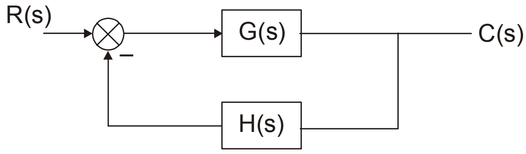
CLASSROOM PRACTICE TEST-01
CONTROL SYSTEM (SOLUTIONS)

ANSWER KEY

1	(c)	**14**	(c)	**27.**	(a)	**40.**	(c)	**53.**	(a)
2	(a)	**15**	(a)	**28.**	(c)	**41.**	(b)	**54.**	(c)
3	(b)	**16.**	(c)	**29.**	(a)	**42.**	(d)	**55.**	(c)
4	(b)	**17.**	(b)	**30.**	(b)	**43.**	(b)	**56.**	(b)
5	(d)	**18.**	(b)	**31.**	(a)	**44.**	(d)	**57.**	(c)
6	(b)	**19.**	(d)	**32.**	(a)	**45.**	(b)	**58.**	(b)
7	(a)	**20.**	(b)	**33.**	(c)	**46.**	(c)	**59.**	(b)
8	(d)	**21.**	(c)	**34.**	(a)	**47.**	(d)	**60.**	(c)
9	(c)	**22.**	(a)	**35.**	(c)	**48.**	(b)	**61.**	(b)
10	(b)	**23.**	(c)	**36.**	(c)	**49.**	(b)	**62.**	(c)
11.	(b)	**24.**	(b)	**37.**	(a)	**50.**	(b)	**63.**	(b)
12	(b)	**25.**	(b)	**38.**	(c)	**51.**	(d)	**64.**	(c)
13.	(d)	**26.**	(a)	**39.**	(b)	**52.**	(b)	**65.**	(b)

1. **(c)**

A control system with feedback taking negative feedback (widely used).



$$\text{Impulse redponse, } T = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Sensitivity w.r.t forward path parameter

$$S_G^T = \frac{dT}{T}\Big/\frac{dG}{G} = \frac{dT}{DG}\cdot\frac{G}{T}$$

$$\frac{dT}{dG} = \frac{1}{\left(1+G(s)H(s)\right)^2}$$

$$\therefore \quad S_G^T = \frac{1}{\left[1+G(s)H(s)\right]^2}\times\frac{G(s)}{G(s)}\left[1+G(s)H(s)\right]$$

$$\text{or} \quad S_G^T = \frac{1}{1+G(s)H(s)}$$

In the same manner, sensitivity w.r.t feedback path parameter.

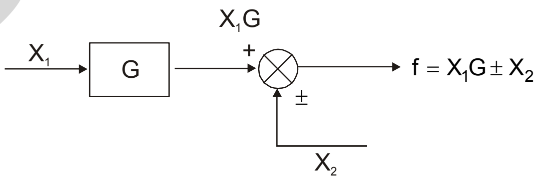
$$S_H^T = \frac{dT}{T}\Big/\frac{dH}{H} = \frac{dT}{dH}\frac{H}{T} = \frac{-G(s)}{1+G(s)H(s)}$$

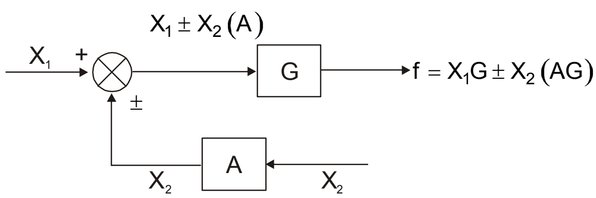
2. **(a)**

To have a differential equation of second degree, we should have all the 3 parameters in the circuit. i.e. R,L,C in electrical circuit and K,D,M in mechanical circuit.

Note: This condition is applicable when there is no missing term in the differential equation.

3. **(b)**





$$\text{As} \quad f = X_1G \pm X_2 = X_1G \pm X_2(AG)$$

$$\therefore \quad AG = 1$$

$$\therefore \quad \boxed{A = \frac{1}{G}}$$

4. **(b)**

By Mason's gain formula

$$T.F = \frac{\sum P_k \Delta_k}{\Delta}$$

$$P_1 = abdeg$$

$$\Delta_1 = 1$$

$$\Delta = 1-(bc+ef)+bcef$$

$$\therefore \quad TF = \frac{ab\,deg}{1-(bc+ef)+bcef}$$

5. **(d)**

$$T(s) = \frac{C(s)}{R(s)} = \frac{25k}{s^2+5s+25k}$$

Therefore,

$$S_K^T = \frac{\partial T}{\partial k}\cdot\frac{k}{T} = \frac{\partial}{\partial k}\left(\frac{25k}{s^2+5s+25k}\right)\times\frac{k}{T}$$

$$= \frac{25(s^2+5s+25k)-25k(25)}{(s^2+5s+25k)^2}\times\frac{k}{25k}(s^2+5s+25)$$

$$= \frac{s^2+5s}{s^2+5s+25k}$$

For normal value of k = 1

$$S_K^T = \frac{s(s+5)}{s^2+5s+25}$$

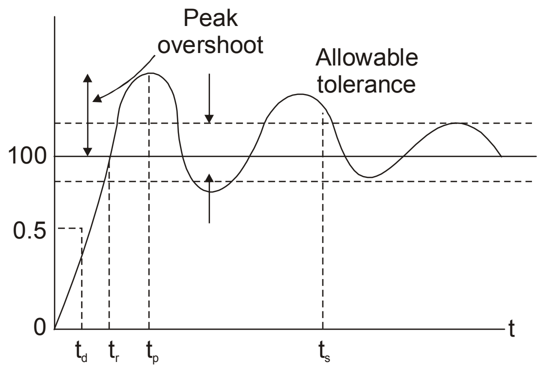
At ω = 5

$$\left|S_K^T\right| = \left|\frac{j5(5+j5)}{-25+j25+25}\right| = \sqrt{2} = 1.414$$

6. **(b)**

Rise time (t_r) : Overdamped system $\rightarrow$ time required to rise from 10 to 90% of the final value underdamped system-time required to rise from 0 to 100% of its final value.

for underdamped system



7. **(a)**

Closed loop transfer function of the system.

$$T(s) = \frac{\dfrac{k_v(K_D's+1)}{s(s\tau+1)}}{1+\dfrac{k_v(k_D's+1)}{s(s\tau+1)}}$$

Characteristic equation

$$s^2 + \left(\frac{1+k_v k_D'}{\tau}\right)s + \frac{k_v}{\tau} = 0$$

$$\therefore \quad \omega_n' = \sqrt{\frac{k_v}{\tau}} = \omega_n$$

$$\xi' = \frac{1+k_v k_D'}{2\sqrt{k_v \tau}} = \xi + \frac{k_D'}{2}\sqrt{k_v/\tau}$$

Without compensation

$$s^2 + \frac{1}{\tau}s + \frac{k_v}{\tau} = 0$$

$$\therefore \quad \omega_n = \sqrt{\frac{k_v}{\tau}}, \quad \xi = \frac{1}{2\sqrt{k_v \tau}}$$

From above it is clear that underdamped natural frequency remains same as that in uncompensated system.

Therefore, statement (1) is corrected. However, damping is increased by $\frac{K_D'}{2}\sqrt{K_V/\tau}$. Therefore, state (2) is not correct.

8. (d)

To obtain time response, transfer function is needed but in case of frequency domain analysis, frequency response can be obtained experimentally without having knowledge of transfer function.

For non rational T.F. i.e. e^{-Ts} etc. time domain methods are used after making such elements rational (using certain approximations) while if we know about the frequency response then step response can be predicated to a great extent. Step response means peak overshoot, rise time etc.

9. **(c)**

Addition of a non zero pole to T.F.

Let the pole is at s = $\frac{1}{T_1}$, then $s + \frac{1}{T_1}$ or $(j\omega T_1 + 1)$ will be the factor in denominator of T.F. effect on angle, can be expressed as

$$\phi = \angle T\cdot F - \tan^{-1}\left(\frac{\omega T_1}{1}\right)$$

at ω = 0, $\quad \phi = \angle T\cdot F$, No effect

at $\omega = \infty$, $\quad \phi = \angle T\cdot F - 90°$

Similarly for pole at origin (s = 0)

$$\phi = \angle T\cdot F - 90°$$

10. **(b)**

Due to phase lead compensation, gain crossover frequency increases, so bandwidth increases.

$$B\cdot W \propto \frac{1}{t_r}$$

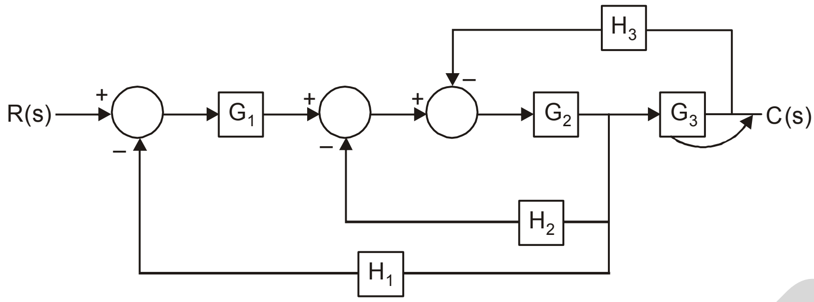
$\therefore$ rise time decreases.

Phase lead compensation improves transient resonse.

