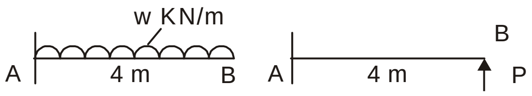
Answer key

1.	(a)	16.	(d)	31.	(a)	46.	(b)	61.	(d)
2.	(a)	17.	(d)	32.	(d)	47.	(a)	62.	(a)
3.	(c)	18.	(d)	33.	(c)	48.	(c)	63.	(a)
4.	(d)	19.	(c)	34.	(d)	49.	(d)	64.	(c)
5.	(a)	20.	(d)	35.	(d)	50.	(d)	65.	(c)
6.	(c)	21.	(a)	36.	(d)	51.	(c)	66.	(a)
7.	(b)	22.	(c)	37.	(a)	52.	(b)	67.	(a)
8.	(a)	23.	(a)	38.	(a)	53.	(c)	68.	(a)
9.	(c)	24.	(b)	39.	(b)	54.	(a)	69.	(b)
10.	(d)	25.	(c)	40.	(b)	55.	(c)	70.	(a)
11.	(b)	26.	(b)	41.	(d)	56.	(a)	71.	(d)
12.	(c)	27.	(b)	42.	(d)	57.	(a)	72.	(a)
13.	(a)	28.	(c)	43.	(d)	58.	(c)	73.	(b)
14.	(d)	29.	(c)	44.	(a)	59.	(c)	74.	(a)
15.	(b)	30.	(d)	45.	(d)	60.	(c)	75.	(a)

SOM CLASS TEST-SOLUTIONS

1. (a)



$$\delta_B(\downarrow) = \frac{wl^4}{8EI}$$

$$\delta_B(\uparrow) = \frac{Pl^3}{3EI}$$

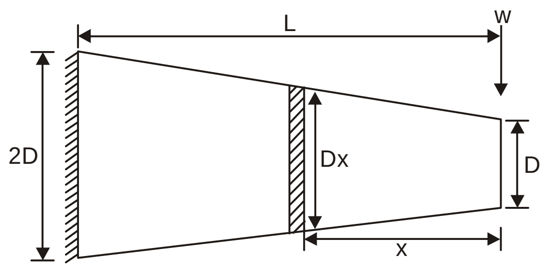
$\delta_B(\downarrow) = \delta_B(\uparrow)$ (for zero vertical deflection at 'B')

$$\Rightarrow \quad \frac{wl^4}{8EI} = \frac{P\times l^3}{3EI}$$

$$\therefore \quad \frac{800}{8} = \frac{P}{3}$$

$$\therefore \quad P = 300 \text{ KN}$$

2. (a)



Assuming a cross section at distance x from free end.

$$D_x = D + \frac{2D-D}{L}x = D + \frac{Dx}{L}$$

$$\therefore \ I_x = \frac{\pi}{64}\left(D+\frac{Dx}{L}\right)^4 = \frac{\pi D^4}{64}\left(1+\frac{x}{L}\right)^4$$

By unit method

$$\Delta_{\text{free end}} = \int_0^L \frac{M.m\,dx}{EI} = \int_0^L \frac{Wx \times x\,dx}{E\times\frac{\pi D^4}{64}\left(1+\frac{x}{L}\right)^4}dx$$

Let $1+\frac{x}{L} = t$

$\Rightarrow dx = L\,dt$

$x^2 = (t-1)^2 L^2$

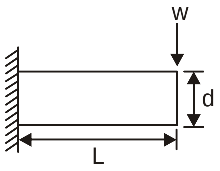
$$\therefore \quad \Delta_{\text{free end}} = \int_1^2 \frac{W\times 64}{\pi ED^4}\frac{(t-1)^2}{t^4}dt \times L^3$$

$$= \frac{64WL^3}{\pi ED^4}\left[\int_1^2\left\{\frac{1}{t^2}+\frac{1}{t^4}-\frac{2}{t^3}\right\}dt\right]$$

$$= \frac{64WL^3}{\pi ED^4}\left[-\frac{1}{t}-\frac{1}{3t^3}+\frac{2}{2t^2}\right]_1^2$$

$$= \frac{64WL^3}{\pi ED^4}\left[-\frac{1}{2}-\frac{1}{24}+\frac{1}{4}+1+\frac{1}{3}-1\right] = \frac{64}{24}\frac{WL^3}{\pi ED^4}$$

$$\therefore \quad \Delta_{\text{free end}} = \frac{64}{24}\frac{WL^3}{\pi ED^4} \qquad \dots (1)$$



$$\Delta_{\text{free end}} = \frac{WL^3\times 64}{3E\times\pi d^4} \qquad \dots (2)$$

