[ECE] CLASSROOM PRACTICE TEST-03
EMFT (SOLUTIONS)

ANSWER KEY

1.	(c)	**14.**	(b)	**27.**	(b)	**40.**	(d)	**53.**	(a)
2.	(b)	**15.**	(d)	**28.**	(b)	**41.**	(c)	**54.**	(c)
3.	(c)	**16.**	(b)	**29.**	(a)	**42.**	(d)	**55.**	(a)
4.	(b)	**17.**	(b)	**30.**	(c)	**43.**	(a)	**56.**	(c)
5.	(a)	**18.**	(d)	**31.**	(b)	**44.**	(c)	**57.**	(d)
6.	(d)	**19.**	(a)	**32.**	(c)	**45.**	(a)	**58.**	(c)
7.	(b)	**20.**	(c)	**33.**	(d)	**46.**	(d)	**59.**	(a)
8.	(a)	**21.**	(a)	**34.**	(c)	**47.**	(a)	**60.**	(d)
9.	(d)	**22.**	(b)	**35.**	(c)	**48.**	(c)	**61.**	(a)
10.	(b)	**23.**	(d)	**36.**	(b)	**49.**	(b)	**62.**	(d)
11.	(b)	**24.**	(a)	**37.**	(b)	**50.**	(c)	**63.**	(c)
12.	(b)	**25.**	(d)	**38.**	(d)	**51.**	(b)	**64.**	(a)
13.	(d)	**26.**	(b)	**39.**	(a)	**52.**	(c)	**65.**	(d)

1. (c)

2. (b)

SWR is used to find out the amount of mismatch between line and load greater the SWR greater the mismatch.

3. (c)

Let the potential of equipotential surface is V_0.

So, $V_0 = 100\ln\{\tan\phi/2\}$

or $\ln\{\tan\phi/2\} = 0.01\ V_0$ ...(i)

Now, as V_0 is consant, so the equation (i) will be of the form ϕ = constant, which represents a plane.

4. (b)

From continuity equation

$$\frac{\partial \rho_v}{\partial t} = -\nabla\cdot\vec{J}$$

$$\vec{\nabla}\cdot\vec{J} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\cdot\frac{25}{\rho}\right\} + \frac{1}{\rho}\frac{\partial}{\partial\phi}(0) + \frac{\partial}{\partial z}\left(\frac{-20}{\rho^2+0.01}\right)$$

$$\vec{\nabla}\cdot\vec{J} = 0 + 0$$

$$\frac{\partial \rho_v}{\partial t} = 0$$

so, we can say that the charge density is constant with respect to time.

5. (a)

For left circular wave, X should lead Y by 90° and both should have equal amplitude for elliptically polarized wave, X and Y can have any phase difference.

6. (d)

Direction of poynting vector is same as of direction of wave propagation, not the direction of electric field. When wave is incident with Brewster angle, no part of the wave is reflected.

7. (b)

For good conductors, $\alpha = \beta = 1/\delta$

also $\lambda = \dfrac{2\pi}{\beta}$

so $\lambda = 2\pi\delta$

$\Rightarrow$ $\delta = 0.159\lambda$

Hence skin depth is very small.

Skin resistance is real part of η

so option (b) is correct.

8. (a)

Waveguide allows waves of frequencies above the cutoff frequency and attenuates all lower frequencies. Hence waveguide acts as a high pass filter. So option (a) is correct.

9. (d)

Microwaves provides higher bandwidth as compared to RF waves

Due to shorter wavelength, microwave requires special components.

Hence option (d) is correct.

10. (b)

For linear polarization.

- Components should have either no phase difference or a phase difference of 180°.
- Resultant vector should not rotate.

Hence option (b) is correct.

11. (b)

The magnetic field intensity is given by

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

$$\vec{H} = \frac{1.2}{2\pi\rho}\hat{a}_\phi$$

at ρ = 1.5 cm,

$$\vec{B} = \mu\vec{H} = \mu_0(6)\vec{H}$$

$$= \frac{4\pi\times10^{-7}\times6\times1.2}{2\pi\times1.5\times10^{-2}}\hat{a}_\phi$$

$$\boxed{\vec{B} = 9.6\times10^{-5}\ \hat{a}_\phi}\ \frac{\text{Weber}}{\text{m}^2}$$

12. (b)

The electric field is given by—

$$E = \nabla V = -80\frac{\partial}{\partial\rho}\rho^{0.6}\ \hat{a}_\rho$$

$$E = -48\rho^{0.4}\hat{a}_\rho \text{ V/m}$$

$$\vec{D} = \varepsilon_0\vec{E}$$

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$\vec{D} = -48\ \varepsilon_0 \rho^{-0.4} \hat{a}_\rho$ C/m²

At $\rho = 0.6$m,

$\vec{D} = -48\ \varepsilon_0 .(0.6)^{-0.4} \hat{a}_\rho$

$\vec{D} = -0.521 \times 10^{-9}\ \hat{a}_\rho$ C/m²

The flux density is constant.

So, the flux $\psi = \vec{D}.\vec{S}$

$= -0.521 \times 10^{-9} \times 2\pi\rho z$

$= -0.521 \times 10^{-9} \times 2\pi \times 0.6 \times 1$

$\psi = -1.96$ nC

From Gaussis law, $Q_{emc.} = \psi = -1.96$ nC.

13. (d)

Ampere Law states that $\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$.