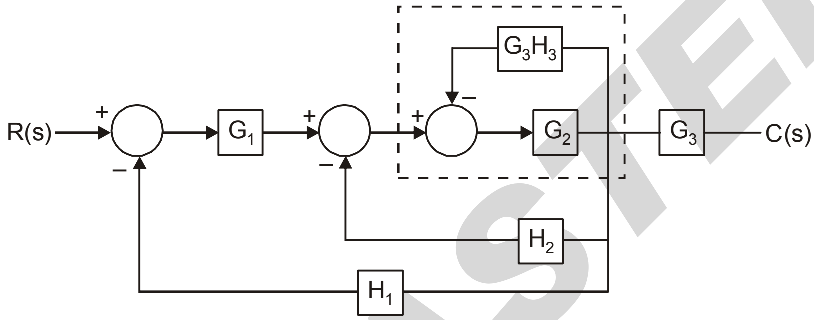
$\therefore$ ξ increases and hence peak overshoot M_p = $e^{-\pi\xi/\sqrt{1-\xi^2}}$ reduces.

11. (b)

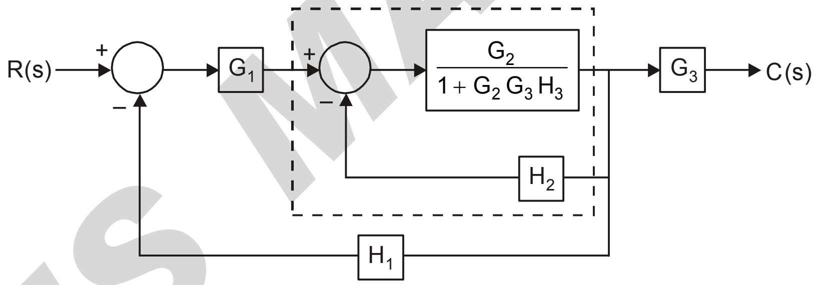
Given



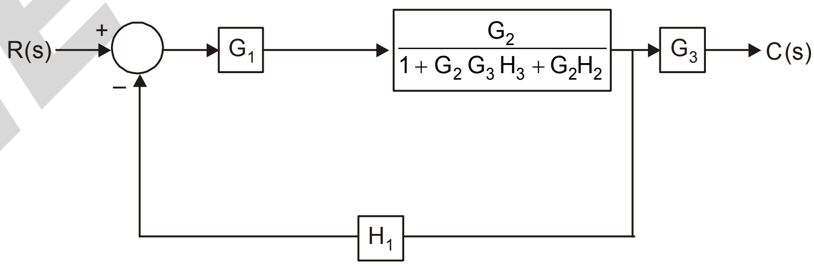
By taking $\boxed{G_3}$ right to the take off point



Solving for given inner loop above, we get



Solving for given another inner loop above, we get



So

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_2 G_1 H_1 + G_2 H_2 + G_2 G_3 H_3}$$

12. (b)

$$\frac{\omega_c(s)}{\omega_d(s)} = \frac{\frac{k}{s(s+2)}}{1+\frac{k}{s(s+2)}} = \frac{k}{s(s+2)+k}$$

$$\omega_d(s) = \frac{100}{s}$$

$$\therefore \quad \omega_c(\text{steady state}) = \lim_{s\to 0} s\left(\frac{k}{s(s+2)+k}\right)\times\frac{100}{s}$$ [From final value theorem $\lim_{t\to\infty}\omega_c(t) = \lim_{s\to\infty} s.\omega_c(s)$]

$$\text{or} \quad \omega_c(ss) = \lim_{s\to 0}\frac{100k}{s(s+2)+k} = \frac{100K}{K} = 100 \text{ rad/s}$$

13. (d)

The feedback paths are

1. $-G_1 H_1$
2. $-G_2 H_2$
3. $-G_3H_3$
4. $-G_4 G_5 H_4$
5. $-H_6$
6. $-G_2G_3G_4G_5H_5$
7. $G_6G_5H_5H_1$
8. $G_6G_5H_4H_3H_2H_1$

14. (c)

The circuit has two storage elements, so there are two state variables.

State variables i_L and e_C

By KCL; $$i_L = \frac{e_c}{R_2} + i_c$$

or $$i_c = i_L - \frac{e_c}{R_2} \quad ...(1)$$

and $$e = R_1 i_L + e_L + e_C$$

or $$e_L = e - R_1 i_L - e_C \quad ...(2)$$

As $e_L = L\frac{di_L}{dt}$

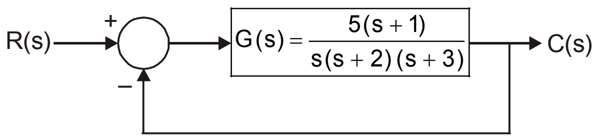
$$\therefore \quad \frac{di_L}{dt} = \frac{e_L}{L} = \frac{e}{L} - \frac{R_1}{L}i_L - \frac{e_c}{L} \quad ...(3)$$

Also, $i_c = c\frac{de_c}{dt}$

$$\therefore \quad \frac{de_c}{dt} = \frac{i_c}{c} = \frac{i_L}{c} - \frac{e_c}{R_2 c} \quad ...(4)$$

15. (a)

Given



$$G(s) = \frac{5(s+1)}{s(s+2)(s+3)}, \quad H(s) = 1$$

Position error const.: $$k_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0}\frac{5(s+1)}{s(s+2)(s+3)} = \infty$$

Velocity error const.: $k_v = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} \frac{s\cdot 5(s+1)}{s(s+2)(s+3)} = 5/6$

Acceleration error const.: $k_a = \lim_{s\to 0} s^2\,G(s)H(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = 0$

16. **(c)**

Characteristic equation is

$$1+G(s) = 0$$

Or $$1+\frac{k}{s(s+8)} = 0 \quad \text{Or} \quad s^2 + 8s + k = 0 \qquad \text{... (i)}$$

Optimum characteristic equation is

$$s^2 + 1.4\omega_n s + \omega_n^2 = 0 \qquad \text{... (ii)}$$

Comparing eq. (i) and (ii)

$$\omega_n = \sqrt{k}$$

and $$1.4\omega_n = 8$$

Thus, $$1.4\sqrt{k} = 8$$

Or $$k = \left(\frac{8}{1.4}\right)^2 = 32.65$$

17. **(b)**

With derivative feedback, the characteristic equation is

$$1+G(s) = 1+\frac{\frac{8}{s(s+2)}}{1+\frac{8as}{s(s+2)}} = 0$$

Or $$s^2 + (2s + 8a)\,s + 8 = 0$$

Comparing with standard characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$2\xi\omega_n = 2 + 8a$$

$$\omega_n^2 = 8$$

$\Rightarrow$ $$\omega_n = 2\sqrt{2}$$

Thus, $$2\times 0.7\times 2\sqrt{2} = 2+8a$$

Or $$a = \frac{2\times 0.7\times 2\sqrt{2} - 2}{8}$$

$$= 0.245$$

18. **(b)**

To obtain peak time,

$$C(t_p) = 1-\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t_p + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] = 1$$

$$\frac{dC(t)}{dt} = 0 \text{ gives, } \sin\left(\omega_n\sqrt{1-\xi^2}\right)t = 0$$

Therefore, the time to various peaks is given by

$$\omega_n\sqrt{1-\xi^2}\,t = 0, \pi, 2\pi, 3\pi \ldots$$

First peak (overshoot); $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

Second peak (undershoot); $t = \dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}}$

Third peak (second overshoot); $t = \dfrac{3\pi}{\omega_n\sqrt{1-\xi^2}}$

Rise time (t_r)

$$t_r = \frac{\pi - \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

For $\xi = 0$; $t_r = \dfrac{\pi - \dfrac{\pi}{2}}{\omega_n} = \dfrac{\pi}{2\omega_n}$

19. (d)

$$T(s) = \frac{\omega_0(s)}{V_r(s)} = \left[\frac{k_t k k_m}{(\tau' s+1)} \Big/ 1 + \frac{k k_t k_m}{(\tau' s+1)}\right]^{k_t}$$

$$T(s) = \frac{k k_t k_m}{\tau' s + 1 + k k_m k_t}$$

$$= \frac{\dfrac{k k_t k_m}{(1 + k k_t k_m)}}{\tau'' s + 1}$$

Where, $\tau'' = \dfrac{\tau'}{(1 + k k_m k_t)}$

Thus, $\tau'' = \dfrac{\tau'}{10} = \dfrac{\tau'}{1 + k k_m k_t}$

Or $1 + k k_m k_t = 10$

Or $1 + k\,(1.25)\,(0.2) = 10$

$$k = \frac{10-1}{1.25 \times 0.2} = 36$$

20. (b)

The number of asymptotes are

$$P - Z \text{ i.e } 4 - 1 = 3$$

P = Number of open loop poles

Z = Number of open loop zero

The angle of asymptotes are = $\dfrac{(2K+1)}{P-Z}180°$ where $K = 0, 1, 2 \ldots$

$$\alpha_1 = \frac{(2\times 0+1)}{4-1}\times 180^\circ = 60^\circ$$

$$\alpha_2 = \frac{(2\times 1+1)\times 180^\circ}{4-1} = 180^\circ$$

$$\alpha_3 = \frac{(2\times 2+1)\times 180^\circ}{4-1} = 300^\circ$$

21. **(c)**

The open-loop transfer function is

$$G(s)H(s) = \frac{k}{(s+1)(s+3)}$$

Characteristic equation is

$$HG(s)H(s) = 0$$

$$\therefore \quad 1+\frac{k}{(s+1)(s+3)} = 0$$

or $$(s+1)(s+3)+k = 0$$

or $$s^2+4s+3+k = 0$$

For $$k = 6.5$$

$$s^2+4s+9.5 = 0$$

Thus, $$\omega_n = \sqrt{9.5} = 3.082$$

$$2\xi\omega_n = 4$$

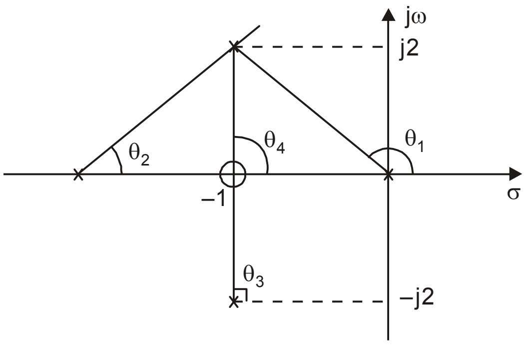
or $$\xi = \frac{4}{2\times\sqrt{9.5}} = 0.649$$