For (1) and (2) to be equal

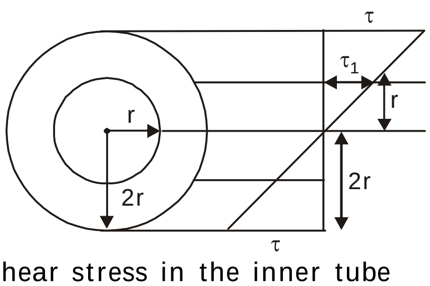
$$\frac{64}{24}\frac{WL^3}{\pi ED^4} = \frac{WL^3\times 64}{3E\pi d^4}$$

$$\Rightarrow d^4 = \frac{24}{3}D^4$$

or $d = 1.682D$

3. (c)

4. (d)

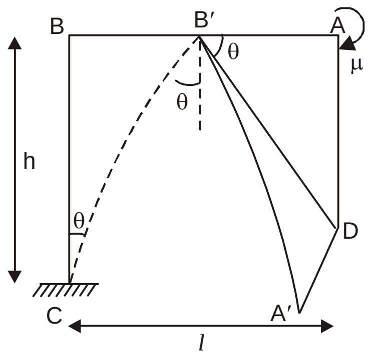


Shear stress in the inner tube

$$\tau_1 = \frac{\tau\times r}{2r}$$

$$\tau_1 = 0.5\tau$$

5. (a)



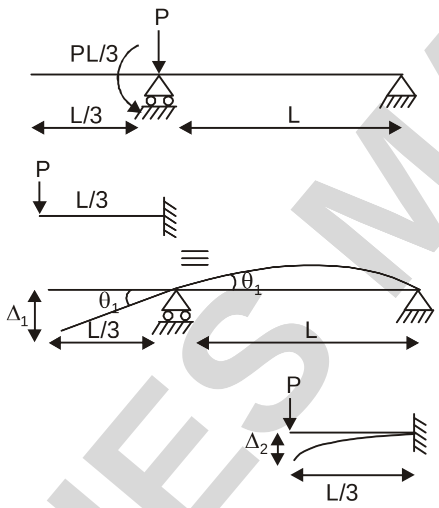
$$\theta = \frac{\mu h}{EI},\ AD = \theta \times l = \frac{\mu h l}{EI}$$

$$DA' = \frac{\mu l^2}{2EI}$$

Due to very small deflection, DC' may be taken as vertical deflection

$$\therefore\ (D_A)_{vertical} = \frac{\mu l}{EI} + \frac{\mu l^2}{2EI} = \frac{\mu l}{EI}\left[h + \frac{l}{2}\right]$$

6. (c)

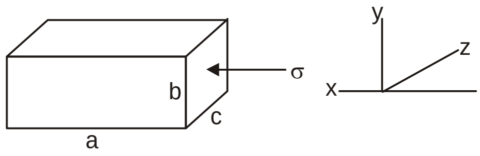


$$\theta_1 = \frac{ML}{3EI} = \frac{PL}{3} \times \frac{L}{3EI}$$

$$\therefore\ \Delta_1 = \theta_1 \times \frac{L}{3} = \frac{PL}{3} \times \frac{L}{3EI} \times \frac{L}{3} = \frac{PL^3}{27EI}$$

$$\Delta_2 = \frac{P \times \left(\frac{L}{3}\right)^3}{3EI} = \frac{PL^3}{81EI}$$

$$\therefore\ \Delta_{Total} = \Delta_1 + \Delta_2 = \frac{PL^3}{27EI} + \frac{PL^3}{81EI} = \frac{4PL^3}{81EI}$$



