean velocity which would be equal to the actual total momentum flux through the entire cross-section.

41. (b)



$$F = P_1 A \quad \dots (i)$$

$$\Rightarrow A_1 V_1 = A_2 V_2$$

$$\Rightarrow 0.2 \times 100 \times 1 \times 10^{-2} = 0.07 \times V_2$$

$$V_2 = \left(\frac{0.2}{0.07} \right) = \frac{20}{7} \text{ m/s} \quad \dots (ii)$$

\(\Rightarrow\) Applying Energy Equation at (i) and (ii)

$$\Rightarrow \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

$$\Rightarrow P_1 = \frac{\rho}{2}(v_2^2 - v_1^2) = \frac{1000}{2} \left[\left(\frac{20}{7} \right)^2 - (10^{-2})^2 \right]$$

$$= 4081.58 \text{ N/m}^2$$

$$F = P_1 \times A = 4081.58 \times 0.2 \times 10^{-4} \text{ N}$$

$$= 8.164 \times 10^{-3} \text{ kg}$$

42. (d)

43. (a)

$$Q = \sqrt{2gH} \cdot \frac{\pi d^2}{4} \times C_d$$

$$\Rightarrow \frac{2}{1000} = \sqrt{2gH} \times \frac{\pi}{4} \times \left(\frac{1.5}{100} \right)^2 \times 0.6$$

$$\Rightarrow H = 18.13 \text{ m}$$

pressure of air = p

$$p = \rho_w g(18.13 - 1) = 168 \text{ kN/m}^2$$

44. (c)

45. (b) Just check by dimensions

$$\tau_0 = \mu \left(\frac{du}{dy} \right).$$

46. (c) Pressure drop = $P_1 - P_2 = \frac{32\mu VL}{D^2}$

$$V = \frac{Q}{A} = \frac{800 \times 4}{\pi \times (0.5)^2} = 4.07 \text{ m/s.}$$

$$2 \times 10^6 = \frac{32 \times \mu \times 4.07 \times 2}{(0.5 \times 10^{-3})^2}$$

$$\mu = 0.0019 \text{ N-s/m}^2$$

47. (a)

Since pressure drop in flow is directly proportional to friction loss

$$\therefore \Delta P \propto h_f \text{ and } h_f = \frac{4fL}{D} \times \frac{V^2}{2g}$$

For laminar flow, the friction factor

$$f = \frac{16}{\text{Re}} = \frac{16\mu}{\rho VD}$$

$$\therefore h_f = \frac{64\mu\ell}{\rho VD^2} \times \frac{V^2}{2g}$$

$$h_f = \frac{64\mu\ell}{2\rho g D^2} \times V$$

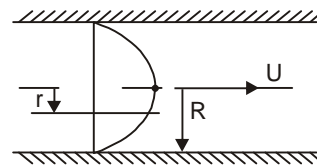
$$\therefore h_f \propto V$$

For laminar flow $\Delta P \propto V$.

Hence flow is laminar.

48. (b)

Assuming the flow in pipe is laminar.



\(\therefore\) Shear stress at radius r ,

$$\frac{\tau}{r} = - \left(\frac{\partial P}{\partial x} \right) = \frac{\tau_0}{R} = \text{constant}$$

The pressure drop across length of the pipe

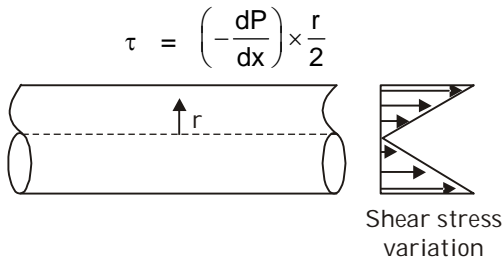
$$\tau_0 = - \frac{R}{2} \left(\frac{\Delta P}{\Delta x} \right)$$

$$= \frac{50 \times 10^{-3}}{2} \times \frac{50 \times 10^3}{10}$$

$$= \frac{250}{2} \text{ N/m}^2$$

$$= 0.125 \text{ kPa}$$

49. (c)

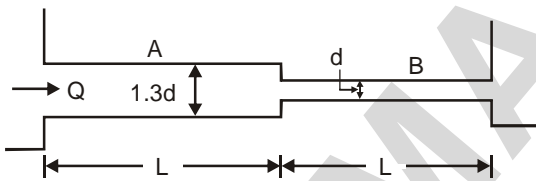


As can be seen from the figure, shear stress is maximum at the pipe wall.