22. **(a)**

Given $$G(s) = \frac{K(s+1)}{s(s+2)(s^2+2s+5)}$$

the poles locations are

$$s(s) = 0, -2, -1-j2, -1+j2$$



$$\theta = \Sigma\,\theta_{zero} - \Sigma\,\theta_{pole} = \theta_4 - (\theta_1+\theta_2+\theta_3)$$

$$\theta = 90^\circ - (\theta_1+\theta_2+90^\circ) \quad \because\ (\theta_4 = \theta_3 = 90^\circ)$$

$$\theta = -(\theta_1+\theta_2)$$

$$\theta_1 = 180 - \tan^{-1}\left(\frac{2}{1}\right)$$

$$\theta_1 = 116.57°$$

$$\theta_2 = \tan^{-1}\left(\frac{2}{1}\right) = 63.43°$$

$$\theta = -(180°)$$

Angle of departure

$$\theta_D = 180° + \theta = 180° - 180° = 0°$$

23. **(c)**

The Routh's array is

s^3	1	$K+2$
s^2	3K	4
s^1	$\frac{3K^2+6K-4}{3K}$	X
s^0	4	X

For stability

$3K > 0$ i.e. $K > 0$

and $3K^2 + 6K - 4 > 0$ i.e. $K > -1 \pm 1.53$

The range of K is $K > 0.53$.

24. **(b)**

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

To check controllability:

$$Q_c = [B \ : \ AB]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\therefore \quad Q_c = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$|Q_c| = 1 - 1 = 0$$

rank is 1(<2) so system is not controllable.

To check observability:

$$Q_c = \left[C^T : A^T C^T\right]$$

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ A^T = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow |Q_0| = 1 \qquad \text{(rank = 2)}$$

System is observable.

25. (b)

$$H(s) = \frac{s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10000}\right)} = \frac{1}{\left(\frac{1}{s}+\frac{1}{10}\right)\left(1+\frac{s}{10000}\right)}$$

$$= \frac{10}{\left(1+\frac{10}{s}\right)\left(1+\frac{s}{10000}\right)} = \frac{10}{\left(1+\frac{10}{j\omega}\right)\left(1+\frac{j\omega}{10000}\right)}$$

For $10 < \omega < 10000$

$$H(s) = 20\log 10$$

$$= 20 \text{ dB}$$

26. (a)

$$G(s) = \frac{K}{s+a}$$

$$G(0) = \frac{K}{a} = 20\text{dB} = 10$$

From plot, $a = 10$

$\Rightarrow$ $K = 10 \times 10 = 100$

27. (a)

$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
1.3	–130°
1.2	–140°
1.0	–150°
0.8	–160°
0.5	–180°
0.3	–200°

at $\angle G(j\omega) = -180°$, $|G(j\omega)| = 0.5$

$\therefore$ G.M. (in dB) $= -20 \log (0.5) = 6.02$ dB

$\because$ At $|G(j\omega)| = 1$, $\angle G(j\omega) = -150°$

P.M $= 180° + \phi = 30°$

28. (c)

Given differential equation are

$$\dot{x}_1 = -x_1 + u$$

$$\begin{bmatrix} \dot{x} = Ax + Bu \rightarrow \text{state equation} \\ y = Cx + Du \rightarrow \text{output equation} \end{bmatrix}$$

and $\dot{x}_2 = x_1 - 2x_2 + u$

considering x_1 as output i.e. $x_1 = y$

$$\dot{x}_1 = -y + u$$

and $\dot{x}_2 = y - 2x_2 + u$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For controlability,

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} \text{ must have rank}$$
$$= 2$$

$$Q_C = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$|Q_C| = -2$$

Rank = 2

Hence output is contrallable.

Again let $x_2 = y$

then $\dot{x}_1 = -x_1 + u$

$$\dot{x}_2 = x_1 - 2y + u$$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$|Q_C| = 1 + 1 = 2$$

Hence Rank = 2 $\rightarrow$ controlable

29. (a)

- No branch terminates at finite zero, so system does not has any finite zero.
- There are three asymptotes for the given system and their intersection is origin i.e. centroid is origin

$$\text{Centroid} = \frac{\sum \text{real part of open loop poles} - \sum \text{real part of open loop zeroes}}{P - Z}$$

$$= \frac{\sum \text{real part of open loop poles}}{P}$$

⇒ Real part of the open loop poles = 0

- Break away point is also at origin

∴ All the three poles are at origin, hence the open loop transfer function for the given root locus will be

$$G(s)H(s) = \frac{K}{s^3}$$

30. (b)

Consider frequencies

- ω_1 = 2.5, change in slope = –40 – (–20) = –20 db/dec, so at ω_1 = 2.5 one pole exist
- Initially slope is –20 dB/dec so one pole exist at origin
- ω_3 = 40, change in slope is = –60–(–40) = –20 db/dec

  so one pole exist at ω_3 = 40 rad/sec.

$$G(s) = \frac{K}{s\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_3}\right)} = \frac{K}{s\left(1+\frac{s}{2.5}\right)\left(1+\frac{s}{40}\right)} = \frac{2.5\times 40\ k}{s(2.5+s)(40+s)}$$

where K = open loop gain

For line whose slope is –20 db/dec

$$y = m(\log \omega) + 20 \log k$$

At ω = 2.5, y = 40 db, m = –20 db/dec.

so 40 = –20 log (2.5) + 20 log K

⇒ K = 250

$$\Rightarrow \quad G(s) = \frac{250\times 2.5\times 40}{s(s+2.5)(s+40)}$$

$$\boxed{G(s) = \frac{25000}{s(s+2.5)(s+40)}}$$

31. (a)

Relative stability here means position of closed loop poles and consequently the peak overshoot, settling time, etc.

By factorisation of characteristic equation closed loop poles can be obtained but it becomes much laborious when degree of characteristic equation is three or higher. Root locus gives the location of closed loop poles with the variation of system parameter. Thus by analysing root locus, system parameter can be adjusted to a level to get desired transient response.

32. (a)

Angle of asymptotes

$$\phi = \frac{(2a+1)180^\circ}{p-z}, \quad a = 0,1,2, ...$$

if number of poles increases then (p–z) increases, hence, ϕ reduces. Asymptotes move towards RHS s-plane i.e root locus is pushed towards RHS s-plane.

33. (c)

Phase angle is not affected by variation of gain of the system. But when gain is varied, GCOF will change hence PM will be changed.

$$PM = 180° + \phi_{\text{at GCOF}}.$$

This will change when GCOF changes.

34. (a)

Overshoot or undershoot = $e^{-\frac{n\pi\delta}{\sqrt{1-\delta^2}}}$

for overshoots, n = 1, 3, 5, 7, ...

for undershoots, n = 2, 4, 6, 8,

35. (c)

Impulse response is the response when the unit impulse input is applied to it when circuit is an initially relaxed condition. Impulse response of circuit will contain only natural response terms. The differencing and differentiation operations will not eliminate the constant terms as the constant terms emerge based on the type of input not on the differentiation (or) differencing operations.

36. (c)

Given Open-loop transfer function

So characteristic equation is

$$1 + G(s)\,H(s) = 0$$

$$G(s) = \frac{K(s-2)}{(s+1)^2}$$

given $H(s) = 1$

$$\Rightarrow \quad 1 + \frac{k(s-2)}{(s+1)^2} = 0$$

$$\Rightarrow \quad (s+1)^2 + k(s-2) = 0$$

$$\Rightarrow \quad s^2 + 2s + 1 + ks - 2k = 0$$

$$\Rightarrow s^2 + s(k+2) + 1 - 2k = 0$$

The routh's tabulation is

s^1	1	1–2k
s^1	k+2	0
	1–2k	

For stability $K + 2 > 0$

$\Rightarrow \quad k > -2$ ...(1)

$\Rightarrow \quad 1 - 2k > 0 \Rightarrow k < \frac{1}{2}$ ...(2)

so from (1) and (2)

$$-2 < k < \frac{1}{2} \text{ system is stable}$$

37. **(a)**

Corner frequencies are at 2.5, 10 and 25. Thus, transfer function will be

$$\text{T.F.} = \frac{k\left(\frac{s}{2.5}+1\right)\left(\frac{s}{10}+1\right)}{s\left(\frac{2}{25}+1\right)}$$

For first asymptote

$$20 \log k - 20 \log w = -20 \text{ at } w = 2.5$$

(**Note :** 6 db/octave = 20 db/decade)

Thus, $$20\log\left(\frac{k}{2.5}\right) = -20$$

or $$k = 0.25$$

Hence, $$\text{T.F.} = \frac{0.25\left(\frac{s}{2.5}+1\right)\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{25}+1\right)}$$

or $$\text{T.F.} = \frac{0.25(s+2.5)(s+10)}{s(s+25)}$$

38. **(c)**

At gain crossover frequency

$$|G(j\omega)| = 1$$

Given, $$G(s) = \frac{ks^2}{(1+0.2s)(1+0.2s)}$$

$$= \frac{ks^2(5\times 50)}{(s+5)(s+50)}$$

$\therefore$ $$G(j\omega) = \frac{k(j\omega)^2(250)}{(j\omega+5)(j\omega+50)}$$

$$|G(j\omega)| = \frac{k(250)\omega^2}{\sqrt{25+\omega^2}\sqrt{2500+\omega^2}}$$

at $\omega = 5$ $$\frac{k(250)(5^2)}{\sqrt{25+25}\,\sqrt{2500+25}} = 1$$

$\therefore$ $$k = 0.05685$$

or $$k = 0.057$$

39. **(b)**

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Solution of homogenous equation is $x(t) = \phi(t)x_0$

where, $\phi(t) = L^{-1}[(sI - A)^{-1}]$

$$sI - A = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s-1)^2}\begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$x(t) = \phi(t)x_0 = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