Final length in x, y, z direction is

$a - ea$, $b + e\mu b$ and $c + \mu ec$

$$\therefore\ \text{Final volume} = (a - ea)(b + e\mu b)(c + e\mu c)$$

$$= abc(1 - e)(1 + e\mu)(1 + e\mu)$$

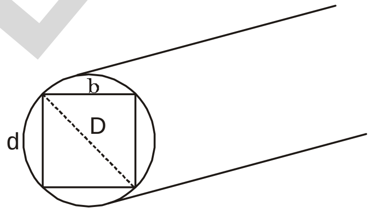
$$= (1 - e)(1 + e\mu)^2\, V \quad \text{where } V = abc$$

7. (b)

8. (a)

9. (c)

10. (d)



Let rectangular section is (b × d)

where $b^2 + d^2 = D^2$

For the beam to be strongest in the bending, Z should be maximum.

$$Z = \frac{bd^2}{6}$$

$$Z = \frac{b\left(D^2 - b^2\right)}{6}$$

For strongest section

$$\frac{dz}{db} = 0 = \frac{1}{6}\left[D^2 - 3b^2\right]$$

$$D^2 = 3b^2$$

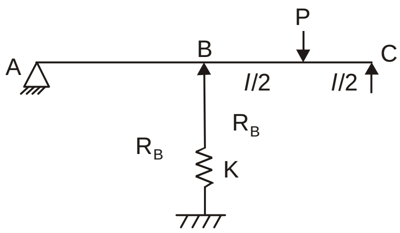
$$b = \frac{1}{\sqrt{3}}D$$

$$\because\ d^2 = D^2 - \frac{D^2}{3} = \frac{2}{3}D^2$$

$$\Rightarrow\ d = \sqrt{\frac{2}{3}}D$$

$$\frac{d}{b} = \sqrt{2}$$

11. (b)



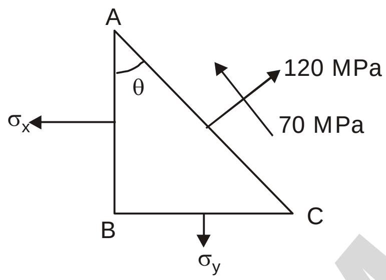
$$R_B + R_C = P$$

$$R_B = R_C = \frac{P}{2}$$

$$\text{Force in spring} = \frac{P}{2}$$

$$\text{Displacement at B} = \frac{P}{2K} \quad \text{(Downward)}$$

12. (c)



$$120 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\Rightarrow 120 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}(2\cos^2\theta - 1)$$

$$= \sigma_x(0.5 + 0.5(2\cos^2\theta - 1)) + \sigma_y(0.5 - 0.5(2\cos^2\theta - 1))$$

$$\cos\theta = \frac{AB}{AC} = \frac{4}{5}$$

$$\Rightarrow 120 = \sigma_x\left(0.5 + 0.5\left(2\times\left(\frac{4}{5}\right)^2 - 1\right) + \sigma_y\left(0.5 - 0.5\left(2\times\left(\frac{4}{5}\right)^2 - 1\right)\right)\right)$$

$$\Rightarrow \quad 120 = \frac{16}{25}\sigma_x + \frac{9}{25}\sigma_y \quad \text{....(1)}$$

$$\tau_{xy} = 70$$

$$= \frac{-(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

$$= \frac{-(\sigma_x - \sigma_y)}{2} 2\sin\theta\cos\theta$$

$$= \frac{-(\sigma_x - \sigma_y)}{2} \times 2 \times \frac{3}{5} \times \frac{4}{5} = -(\sigma_x - \sigma_y) \times \frac{12}{25} \quad \text{..(2)}$$

From (1) and (2)

$$\sigma_x = 67.5 \text{ MPa}$$

$$\sigma_y = 213.33 \text{ MPa}$$

13. (a)