50. (c)

51. (b)

52. (d)



$$\therefore h_{fA} = \frac{fLQ^2}{12.1(1.3d)^5}$$

and

$$h_{fB} = \frac{fLQ^2}{12.1(d)^5}$$

$$\therefore \frac{h_{fA}}{h_{fB}} = \frac{d^5}{(1.3d)^5}$$

$$= \frac{1}{1.3^5} = 0.2693$$

$$\approx 0.27$$

53. (a)

54. (c) For laminar flow,

By Hagen-Poiseuille equation

$$\text{Head loss, } h_f = \frac{32\mu VL}{\gamma D^2} \Rightarrow \frac{h_f}{L} = \frac{32\mu V}{\gamma D^2}$$

$$\therefore 1 = \frac{32\mu V}{\gamma D^2}$$

$$\text{and } \frac{h_f}{L} = \frac{32\mu(2V)}{\gamma D^2} = 1 \times 2 = 2 \text{ m}$$

55. (a)

$$\frac{u_{\max}}{u_{\text{avg}}} = 1.33\sqrt{f} + 1$$

$$\frac{3.635}{3} = 1.33\sqrt{f} + 1$$

$$f = 0.027$$

56. (c)

$$h \propto \frac{1}{d}$$

$$3 \times 1 = 0.2 \times h$$

$$h = 15 \text{ cm} = 150 \text{ mm}$$

57. (d)

$$u_* = \sqrt{\tau_0/\rho}$$

$$\frac{u}{u_*} = \sqrt{\frac{8}{f}}$$

$$\frac{u^2}{u_*^2} = \frac{8}{f}$$

$$\frac{\rho u^2}{\tau_0} = \frac{8}{f}$$

$$\tau_0 = \frac{\rho f u^2}{8} = \frac{1.2 \times 0.02 \times (20)^2}{8} = 1.2 \text{ N/m}^2$$

58. (*)

59. (c)

$$R_e = \frac{\rho V x}{\mu}$$

$$3.2 \times 10^5 = \frac{1000 \times 3 \times x}{0.001}$$

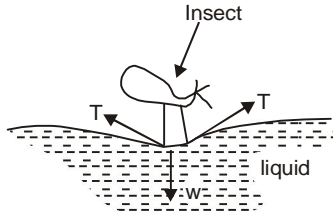
$$x = 0.1066 = 10.67 \text{ cm} \approx 11 \text{ cm}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{R_e}}$$

$$\delta = \frac{5 \times 10.67}{\sqrt{3.2 \times 10^5}} = 0.095 \approx 0.1 \text{ cm}$$

60. (c)

61. (b) The small insect can sit on liquid surface despite having higher density than liquid. The reason being surface tension as shown in figure.



The liquid has viscosity but no role in phenomenon.

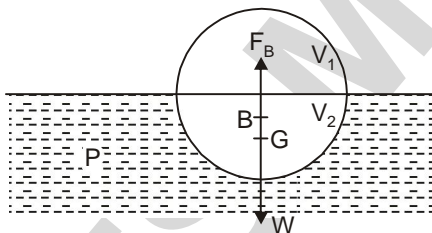
62. (a) Viscosity of liquids varies inversely with temperature so as the hot glue cools down (dry up) it's viscosity increases so movement of two blocks of wood wetted with hot glue would require greater effort.

63. (c) The depth of center of pressure,

$$\bar{h} = \frac{I_G}{AZ} + \bar{Z}$$

Hence independent of density. Center of pressure always lies below geometric center.

64. (a)



Buoyant force,

$$F_B = \rho V_2 g = W$$

V_2 = Volume of solid body submerged in liquid or volume of liquid displaced by solid body.

V_1 = Volume of solid body outside the liquid.

So F_B is weight of liquid displaced and always vertically up against the weight of body.

65. (d)

66. (a) A flow net is a graphical solution to the equations of steady ground water flow. A flow net consists of two sets of lines which must always be orthogonal (perpendicular to each other). Flow lines, which show the

direction of water flow, and equipotential lines (lines of constant head), which show the distribution of potential energy. Flow nets are usually constructed through trial and error sketching.

In an isotropic medium the hydraulic gradient has to be the steepest possible meaning thereby that the length of flow shall be the shortest. Thus, flow lines have to cross equipotential lines orthogonally. The space between two adjacent flow lines is called path and the figure formed on the flownet between any two adjacent flow lines and adjacent equipotential lines is referred to as field.