40. **(c)**

Since we know

$$\dot{x}(t) = A\,x(t) + Bu(t) \quad \ldots(1)$$

taking laplace transform both side

$$s\,X(s) - X(0) = A\,X(s) + Bu(s)$$

$$(sI - A)\,X(s) = BU(s) + X(0)$$

$$X(s) = (sI - A)^{-1}X(0) + (sI - A)^{-1}BU(s)$$

taking inverse laplace

$$X(t) = L^{-1}[(SI-A)^{-1}X(0)] + L^{-1}(sI - a)^{-1}BU(s)] \quad \ldots(2)$$

given state equation as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t) \quad \ldots(3)$$

From eqn (1) and (3), on comparing

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Therefore

$$sI-A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

The inverse matrix of sI–A is

$$(sI-A)^{-1} = \frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$\phi(t) = L^{-1}\left[(sI-A)^{-1}\right]$$

$$\phi(t) = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}$$

and

$$(sI-A)^{-1}BU(s) = \frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{1}{s}$$

$$= \frac{1}{s^2+3s+2}\begin{bmatrix} \frac{1}{s} \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad L^{-1}\left[(sI-A)^{-1}BU(s)\right] = \begin{bmatrix} 0.5-e^{-t}+0.5e^{-2t} \\ e^{-t}-e^{-2t} \end{bmatrix};\ t \geq 0 \qquad \text{...(4)}$$

and

$$L^{-1}\left[(sI-A)^{-1}\right]X(0) = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{2t} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t}-e^{-2t} \\ -e^{-t}+2e^{-2t} \end{bmatrix} \qquad \text{...(5)}$$

From eqn (2), (4) and (5)

$$x(t) = \begin{bmatrix} e^{-t}-e^{-2t} \\ -e^{-t}+2e^{-2t} \end{bmatrix} + \begin{bmatrix} 0.5-e^{-t}+0.5e^{-2t} \\ e^{-t}-e^{-2t} \end{bmatrix}$$

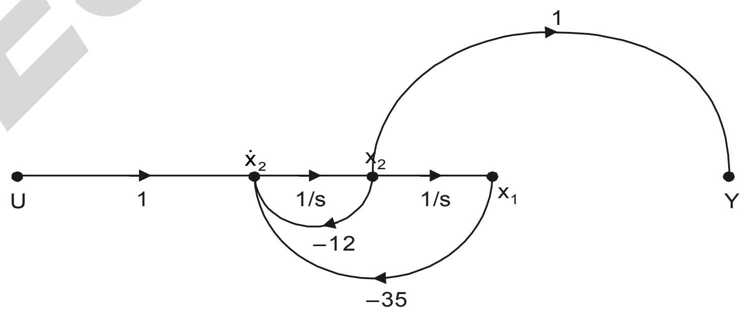
$$x(t) = \begin{bmatrix} 0.5-1.5e^{-2t} \\ e^{-2t} \end{bmatrix}$$

41. **(b)**

Given function is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s}{(s+5)(s+7)} = \frac{s}{s^2+12s+35} = \frac{1/s}{1+\frac{12}{s}+35/s^2}$$

signal flow graph of G(s) in phase variable form



assuming node x_1, x_2 and $\dot{x}_2$ as given an signal flow graph above, we get

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -35x_1 - 12x_2 + u$$

so

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -35 & -12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$$

42. **(d)**

Lag compensator transfer function

$$G_c(s) = \frac{\tau s+1}{\beta\tau s+1}, \beta > 1$$

at low frequencies : Gain is 1.

att higher frequencies : Gain is $\frac{1}{\beta}$ i.e., it attenuates the higher frequencies, hence it is similar to low pass filter.

It also improves the steady state response due to addition of a dominant pole similar to PI controller.

43. (b)

$$G_c(j\omega) = \alpha\left(\frac{1+j\omega z}{1+j\omega\alpha\tau}\right); \alpha < 1$$

At zero frequency $|G_c(j\omega)| = \alpha$

At any frequency, for phase-lead network

$$\phi = \tan^{-1}\omega z - \tan^{-1}\alpha\omega z$$

$$\frac{d\phi}{d\omega} = 0 = \frac{z}{1+(\omega z)^2} - \frac{\alpha z}{1+(\alpha\omega z)^2}$$

or $$\omega^2 z^2\alpha(\alpha-1) = \alpha - 1$$

or $$\omega_m = \omega = \frac{1}{\tau\sqrt{\alpha}}$$

At $\omega = \omega_m$

$$\tan\phi_m = (1-\alpha)\big/2\sqrt{\alpha} \text{ or } \sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\therefore \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

44. (d)

Given,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

If zero at s = –z is added to above system.

$$\frac{C(s)}{R(s)} = \frac{(s+z)(\omega_n^2)}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\frac{C}{R}(0) = \frac{z\omega_n^2}{\omega_n^2} = z$$

therefore to keep $\frac{C}{R}(0)$ to unity, $\frac{C(s)}{R(s)}$ is adjusted to

$$\frac{C(s)}{R(s)} = \frac{(s+z)\left(\frac{\omega_n^2}{z}\right)}{s^2+2\xi\omega_n s+\omega_n^2}$$

or $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} + \frac{s}{z}\left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\right)$

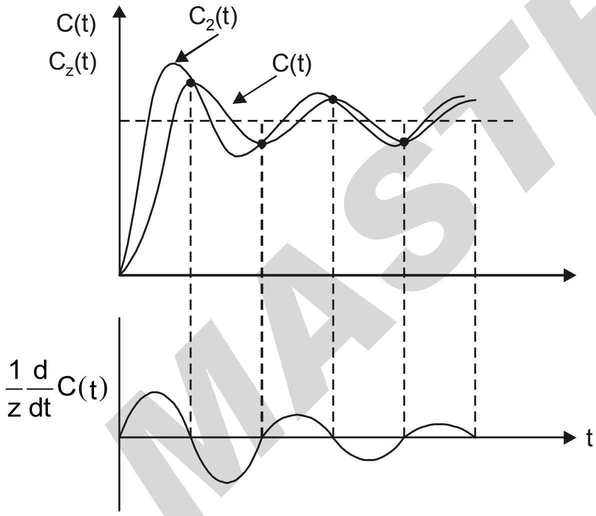
Let $C_2(t)$ be the response of the system with zero at s = –2, then

$$C_2(t) = C(t) + \frac{1}{z}\frac{d}{dt}C(t)$$

$$\because \quad \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \to C(t)$$

$$\& \; C(s) \to \frac{d}{dt}c(t)$$

where $C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$



45. (b)

$$T(s) = \frac{C(s)}{R(s)} = \frac{\alpha/s(s+1)}{1+[\alpha/s(s+1)]}$$

$$= \frac{\alpha}{s(s+1)+\alpha}$$

Sensitivity,

$$S_\alpha^T = \frac{\partial T}{\partial \alpha}\cdot\frac{\alpha}{T}$$

or $S_\alpha^T = \frac{s(s+1)+\alpha-\alpha}{[s(s+1)+\alpha]^2}\cdot\frac{\alpha}{\alpha}[s(s+1)+\alpha]$

$$= \frac{s(s+1)}{s(s+1)+\alpha}$$

At $s = j\omega = j1$

$$\left|S_\alpha^T(j1) = \left|\frac{j1(j1+1)}{j1(j1+1)+\alpha}\right| = \left|\frac{-1+j1}{(\alpha-1)+j1}\right| = 1\right|$$

or $\frac{\sqrt{2}}{\sqrt{(\alpha-1)^2+1}}=1$

or $(\alpha-1)^2+1=2$

or $(\alpha-1)^2=1$

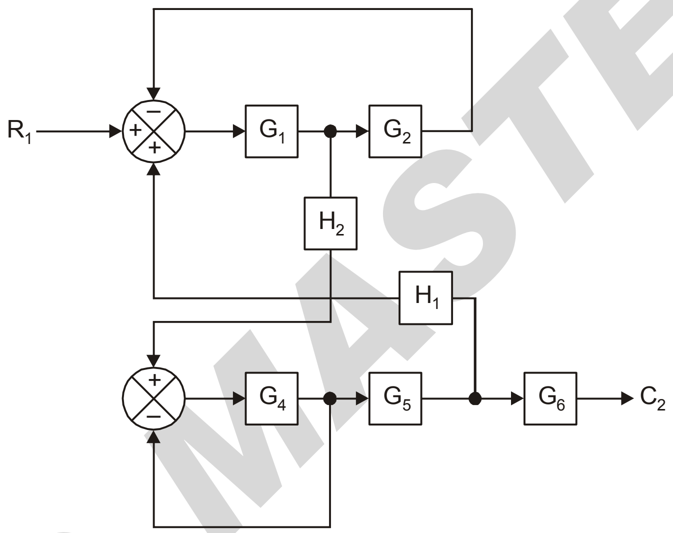
$\alpha-1=\pm 1$

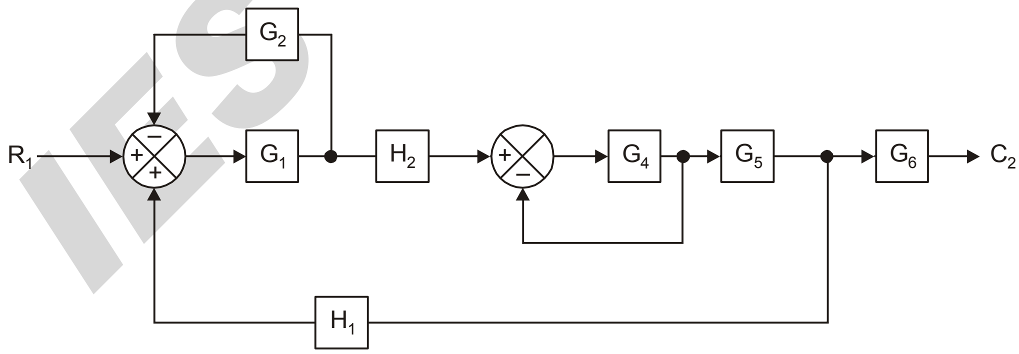
$\alpha=+1\pm 1=0$ or 2

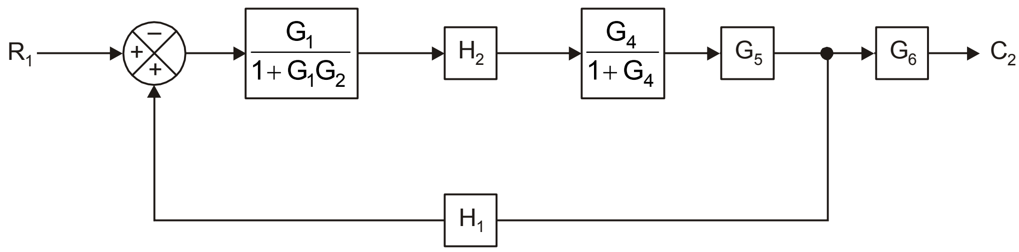
$\alpha=2$ (non zero value)