T = 16 kN-M

M = 20 kN-M

as per Tresca theory

$$\tau_{max} \le \frac{f_y}{2}$$

$$\frac{16}{\pi d^3}\sqrt{m^2 + T^2} \le \frac{f_y}{2}$$

$$D^3 = \frac{16 \times 2}{\pi \times f_y} \times \sqrt{M^2 + T^2}$$

D = 101.45 mmm

D = 102 mm

14. (d)

$$\frac{t_c}{t_s} = \frac{2 - \mu_c}{1 - \mu_h}$$

$$\frac{t_c}{t_s} = \frac{2 - 0.35}{1 - 0.25}$$

$$\frac{t_c}{t_s} = 2.2$$

15. (b)

$$\sigma_h = \frac{F}{A} - \sigma_l$$

$$\Rightarrow \quad \frac{P_r}{t} = \frac{F}{2\pi rt} - \frac{P_r}{2t}$$

$$\Rightarrow \quad F = 3\pi r^2 p$$

16. (d)

$$\text{Strain energy stored, } v = \frac{LT^2}{2GI_p}$$

$$v_1 = \frac{T^2 \times 2L \times 32}{2G \times \pi D^4}$$

$$v_2 = \frac{T^2 L \times 32}{2 \times G \times \pi \times 16 D^4}$$

$$\frac{v_1}{v_2} = 32$$

17. (d)

twist angle at C

$$\theta_c = \frac{T_{AC} \times L_{AC}}{GI_P}$$

$$T_{AC} = \frac{T \times \frac{2L}{3}}{L} = \frac{2T}{3}$$

$$\therefore \quad \theta_c = \frac{\frac{2}{3}T \times \frac{L}{3}}{GI_P}$$

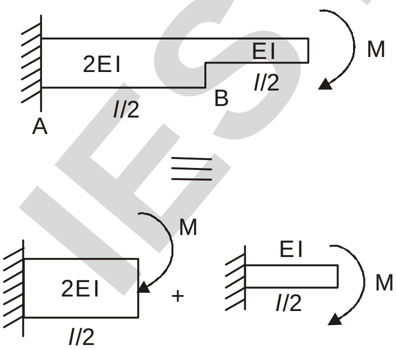
$$\Rightarrow \quad \theta_c = \frac{2TL}{9GI_P}$$

18. (d)

- Effective length of column is the distance between adjacent points of contraflexure or point of zero bending moment.
- Rankine theory of failure in applicable for both short and long column.
- $P_{CR} = \frac{\pi^2 EI}{L^2}$

  for all grade of steel, E is same. So various grades of steel will fail at same buckling load.

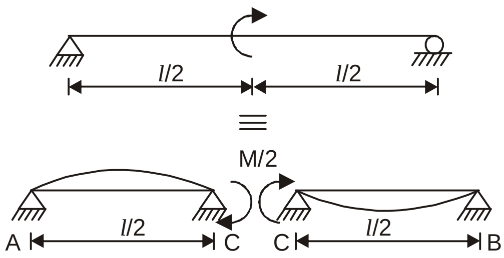
19. (c)



$$\Delta_C = \frac{M\left(\frac{l}{2}\right)^2}{2EI} + \frac{M\left(\frac{l}{2}\right)^2}{2\times 2EI} + \frac{M\left(\frac{l}{2}\right)}{2EI} \times \frac{l}{2}$$

$$= \frac{Ml^2}{8EI} + \frac{Ml^2}{16EI} + \frac{Ml^2}{2\times 4EI} = \frac{(2+1+2)Ml^2}{16EI} = \frac{5}{16}\frac{Ml^2}{EI}$$

20. (d)

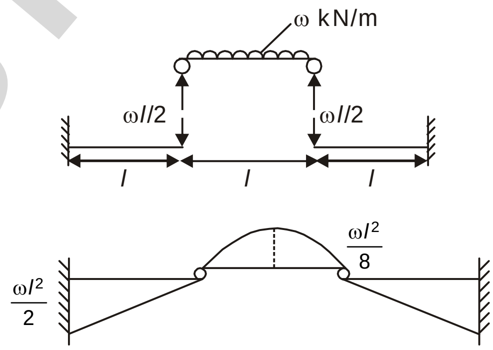


$$\theta_A = \frac{\frac{M}{2} \times \frac{l}{2}}{6EI} = \frac{Ml}{24EI}$$

$$\theta_C = \frac{\frac{M}{2} \times \frac{l}{2}}{3EI} = \frac{Ml}{12EI}$$

$$\theta_B = \frac{\frac{M}{2} \times \frac{l}{2}}{6EI} = \frac{Ml}{24EI}$$

21. (a)

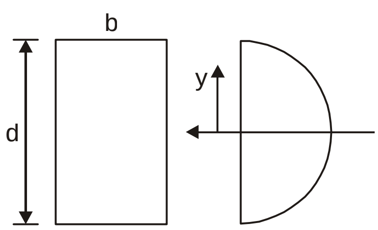


22. (c)

Theory of simple bending is only applicable to sections of beam in which plane of loading is axis of symmetry. △ and T have symmetry about loading axis (vertical axis) so theory of simple bending is applicable only to these sections.