67. (d)

68. (d)

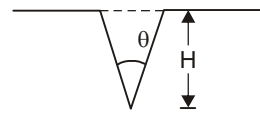
The discharge through a triangular weir is given by

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{5/2}$$

where C_d is the discharge coefficient

H is the head of flow

θ is the vertex angle



$$\text{Area of the triangular notch} = \frac{1}{2} (2H \tan \frac{\theta}{2}) H$$

$$= H^2 \tan \frac{\theta}{2}$$

$$\therefore \text{Avg. velocity} = \frac{Q}{A} = \frac{\frac{8}{15} C_d \sqrt{2g} H^{5/2} \tan \frac{\theta}{2}}{H^2 \tan \frac{\theta}{2}}$$

$$= \frac{8}{15} C_d \sqrt{2gH}$$

69. (a) The shear stress distribution in fully developed laminar flow in pipe,

$$\tau = -\frac{r}{2} \left(\frac{dP}{dx} \right)$$

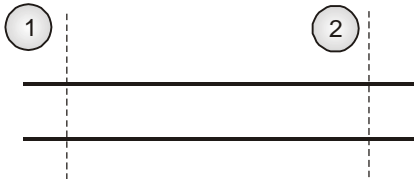
The velocity profile obtained by replacing τ

by $\mu \frac{du}{dr}$ results in parabolic form.

70. (a) $\frac{P}{\gamma} + \frac{V^2}{2g} + Z = \text{constant}$

Above equation is known as Bernoulli's equation in which the term $\frac{P}{\gamma}$ is known as the pressure head or static head which is a form of energy hence pressure intensity in a liquid is a form of energy.

Considering a horizontal pipe as shown in figure having steady uniform flow.



Applying energy equation between (1) and (2) for steady flow

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 =$$

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

where h_L is head loss or energy dissipated between section (1) and (2)

As the flow is uniform $\Rightarrow V_1 = V_2$

and pipe is horizontal $\Rightarrow Z_1 = Z_2$

$$\Rightarrow \frac{P_1}{\gamma} = \frac{P_2}{\gamma} + h_L$$

$$\frac{P_1 - P_2}{\gamma} = h_L - (Z_1 - Z_2) \dots (A)$$

Thus pressure gradient $\frac{(P_1 - P_2)}{\gamma}$ is not a function of energy dissipation alone. It also depends on datum head.

For example, if $h_L = Z_1 = Z_2$ then

$$\Rightarrow \frac{P_1 - P_2}{\gamma} = h_L$$

Means pressure gradient is a measure of the rate of energy dissipation and pressure intensity is a form of energy. So Reason is correct explanation of Assertion.

71. (c)

72. (b)

If ideal conditions are considered, the thickness of boundary layer will be ever increasing whether the boundary layer is laminar or turbulent. The boundary layer breaks when there is flow separation.

But in practice 99% depth of boundary layer develops within short distance from leading edge. This short distance depends upon flow and surface conditions.

73. (a) At outer layer of boundary layer,

$$\frac{du}{dy} = 0$$

So shear stress,

$$\tau = u \frac{du}{dy} = 0$$

74. (b) When two systems are geometrically, kinematically and dynamically similar then they are said to be completely similar or complete similitude exists between the two systems. The dynamic similarity implies geometric and kinematic similarities and hence if two systems are dynamically similar, they may be said to be completely similar. More over, for complete similitude to exist between the two systems viz. model and prototype, the dimensionless or π terms, formed out of the complete set of variables involved in that phenomenon.

Since inertia force always exists when any mass is in motion, the conditions for dynamic similarity are developed by considering the ratio of the inertia force and any one of the remaining forces. Each of these ratios will obviously be a non-dimensional factor.

Inertia and elasticity force ratio is Mach number

$$F_i = \rho L^2 V^2$$

$$F_e = \text{Bulk modulus of elasticity} \times \text{area} \\ = k \times A = k \times L^2$$

Where k is the bulk modulus of elasticity of the flowing fluid

$$\frac{F_i}{F_e} = \frac{\rho L^2 V^2}{k L^2} = \frac{V^2}{K/\rho} = \frac{V^2}{C^2}$$



Where $C = \sqrt{k/\rho}$, which represents the velocity of sound in that fluid medium whose

k and ρ are being considered. The ratio $\frac{V^2}{C^2}$

or $\frac{V^2}{\sqrt{k/\rho}}$ is known as Cauchy number. The

square root of this ratio i.e. $\frac{V}{C}$ or $\frac{V}{\sqrt{k/\rho}}$ is

known as Mach number (Ma).

- 75. (b)** Reynolds was the first to propose a criterion for differentiation between laminar and turbulent flows in his classic dye visualisation

$R_e = \frac{DV\rho}{\mu}$ and suggested a critical value of

$R_e = 2100$ for the upper limit of laminar flow.

All experimental and theoretical evidence points to the fact that the laminar flow state (which exists potentially for all flow rates) is linearly stable to any infinitesimal disturbance. The clear implication is that the observed transition process can then only be initiated by finite amplitude disturbances. Reynolds himself observed that turbulence was triggered by inlet disturbances to the pipe and the laminar state could be maintained to $R_e \approx 12,000$ if he took great care in minimizing external disturbances to the flow. By careful design of pipe entrances, laminar flow can be maintained upto Reynolds number of 40,000 and even more upto 1,00,000 by minimising ambient disturbances.