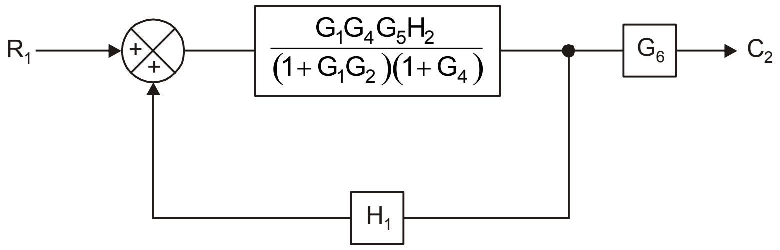
46. (c)

To obtain $\left.\frac{C_2}{R_1}\right|_{R_2=0}$





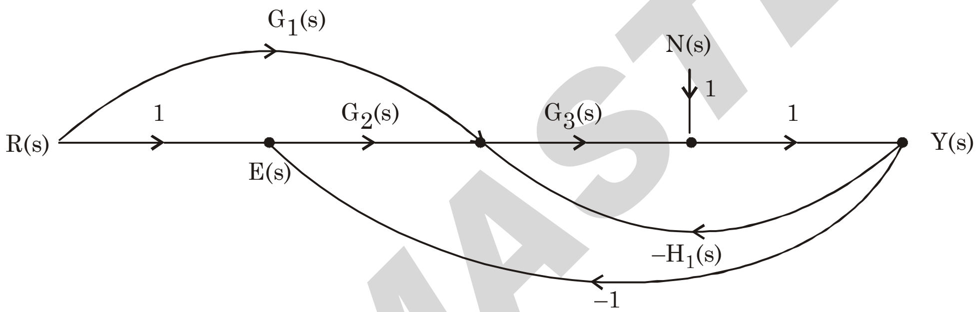




$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_1G_2)(1+G_4) - H_1H_2G_1G_4G_5}$$

47. (d)

The signal flow graph corresponding to block diagram is



To find $\left.\frac{E(s)}{N(s)}\right|_{R(s)=0}$, first find $\left.\frac{Y(s)}{N(s)}\right|_{R(s)=0}$

Since

$$E(s) = R(s) \cdot 1 - Y(s)$$

$\because$ $R(s) = 0$ given so $E(s) = -Y(s)$ ...(1)

Let $K = \left.\frac{Y(s)}{N(s)}\right|_{R(s)=0}$ ...(2)

then from eqn (1)

$$\left.\frac{E(s)}{N(s)}\right|_{R(s)=0} = -K \quad ...(3)$$

using mason gain formula

$$\left.\frac{Y(s)}{N(s)}\right|_{R(s)=0} = \frac{P_1\Delta_1}{\Delta} \quad ...(4)$$

$\because$ $P_1 = 1$

$\Delta_1 = 1$

$\Delta = 1 + G_3(s)H_1(s) + G_2(s)G_3(s)$

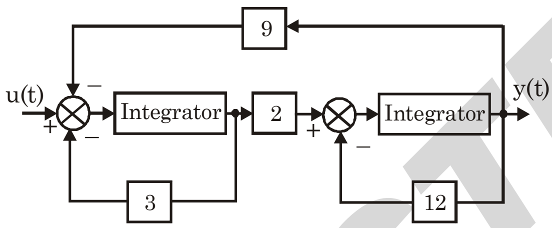
so from eqn (3)

$$\left.\frac{Y(s)}{N(s)}\right|_{R(s)=0} = \frac{1}{1+G_3(s)H_1(s)+G_2(s)G_3(s)}$$

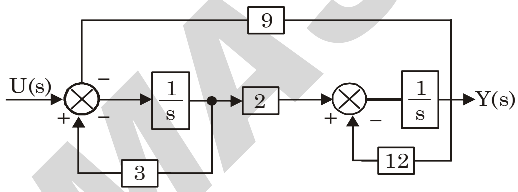
from eqn (3) and (2)

$$\left.\frac{E(s)}{N(s)}\right|_{R(s)=0} = \frac{-1}{1+G_3\left[H_1(s)+G_2(s)\right]}$$

48. (b)



$\because$ We know that transfer function of integrator $= \dfrac{1}{s}$



$$\Rightarrow \quad T(s) = \frac{\dfrac{2}{(s+3)(s+12)}}{1+\dfrac{2\times 9}{(s+3)(s+12)}}$$

$$= \frac{2}{(s+3)(s+12)+18}$$

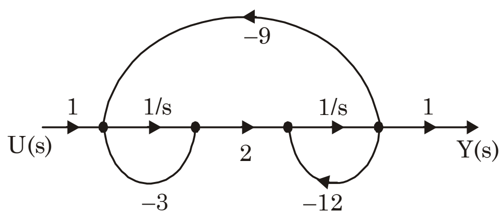
$$= \frac{2}{s^2+15s+36+18}$$

$$= \frac{2}{(s^2+15s+54)} = \frac{2}{(s+6)(s+9)}$$

$$= \frac{1}{27\left(1+\dfrac{s}{6}\right).\left(1+\dfrac{s}{9}\right)}.$$

Alternate :

SFG of Block diagram



Integrator $= \dfrac{1}{s}$

One forward path Gain $= \frac{1}{s} \times 2 \times \frac{1}{s} = \frac{2}{s^2} = P_1$

There are 3 loops and two loops are non-touching

$$L_1 = \frac{-3}{s}, L_2 = \frac{-12}{s}, L_3 = \frac{18}{s^2}$$

L_1 and L_2 are non-touching.

$$\Delta = 1 - \left(-\frac{3}{s} - \frac{12}{s} - \frac{18}{s^2}\right) + \left(-\frac{3}{s}\right)\left(-\frac{12}{s}\right)$$

$$= 1 + \frac{15}{s} + \frac{18}{s^2} + \frac{36}{s^2}$$

$$\Delta = \frac{s^2 + 15s + 54}{s^2}$$

$$\Delta_1 = 1$$

So $$\frac{Y(s)}{U(s)} = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{\frac{2}{s^2}}{\frac{s^2 + 15s + 54}{s^2}} = \frac{2}{(s+9)(s+6)}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{27\left(1 + \frac{s}{9}\right)\left(1 + \frac{s}{6}\right)}.$$

49. (b)

Given, $$G(s) = \frac{k}{s(Ts+1)}$$

Characteristic equation is

$$1 + G(s) = 0$$

$\therefore$ $$1 + \frac{k}{s(Ts+1)} = 0$$

Or $$s^2 + \frac{1}{T}s + \frac{k}{T} = 0$$

$\therefore$ $$\omega_n = \sqrt{\frac{k}{T}} \text{ and } \xi = \frac{1}{2\sqrt{kT}}$$

If peak overshoot is 75%, then

$$e^{-\pi\xi_1/\sqrt{1-\xi_1^2}} = 0.75$$

Or $$\frac{\pi\xi_1}{\sqrt{1-\xi_1^2}} = -\ln(0.75)$$

Or $$\xi_1^2 = 8.385 \times 10^{-3} \times \left(1 - \xi^2\right)$$

Or $$\xi_1 = 0.0912$$

For peak overshoot of 25%

$$e^{-\pi\xi_2/\sqrt{1-\xi_2^2}} = 0.25$$

Or $$\frac{\pi\xi_2}{\sqrt{1-\xi_2^2}} = -\ln(0.25)$$

Or $$\xi_2^2 = 0.1947\left(1-\xi_2^2\right)$$

Or $$\xi_2 = 0.4037$$

$$\frac{\xi_1}{\xi_2} = \frac{\dfrac{1}{2\sqrt{k_1T}}}{\dfrac{1}{2\sqrt{k_2T}}} = \sqrt{\frac{k_2}{k_1}} = \frac{0.0912}{0.4037}$$

Or $$\frac{k_2}{k_1} = 0.05104$$

Or $$\frac{k_1}{k_2} = 19.594$$

Thus, k is reduced by a factor of 19.59

50. (b)