23. (a)

24. (b)



$$\tau = \frac{6V\left(\frac{d^2}{4} - y^2\right)}{bd^3}$$

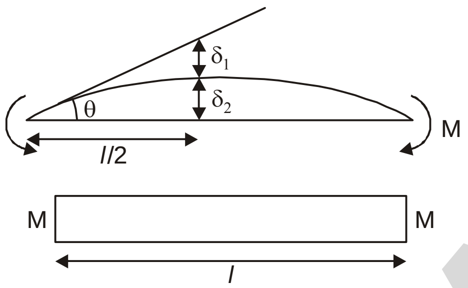
$$\tau_{avg} = \frac{V}{bd}$$

$$\tau = \tau_{avg}$$

$$\frac{6V\left(\frac{d^2}{4} - y^2\right)}{bd^3} = \frac{V}{bd}$$

$$y = \frac{d}{\sqrt{12}} = \frac{d}{2\sqrt{3}}$$

25. (c)



$$\theta = \frac{Ml}{2EI}$$

$$\delta_1 + \delta_2 = \frac{\theta l}{2} = \frac{Ml}{3EI} \times \frac{l}{2} = \frac{Ml^2}{4EI}$$

$$\delta_1 = \frac{ML}{2EI} \times \frac{l}{4} = \frac{Ml^2}{8EI}$$

$$\delta_2 = \frac{Ml^2}{4EI} - \frac{Ml^2}{8EI} = \frac{Ml^2}{8EI}$$

26. (b)

On the plane of maximum shear stress normal stress

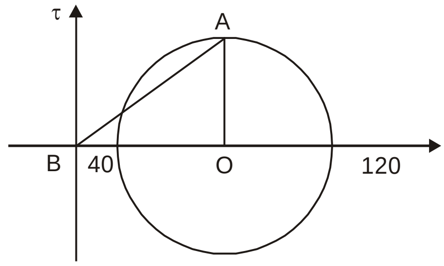
$$\sigma_{n_1} = \sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2}$$

$$= \frac{100 + 20}{2}$$

$$= 60 \text{ MPa}$$

.

27. (b)



$$\sqrt{OA + OB^2} = \sigma_R$$

$$OA = \frac{120 - 40}{2} = 40 \text{ MPa}$$

$$OB = \frac{120 + 40}{2} = 80 \text{ MPa}$$

$$\sigma = \sqrt{80^2 + 40^2}$$

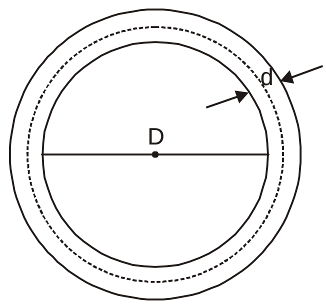
$$= 89.44 \text{ MPa}$$

$$\simeq 90 \text{ MPa}$$

28. (c)

- In non circular section apart from shearing stress, torsion also induces warping stress due to unsymmetrical distribution of shear stress.
- In rectangular section, due to torsion maximum shear stress induced is at the midpoint of the longest sides. This shear stress distribution is non liner from the centre.

29. (c)



$$\therefore \quad \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \quad \frac{\sigma}{d/2} = \frac{E}{\frac{D}{2} + \frac{d}{2}}$$

$$\Rightarrow \quad \sigma = E\left(\frac{d}{D+2}\right)$$

30. (d)

$$\varepsilon_1 = 80 \times 10^{-6}$$

$$\varepsilon_2 = 20 \times 10^{-6}$$

Maximum shear strain = γ_{max}

$$\frac{\gamma_{max}}{2} = \frac{(|\varepsilon_1 - \varepsilon_2|)}{2}$$

$$= 80 \times 10^{-6} - 20 \times 10^{-6}$$

$$\gamma_{max} = 60 \times 10^{-6}$$

31. (a)

32. (d)

$$\text{shear strain} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$= 6 \times 10^{-6} + 6 \times 10^{-6}$$

$$= 12 \times 10^{-6} \text{ unit}$$

33. (c)

$$\frac{\gamma_{yz}}{2} = 0.002$$

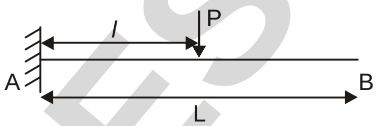
$$\gamma_{yz} = 0.004$$

Shear stress in yz plain = $\gamma_{yz} \times G$

$= 0.004 \times 100$ GPa

$= 400$ MPa

34. (d)



$$\Delta_B = \frac{Pl^3}{3EI} + \frac{Pl^2}{2EI} \times (L - l)$$

$$= \frac{Pl^2}{EI}\left(\frac{l}{3} + \frac{L-l}{2}\right)$$

$$= \frac{Pl^2}{2EI}\left(L - \frac{l}{3}\right)$$

35. (d)

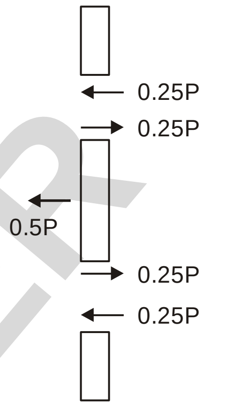
36. (d)

In traiangular section, maximum shear stress occurs at h/2 from vertex.

Therefore statement 1 is wrong.

In circular section, $\tau_{max} = \frac{4}{3}\tau_{avg}$

37. (a)

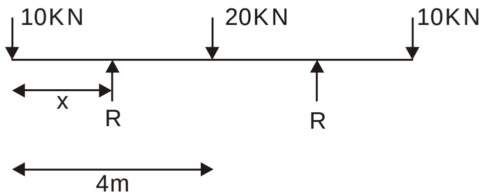


For carried by each pin = 0.5P

$\therefore$ Shear in each pin = 0.25P

38. (a)

39. (b)



R = 20 KN

Max negative BM = 10 × x

Max positive BM = $-40 \times 4 + 20(4 - x)$

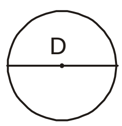
$\Rightarrow \quad 10 \times x = -10 \times 4 + 20(4 - x)$

$\Rightarrow \quad 10x = -40 + 80 - 20x$

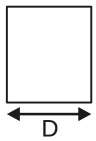
$\Rightarrow \quad 30x = 40 \Rightarrow x = 1.33\,m$

40. (b)

41. (d)



$$Z_c = \frac{\pi}{32} D^3$$



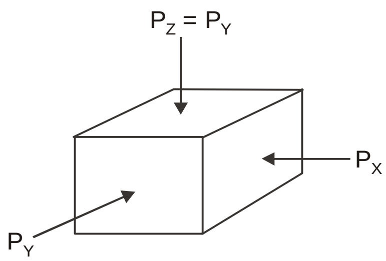
$$Z_S = \frac{D^3}{6}$$

$$\because \quad M = \sigma z$$

for same σ, $$\frac{M_c}{M_s} = \frac{Z_C}{Z_S} = \frac{\frac{\pi}{32} \times D^3}{\frac{D^3}{6}}$$

$$\frac{Z_C}{Z_S} = \frac{3\pi}{16}$$

42. (d)



$\because$ strain in lateral direction $= 0$

$$\Rightarrow \frac{P_y}{E} - \frac{\mu P_x}{E} - \frac{\mu P_y}{E} = 0$$

$$\Rightarrow P_y(1-\mu) = \mu P_x$$

or $$P_y = \frac{\mu P_x}{1-\mu}$$

43. (d)

44. (a)

45. (d)

$$F_1 + F_2 = 1000 \text{ Kg}$$

Taking moment about wire 2.

$$F_1 \times 40 = 1000 \times 20$$

$$\Rightarrow \quad F_1 = 500 \text{ Kg}$$

$$F_2 = 500 \text{ Kg}$$

$$\frac{\Delta_1}{\Delta_2} = \frac{F_1}{A_1E_1} \times \frac{A_2E_2}{F_2}$$

$$= \frac{2}{4} \times \frac{2}{1} = 1$$

46. (b)