Given,

$$G(s) = \frac{16}{s(s+2)}$$

Characteristic equation

$$s^2 + 2s + 16 = 0$$

$\therefore$ $$\omega_n = 4,\ \xi = \frac{2}{2\times 4} = 0.25$$

For unit step input, $$R(s) = \frac{1}{s}$$

$$C(s) = \frac{16}{s\left(s^2+2s+16\right)}$$

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t+\theta\right]$$

Where $$\phi = \tan^{-1}\sqrt{\frac{1-\xi^2}{\xi}}$$

$$A = \frac{1}{\sqrt{1-\xi^2}} = \frac{1}{\sqrt{1-0.25^2}}$$

$$= 1.033$$

$$B = \xi\omega_n = 0.25 \times 4 = 1$$

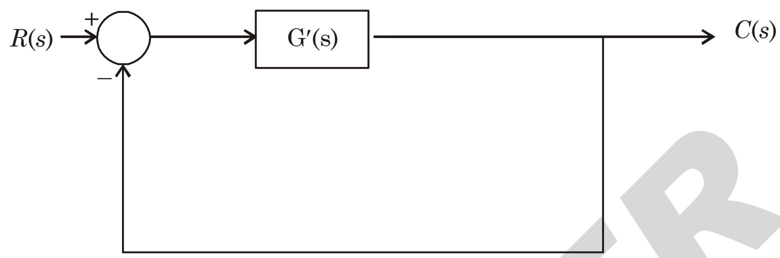
$$C = \omega_n\sqrt{1-\xi^2} = \sqrt[4]{1-0.25^2} = 3.873$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 75.52°$$

$\therefore$ D = 75.52

51. **(d)**

Given system can be represented as



where $$G'(s) = \frac{KG(s)}{20s[1+K_1G(s)]} \quad \ldots(1)$$

putting the value of G(s), we get

$$G'(s) = \frac{100K}{20s[(1+0.1s)(1+0.5s)+100K_1]} \quad \ldots(2)$$

given $r(t) = t\,u(t)$ Unit ramp input

error constant $K_V = \lim_{s\to 0} s\cdot G'(s)$

From equation (2)

$$\Rightarrow \quad K_V = \lim_{s\to 0}\frac{s(100\,K)}{20s[(1+0.1s)(1+0.5s)+100K]} = \frac{5K}{1+100K_1}$$

Steady state error $$e_{ss} = \frac{1}{K_V} = \frac{1+100K_1}{5K} \quad \ldots(3)$$

Since the given system is of third order, the value of K and K_1 must be constrained so that the system is stable.

The characteristic equation is

$$1+\ G'(s)H(s) = 0$$

$\Rightarrow \quad s^3 + 12s^2 + (20 + 2000\ K_1)\ s + 100K = 0$

Routh Tabulation

s^3	1	$20+2000K_1$
s^2	12	$100K$
s^1	$\frac{240+24000K_1-100K}{12}$	0
s^0	$100\ K$	0

Stability condition

$K > 0$ and

$$\frac{240+24000K_1-100K}{12} > 0$$

$\Rightarrow \quad 240 + 2400\ K_1 > 100\ K$

$\Rightarrow \quad 12\ (1 + 100\ K_1) > 5\ K$

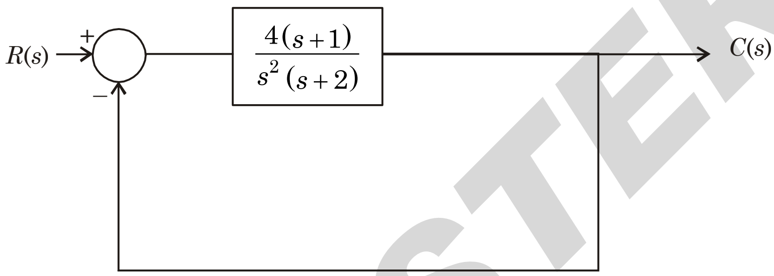
$\Rightarrow \quad \dfrac{1+100K_1}{5K} > \dfrac{1}{12}$

From equation (3) $e_{ss} > \dfrac{1}{12}$

Thus the minimum steady state error that can be obtained with a unit ramp input is $\dfrac{1}{12}$.

52. (b)

Given



$$R(s) = \frac{5}{2s} - \frac{3}{s^2} + \frac{4}{s^3}$$

$$= \frac{5}{2}R_1(s) - 3R_2(s) + 4R_3(s)$$

where $R_1(s) = \dfrac{1}{s}$ (unit step input)

$R_2(s) = \dfrac{1}{s^2}$ (unit ramp input)

$R_3(s) = \dfrac{1}{s^3}$ (unit parabola input)

Since the system is linear, then the effect of R(s) is the summation of effect of each individual input

That is : $e(\infty) = \dfrac{5}{2}e_1(\infty) - 3e_2(\infty) + 4e_3(\infty)$

Since step error constant : $K_p = \lim\limits_{s\to 0} G(s)$

$$= \lim_{s\to 0} \frac{4(s+1)}{s^2(s+2)} = \infty$$

Ramp error constant : $K_v = \lim\limits_{s\to 0} sG(s)$

$$= \lim_{s\to 0} \frac{s4(s+1)}{s^2(s+2)} = \infty$$

Parabola error constant : $K_a = \lim\limits_{s\to 0} s^2 G(s)$

$$= \frac{4(s+1)}{(s+2)} = 2$$

so

$$e_1(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

$$e_2(\infty) = \frac{1}{K_V} = \frac{1}{\infty} = 0$$

$$e_3(\infty) = \frac{1}{K_a} = \frac{1}{2}$$

$\Rightarrow$ $$e_{ss} = 0 + 0 + 4\cdot\frac{1}{2} = 2.$$

53. (a)

Let $$M(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Put $s = j\omega$

$$M(j\omega) = \frac{K\omega_n^2}{(j\omega)^2 + 2\xi\omega_n (j\omega) + \omega_n^2}$$

at $\omega = 0$

$$M(0) = \frac{K\omega_n^2}{0 + 0 + \omega_n^2}$$

$\Rightarrow$ $M(0) = K$ ...(1)

from given frequency plot

$M(0) = 0.9$

so $K = 0.9$

$$\frac{C(s)}{R(s)} = M(s) = \frac{0.9\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

if $R(s) = \frac{1}{s}$ (unit step)

then $C(s) = M(s)\cdot R(s)$

so steady state value of C(s) is

$$C_{ss}(t) = \lim_{s\to 0} s\cdot M(s)\cdot R(s) = \lim_{s\to 0} s\cdot M(s)\cdot R(s) = \lim_{s\to 0} M(s) = M(0) = k$$

$C_{ss}(t) = 0.9$

but steady state value should be = 1 (because input is unit step)

so

Steady state error = 1 – 0.9 = 0.1

54. (c)

By angle criterion for the root locus is

$$\angle\left[\frac{(s+b)}{s(s+a)}\right] = \pm 180^\circ (2q+1), \quad q = 0, 1, 2, \ldots$$

with $s = (\sigma + j\omega)$

$$\mathcal{L}\left[\frac{(s+b)}{s(s+a)}\right] = \mathcal{L}\left[\frac{(\sigma+j\omega+b)}{(\sigma+j\omega)(\sigma+j\omega+a)}\right]$$

Therefore

$$\tan^{-1}\left(\frac{\omega}{\sigma+b}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right) - \tan^{-1}\left(\frac{\omega}{\sigma+a}\right) = -\pi$$

or
$$\tan^{-1}\left(\frac{\omega}{\sigma}\right) + \tan^{-1}\left(\frac{\omega}{\sigma+a}\right) = \pi + \tan^{-1}\left(\frac{\omega}{\sigma+b}\right)$$

Taking tan on both sides,

$$\frac{\frac{\omega}{\sigma}+\frac{\omega}{\sigma+a}}{1-\frac{\omega^2}{\sigma(\sigma+a)}} = \frac{\frac{\omega}{\sigma+b}+\tan\pi}{1-(\tan\pi)\left(\frac{\omega}{\sigma+b}\right)}$$

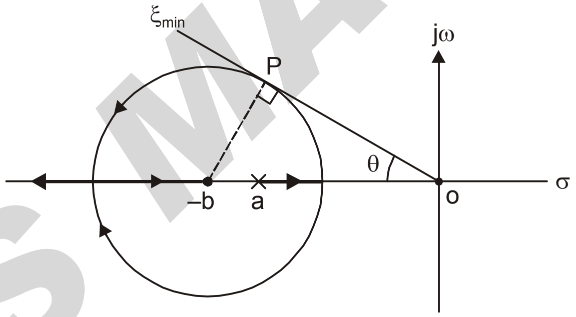
or
$$\frac{\omega}{\sigma}+\frac{\omega}{\sigma+a} = \frac{\omega}{\sigma+b} - \frac{\omega^3}{\sigma(\sigma+a)(\sigma+b)}$$

Solving further

$$(2\sigma+a)(\sigma+b) = \sigma(\sigma+a) - \omega^2$$

or
$$(\sigma+b)^2 + \omega^2 = b^2 - ab$$

It is a circle centered at (–b, 0) and radius $\sqrt{b^2-ab}$



As the k is varied, the least damped (minimum damping factor) complex conjugate poles are obtained by drawing OP tangential to the circular locus.

Thus,

$$OP = \sqrt{b^2-(b^2-ab)} = \sqrt{(ab)}$$

$$\xi_{min} = \cos\theta = \sqrt{(ab)}/b = \sqrt{\frac{a}{b}}$$

55. **(c)**

Open loop poles are

$$s^2(s+2) = 0$$

$$\Rightarrow \quad s = 0,\ 0,\ -2$$

and open loop zeros are

$$(s+1) = 0$$

$\Rightarrow$ $$s = -1$$

Since open loop poles are three, the three root loci will originate from three poles out of three, one root locus path will terminate at the zero s = – 1 and other two root loci will terminate at infinity.