47. (a)

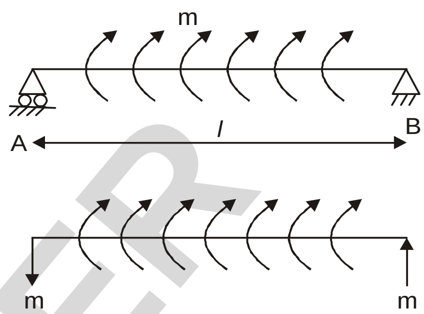
$$\delta_1 = \frac{P_1L}{AE}, \delta_2 = \frac{P_2L}{AE}$$

$$\delta = (P_1 + P_2)\frac{l}{AE}$$

So, $$\delta = \delta_1 + \delta_2$$

48. (c)

49. (d)



S.F. at any point.

$$(S.F.)x = R_A = m$$

So S.F.D.

50. (d)

51. (c)

52. (b)

53. (c)

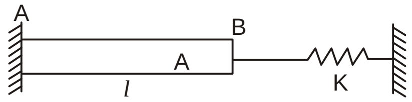
54. (a)

55. (c)

56. (a)

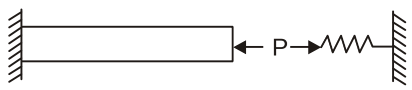
57. (a)

58. (c)



Free expansion of bar (without spring presence)

$= l\alpha t$



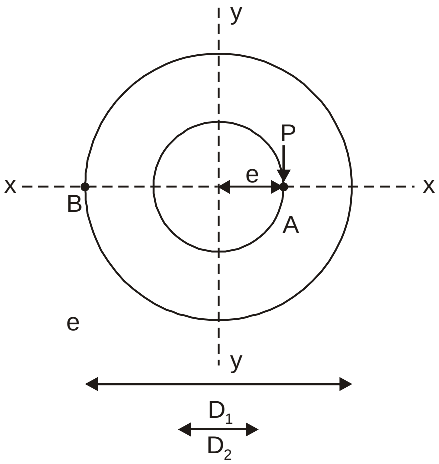
Compression of spring $= \dfrac{P}{K}$

$$\therefore \quad l\alpha t - \frac{P}{K} = \frac{Pl}{AE} \Rightarrow P\left(\frac{1}{K} + \frac{l}{AE}\right) = l\alpha t$$

$$\Rightarrow \frac{P}{A} = \frac{\alpha t E}{1 + \frac{AE}{Kl}} = \sigma$$

59. (c)

60. (c)



Suppose load P is acting at point 'A' at eccentricity of 'e', then

$$\sigma_B = \frac{P}{A} - \frac{My}{I}$$

$$= \frac{P \times 4}{\pi\left(D_1^2 - D_2^2\right)} = \frac{Pe \times \frac{D_1}{2}}{\frac{\pi\left(D_1^4 - D_2^4\right)}{64}}$$

For no tension at point B,

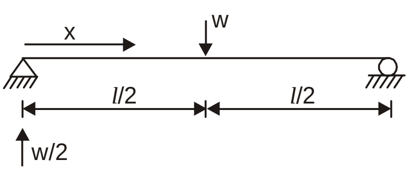
$$\sigma_B = 0$$

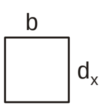
$$\Rightarrow \frac{P \times 4}{\pi\left(D_1^2 - D_2^2\right)} = \frac{Pe \times \frac{D_1}{2}}{\frac{\pi\left(D_1^4 - D_2^4\right)}{64}}$$

$$\Rightarrow e = \frac{D_1^4 - D_2^4}{8D_1\left(D_1^2 - D_2^2\right)}$$

$$\Rightarrow e = \frac{D_1^2 + D_2^2}{8D_1}$$

61. (d)





$$M = \sigma z$$

$$\Rightarrow \frac{w}{2}x = \frac{f \times bd_x^2}{6}$$

$$\Rightarrow d_x^2 = \frac{3wx}{fb}$$

$$\text{or } d_x = \sqrt{\frac{3wx}{fb}}$$

62. (a)

$$K = \frac{E}{\sigma_y}$$

$$\alpha \propto \frac{\sigma_y}{\pi^2 E}$$

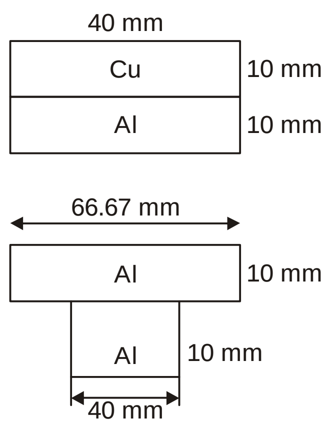
Rankine constant

$$\alpha = \frac{1}{\pi^2 \frac{E}{\sigma_y}}$$

$$\alpha \propto \frac{1}{K}$$

63. (a)

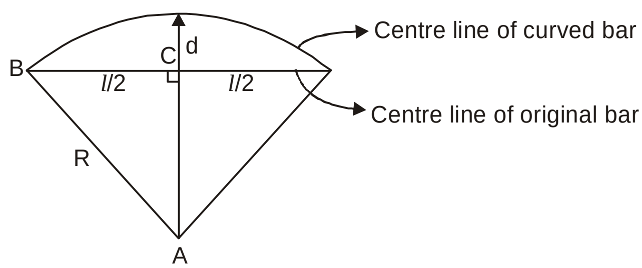
64. (c)



$$\bar{y} \text{ from top} = \frac{(66.67 \times 10 \times 5 + 40 \times 10 \times 15)}{66.67 \times 10 + 40 \times 10}$$

$$= 8.75 \text{ mm}$$

65. (c)



$$R^2 = (R-d)^2 + \frac{l^2}{4}$$

$$\Rightarrow R^2 = R^2 + d^2 - 2Rd + \frac{l^2}{4}$$

$$\text{or } R = l\left(\frac{d}{2l} + \frac{l}{8d}\right)$$

$\because$ d << l

$$\therefore R = \frac{l^2}{8d}$$

$$\because \frac{\sigma}{E} = \frac{y}{R} \Rightarrow \varepsilon = \frac{y}{R}$$

$$\therefore \varepsilon = \frac{\frac{t}{2}}{\frac{l^2}{8d}} = \frac{4td}{l^2}$$

66. (a)

67. (a)

68. (a)

$$\varepsilon_1 = 8 \times 10^{-4}$$

$$\varepsilon_2 = 4 \times 10^{-4}$$

$$\sigma_1 = \frac{E}{1-\mu^2} \times (\varepsilon_1 + \mu\varepsilon_2)$$

$$\sigma_1 = \frac{75000}{1-0.25}(8\times10^{-4} + 2\times10^{-4})$$

$$\sigma_1 = \frac{75000\times10\times10^{-4}}{0.75}$$

$$\sigma_1 = 100 \text{ KPa}$$

69. (b)

70. (a)

Deflection at B

$$6 \times 10^{-3} = \frac{wl^4}{8EI} - \frac{R_B L^3}{3EI}$$

$$6 \times 10^{-3} \times 625000 = \frac{100\times5^4}{8} - \frac{R_B\times5^3}{3}$$

$$6 \times 625 = 125\left[\frac{100\times5}{8} - \frac{R_B}{3}\right]$$

$$30 = \frac{500}{8} - \frac{R_B}{3}$$

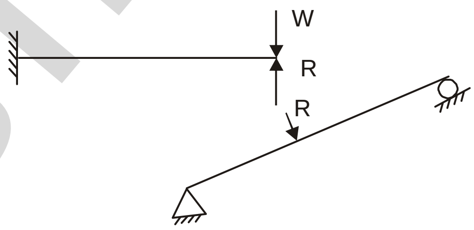
$$\frac{R_B}{3} = 32.5$$

$$\boxed{R_B = 97.5\text{kN}}$$

$$\boxed{\therefore R_A = 5\times100 - 97.5}$$

$$\boxed{R_A = 402.5\text{kN}}$$

71. (d)



$$\frac{(w-R)\ell^3}{3EI} = \frac{R\ell^3}{48EI}$$

$$16\text{ W} - 16\text{ R} = \text{R}$$

$$R = \frac{16}{17}W$$

72. (a)

73. (b)

74. (a)

75. (a)