Asymptotes angle are:

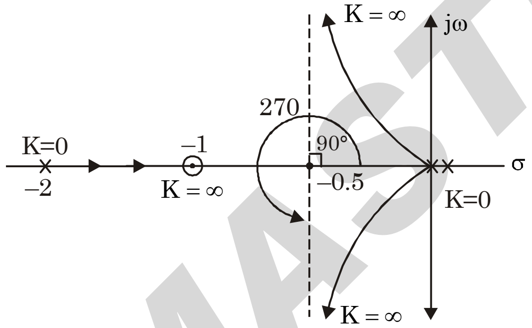
$$\alpha_1 = \frac{180°}{3-1} = 90°$$

$$\alpha_3 = \frac{3 \times 180°}{3-1} = 270°$$

Intersection of the asymptotes with real axis

$$s = \frac{-2-(-1)}{3-1} = -0.5$$

The root locus is plotted as



56. **(b)**

Characteristic equation is

$$s^3 + as^2 + 2s + 1 + k(s + 1) = 0$$

or $$s^3 + as^2 + (2 + k)s + (1 + k) = 0$$

By Routh's criteria:

s^3	1	$(2+k)$
s^2	a	$(1+k)$
s^1	$\frac{a(2+k)-(1+k)}{a}$	
s^0	$(1+k)$	

Condition for sustained oscillation:

$$\frac{a(2+k)-(1+k)}{a} = 0$$

or $$a = \frac{1+k}{2+k}$$

Auxiliary equation

$$A(s) = as^2 + (1 + k) = 0$$

or $$\left(\frac{1+k}{2+k}\right)s^2+(1+k)=0$$

or $$s^2 + (2 + k) = 0$$

Frequency of oscillation,

$\omega=\sqrt{2+k}$

Given that

$\omega=2$ rad/s

$\therefore\ \sqrt{2+k}=2$

or $\boxed{k=2}$

Therefore

$$a = \frac{1+k}{2+k}=\frac{1+2}{2+2}=\frac{3}{4}$$

or $\boxed{a=0.75}$

57. **(c)**

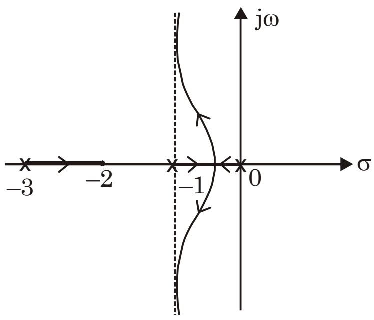
Characteristic equation :

$$s(s + 1)(s + 3) + K(s + 2) = 0$$

Open loop transfer function :

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)}$$

- Number of open loop poles,
  $P = 3$ at $s = 0, -1, -3$.
- Number of open loop zeros, $Z = 1$ at $s = -2$
- Number of roots at infinity = P – Z = 2
- Part of real axis on root locus is from –3 to –2 and from –1 to origin.
- Angle of asymtotes, $$\theta = \frac{(2q+1)180^\circ}{P-Z} = \frac{(2q+1)180^\circ}{3-1} = 90^\circ, 270^\circ$$



- Centroid = $\frac{(-1-3+0)-(-2)}{3-1} = -1$
- Break-away point :

$$K = -\frac{s(s+1)(s+3)}{(s+2)}$$

By equating $\frac{dK}{ds} = 0$ gives value of s

$$s = -0.534, \ -2.23, -2.23$$

The valid break-away point is s = –0.534

∴ Option (c) is correct.

58. **(b)**

Settling time, $t_s = \frac{4}{\xi\omega_n} = 4$

$\therefore \quad \xi\omega_n = 1$

Thus, the real part of dominant root (or complex root pair) should be –1 or more.

To satisfy this condition and to find corresponding value of k, we get s = z – 1

Thus,

$$(z-1)^3 + 8.5(z-1)^2 + 20(z-1) + 12.5(1+k) = 0$$

or $$z^3 + 5.5z^2 + 6z + 12.5k = 0$$

By Routh's criterion

z^3	1	6
z^2	5.5	12.5 k
z^1	$\frac{5.5\times 6-12.5k}{5.5}$	
z^0	12.5 k	

For limiting stability

$$6 \times 5.5 - 12.5k = 0 \quad \text{or} \quad k = 2.64$$

59. (b)

Corner frequency ω = 1 rad/sec, 20 rad/sec, 40 rad/sec

- ω_1 = 1 rad/sec, change in slope is 0 – (–20) = 20 db/dec so one zero exist at ω_1=1 rad/sec.
- Initially slope is –20 db/dec so one pole exist at origin.
- At ω_2 = 20 rad/sec, change in slope is (20 – 0) = 20 db/dec, so one zero exist at ω_2 = 20.
- At ω_3 = 40 rad/sec, change in slope is (0 – 20) = –20 db/dec, so one pole exist at ω_3 = 40.

$$\Rightarrow \quad G(s) = \frac{k\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}{s\left(1+\frac{s}{\omega_3}\right)} = \frac{k(1+s)\left(1+\frac{s}{20}\right)}{s\left(\frac{s}{40}+1\right)}$$

$$\Rightarrow \qquad G(s) = \frac{2k(s+1)(s+20)}{s(s+40)}$$

for calculation of k

line with slope = –20 db/dec

$$y = M \log w + 20 \log k$$

at $\omega = 1$, $y = -9$ and $M = -20$ db/dec

$$\Rightarrow \qquad -9 = -20 \log (1) + 20 \log k$$

$$\Rightarrow \qquad k = 0.35$$

$$\boxed{G(s) = \frac{0.7(s+1)(s+20)}{s(s+40)}}$$

60. (c)

Given

$$G(s) = \frac{K(s-1)}{s(s+1)};\ k < 0$$

since on zero lies in Right hand side of s-plane so given system is non-minimum phase system put

$$s = j\omega$$

$$G(j\omega) = \frac{K(j\omega - 1)}{(j\omega)(j\omega+1)};\ k < 0$$

$\because \qquad K = -|K|$ if $k < 0$,

so $$G(j\omega) = \frac{|K|(1-j\omega)}{(j\omega)(j\omega+1)}$$

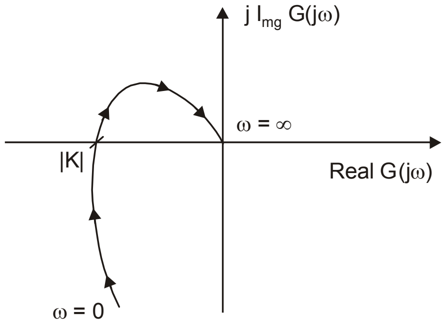
so $$G(j\omega) = \frac{|K|\sqrt{1+\omega^2}}{(\omega)\sqrt{1+\omega^2}} = \frac{|K|}{|\omega|}$$

and $$\angle G(j\omega) = -\tan^{-1}(\omega) - 90^\circ - \tan^{-1}(\omega) = -90^\circ - 2\tan^{-1}(\omega)$$

By putting different values of ω, we get

	$\|G(j\omega)\|$	$\angle G(j\omega)$
$\omega = 0$	∞	–90°
$\omega = 0.01$	100|K|	–91.14°
$\omega = 0.1$	10|K|	–101.42°
$\omega = 1$	1|K|	180°
$\omega = 10$	0.1 |K|	101.421°
$\omega = 100$	0.01 |K|	91.14
$\omega = 1000$	0.001 |K|	90.11
$w = \infty$	0	90°

By using all values, we get



61. 125.264

$$Pm = 180° + \phi \quad ...(1)$$

where

$$\phi = \angle G(j\omega)\big|_{\omega = \omega_{gc}} \quad ...(2)$$

$$\omega_{gc} = \text{gain cross over frequency}$$

At $\omega = \omega_{gc}$

$$|G(j\omega)|_{\omega = \omega_{gc}} = 1 \quad ...(3)$$

from equation (3)

$$\left|\frac{1 + j\omega_{gc}}{\sqrt{3}\, j\omega_{gc}}\right| = 1$$

$$\Rightarrow \quad \left(1 + \omega_{gc}^2\right) = \left(\sqrt{3}\ \omega_{gc}\right)^2$$

$$\Rightarrow \quad 1 + \omega_{gc}^2 = 3\omega_{gc}^2$$

$$\Rightarrow \quad \omega_{gc} = \frac{1}{\sqrt{2}} \text{ rad/sec}$$

from equation (2)

$$\angle \angle G(j\omega_{gc}) = \tan^{-1}(\omega_{gc}) - 90° = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - 90°$$

$$\phi = -54.7356°$$

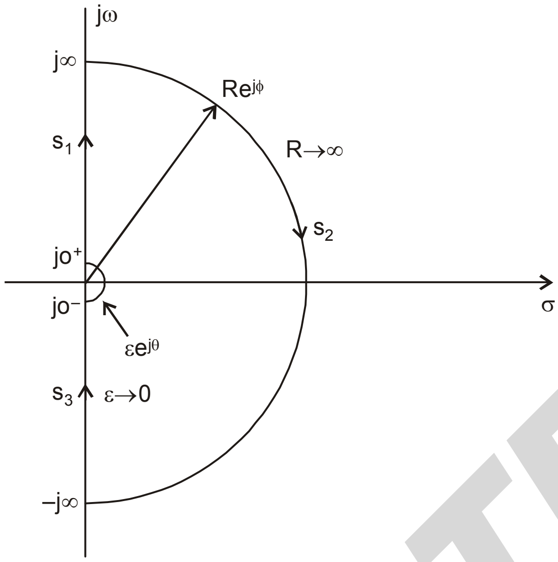
from equation (1)

$$PM = 180 + \phi$$

$$PM = 125.264$$

62. **(c)**

To obtain Nyquist Plot :



Nyquist Contour

Corresponding to S_1 : polar plot

$\omega = 0^+$ $\quad |G(s)H(s)| = \infty \quad \angle G(s)H(s) = -180°$

$\omega = \infty$ $\quad |G(s)H(s)| = 0 \quad \angle G(s)H(s) = -270°$

Note that, there is a zero in open loop transfer function so polar plot intersects real axis between $\omega = 0$ to $\omega = \infty$.

Thus, for $\angle G(s)H(s) = -180°$

$$\tan^{-1}(4\omega) - 180° - \tan^{-1}\omega - \tan^{-1}2\omega = -180°$$

or $$\tan^{-1}4\omega = \tan^{-1}\omega + \tan^{-1}2\omega$$

or $$4\omega = \frac{\omega + 2\omega}{1 - 2\omega^2}$$

or $$8\omega^3 - 4\omega = -3\omega$$

or $$\omega^2 = \frac{1}{8}$$

or $$\omega = \frac{1}{2\sqrt{2}}$$

Corresponding to s_2; $\quad s = Re^{j\phi}$

$$G(s)H(s) = \frac{(4Re^{j\phi} + 1)}{R^2e^{2j\phi}(Re^{j\phi} + 1)(2Re^{j\phi} + 1)}$$

as $R \to \infty$ $|G(s)H(s)| = 0$

Corresponding to s_3 : Reverse polar plot

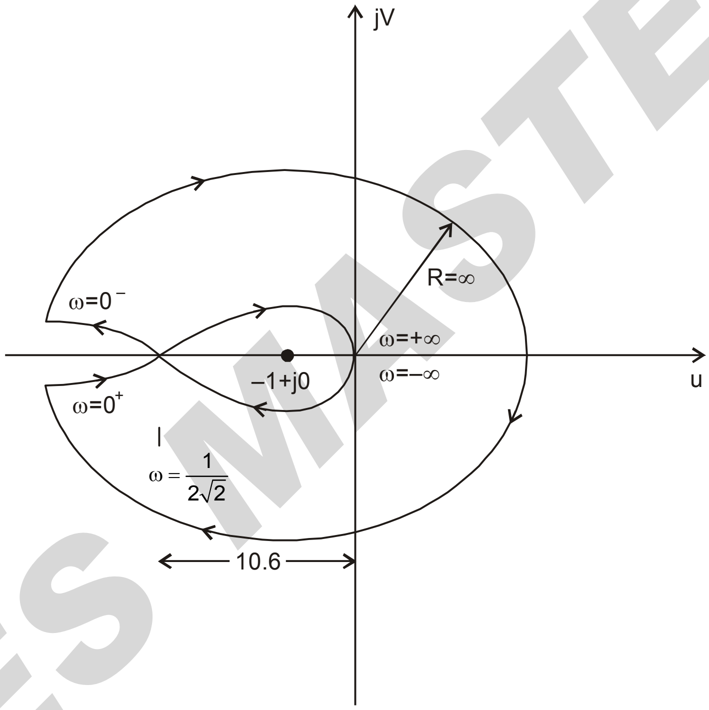
Corresponding to s_4 : $s = \in e^{j\theta}$ $\in \to 0$ and $\theta; -\frac{\pi}{2}$ to $\frac{\pi}{2}$

$$\therefore \quad G(s)H(s) = \frac{1}{\in^2 e^{j2\theta}(1)(1)}$$

as $\in \to 0$ $|G(s)H(s)| = \infty$

$\angle G(s)H(s)$ varies from π to $-\pi$ for $\theta = -\frac{\pi}{2}$ to $\frac{\pi}{2}$

Thus, Nyquist Plot



Thus, clockwise encirclements of $-1+j0$ are 2 (system is unstable).

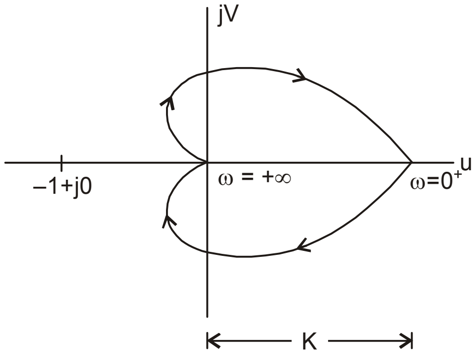
63. **(b)**

To draw polar plot :

$$G(j\omega)H(j\omega) = \frac{K}{(j\omega T_1+1)(j\omega T_2+1)} \Rightarrow \text{The plot meets real axis at K}$$

$\omega = 0$ $|G(j\omega)H(j\omega)| = K$ $\angle G(j\omega)H(j\omega) = 0°$

$\omega = \infty$ $|G(j\omega)H(j\omega)| = 0$ $\angle G(j\omega)H(j\omega) = -180°$



Till K < – 1, the number of encirclements of (–1, 0) = 0, so system is stable for K < – 1

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

To find the point on imaginary axis

$$\angle G(j\omega)H(j\omega) = -90^\circ$$

$$\therefore \quad \tan^{-1}\omega T_1 + \tan^{-1}\omega T_2 = 90^\circ$$

or

$$\frac{\omega T_1 + \omega T_2}{1-\omega^2 T_1 T_2} = \infty$$

$$\therefore \quad \omega = \frac{1}{\sqrt{T_1 T_2}}$$

64. **(c)**

Let the compensator transfer function is

$$G_c(s) = \frac{s+a}{s+b} \text{ where } b > a$$

then,

$$G(s) = \frac{k(s+a)}{s^2(s+b)}$$

From given specifications

$$t_s = \frac{4}{\xi\omega_n} = 4 \quad \Rightarrow \quad \xi\omega_n = 1$$

$$\mu_P = 0.2 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

or

$$\frac{\pi\xi}{\sqrt{1-\xi^2}} = \ln\left(\frac{1}{0.2}\right)$$

or

$$\xi = 0.456$$

It gives

$$\omega_n = \frac{1}{\xi} = \frac{1}{0.456} = 2.193$$

To obtain the root locus

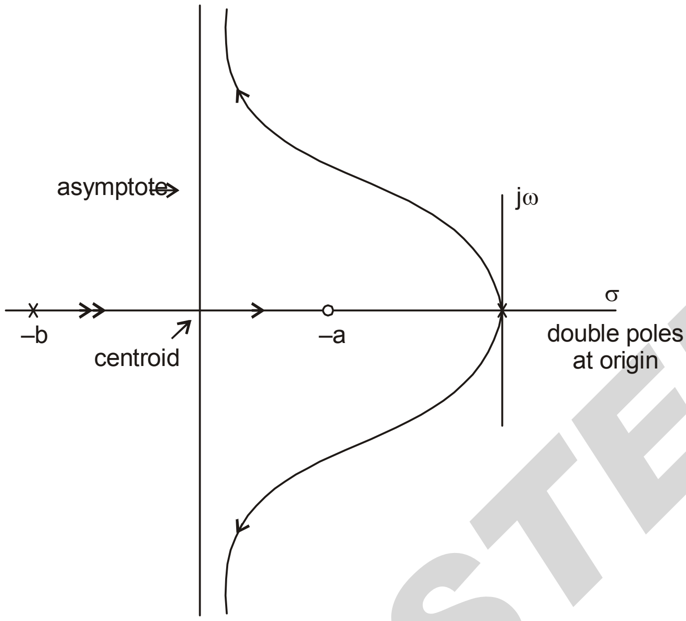
P = 3, Z = 1

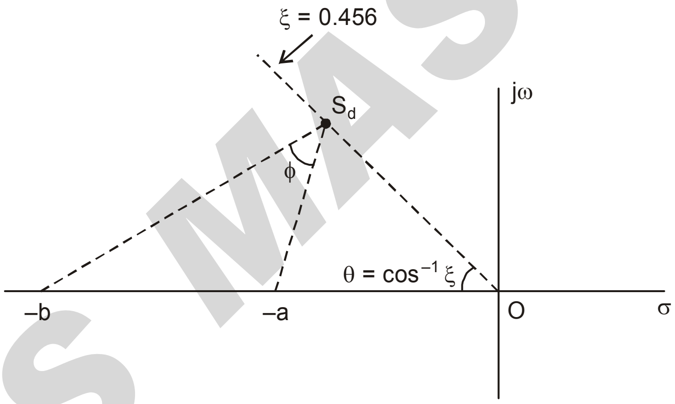
$$\text{Angle of asymptotes} = \frac{180^\circ(2q+1)}{P-z} = 90^\circ(2q+1)$$

$$= 90^\circ, 270^\circ$$

$$\text{Centroid} = \frac{-a-(-b)}{2} = \frac{b-a}{2}$$

i.e., between b and a (i.e., zero and pole of compensator)





The angle at s_d because of the two poles at origin of the uncompensated system is

$$\angle G_f(s) = -2\times(180^\circ - \cos^{-1}\xi)$$
$$= -2 \times 117.129 = -234.259$$

The angle contribution at s_d required of the lead compensator pole-zero pair is

$$\phi = -180^\circ - (-234.259)$$
$$= 54.259^\circ$$

65. **(b)**

Compensated forward path transfer function is

$$G_c(s)G(s) = \frac{k(s+\alpha)}{s(s+4)}$$

Its characteristic equation is

$$1 + G_c(s)G(s) = 0$$

or $$s(s + 4) + k(s + \alpha) = 0$$

or $$s^2 + (4+k)s + k\alpha = 0 \quad ...(1)$$

Peak overshoot

$$\mu_P = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.2 \quad ...(2)$$

and settling time

$$t_s = \frac{4}{\xi\omega_n} = 1 \quad ...(3)$$

From eqn. (2)

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.1$$

or $$\frac{\xi}{\sqrt{1-\xi^2}} = \ln\left(\frac{1}{0.2}\right)\Big/\pi$$

or $$\xi = 0.456$$

Thus, from eqn. (3)

$$\omega_n = \frac{4}{0.456} = 8.773 \text{ rad/s}$$

Using characteristic equation from eqn. (1)

$$k\alpha = \omega_n^2 = (8.773)^2 \quad ...(4)$$

$$k + 4 = 2\xi\omega_n = 8 \quad ...(5)$$

$\therefore$ $$k = 4$$

$$\alpha = (8.773)^2\Big/4$$

$$= 19.24$$