$\therefore \oint \vec{H} \cdot \vec{dl} = -30 + 10 = -20$A

–30A is taken because the path taken is opposite to the direction of magnetic field due to 30A current.

14. (b)

$J = b\rho\, \hat{a}_z$

The current in a small circular ring, of radius ρ^1, and thickness $d\rho^1$, is given by

$I = \vec{J}.d\vec{s} = b\rho' a_z \cdot (\rho' d\rho' d\phi)\hat{a}_z$

$= b\,(\rho')^2 \cdot d\rho'\, d\phi$

$\therefore$ Total current upto a radius ρ

$$I = \int_0^{2\pi}\int_0^{\rho} b(\rho')^2\ d\rho'\ d\phi$$

$$= b\int_0^{2\pi} d\phi \int_0^{\rho} (\rho')^2\ d\rho'$$

$$I = \frac{2\pi\, b\rho^3}{3}$$

For a circular closed path, with a radius ρ, using Ampere's law,

$\oint H.dl = I_{enc}$.

$$H.2\pi\rho = \frac{2\pi b\rho^3}{3}$$

$$\therefore\ H = \frac{b\rho^2}{3}\ \text{A/m.}$$

The direction of $\vec{H}$ will be along $\hat{a}_\phi$.

So, $H = \frac{b\rho^2}{3}\hat{a}_\phi$.

15. (d)

The net capacitance can be considered as two capacitances in series.

$C_1 = \frac{\varepsilon_1 A}{(d/2)}$ and $C_2 = \frac{\varepsilon_2 A}{(d/2)}$

$$\therefore\ C = \frac{C_1 C_2}{C_1 + C_2} = \frac{4\varepsilon_1\varepsilon_2 A^2}{d^2 \times \frac{2A}{d}(\varepsilon_1 + \varepsilon_2)}$$

$$C = \frac{2A}{d}\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}\right)$$

16. (b)

In TM mode, the H field is transverse to the direction of wave propagation. Hence, the H field will be zero in the direction of wave propagation : $E_z \neq 0$ and $H_z = 0$

17. (b)

s = 3

So, $\Gamma = \frac{s-1}{s+1} = \frac{1}{2}$; $\Gamma^2 = \frac{1}{4}$

$P_r = \Gamma^2 \times P_i = \frac{1}{4} \times 6 = 1.5$ watts/m²

$P_t = P_i - P_r$

$P_t = 6 - 1.5$

$P_t = 4.5$ watts/m²

18. (d)

$H_x = H_\rho \cos\phi - H_\phi \sin\phi$

At point P (2, 30°, –1)

$\rho = 5, \ \phi = 30^\circ, \ z = -1$

$H_r = 5 \times 5 \times \sin 30^\circ = 12.5$

$$H_\phi = -\rho z \cos\phi = -5 \times (-1) \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad H_x = 12.5 \cos 30^\circ - \frac{5\sqrt{3}}{2} \sin 30^\circ$$

$$= 12.5\left(\frac{\sqrt{3}}{2}\right) - 2.5\left(\frac{\sqrt{3}}{2}\right) = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$$

Component of $\vec{H}$ parallel to $\hat{a}_x$ is

$$\vec{H}_x = H_x \hat{a}_x = 5\sqrt{3}\ \hat{a}_x$$

19. **(a)**

In the first equation, the right hand side should have dot product between $(\nabla \times \vec{A})$ and $d\vec{S}$

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

20. **(c)**

Gauss's law for the magnetic field states that the magnetic flux emanating from a closed surface is equal to zero.

$$\oint B.ds = 0$$

Above relation signifies that magnetic charges do not exist and magnetic flux lines are closed.

From Divergence theorem,

$$\oint B.ds = \int \nabla.B dv = 0$$

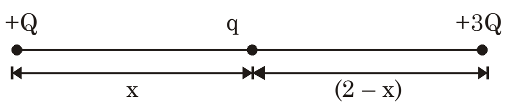
$\nabla.B = 0$

$\nabla.\mu H = 0$

$\mu \nabla.H = 0$

$\nabla.H = 0$

21. **(a)**



Let a charge q is placed in between these two charges this charge q must be negative for this system to be in equilibrium. Let it is placed x metre away from charge +Q.

Net force on this charge must be zero

$$K\frac{Q \cdot q}{x^2} = K\frac{3Q \cdot q}{(2-x)^2}$$

$\Rightarrow (2 - x)^2 = 3x^2$

$\Rightarrow 2 - x = \pm\sqrt{3}x$

Taking positive sign

$2 = (1 + \sqrt{3})x$

$\Rightarrow x = 0.732$ m

Taking negative sign

$2 = x - \sqrt{3}x$

$\Rightarrow x = -2.732$ m

x negative means the charge q in left side of +Q charge not in between +Q and +3Q

$\therefore x = 0.732$

22. **(b)**

For a lossless dielectric

$\sigma = 0, \quad \mu = \mu_0\mu_r \qquad \varepsilon = \varepsilon_0\varepsilon_r$

$\omega = 2\pi \times 9375\,\text{MHz}$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$

$$= \omega\sqrt{4\pi \times 10^{-7} \times 1 \times \frac{1}{36\pi} \times 10^{-9} \times 9}$$

$$= \omega\sqrt{10^{-7} \times \frac{1}{9} \times 10^{-9} \times 9}$$

$$= \omega\sqrt{10^{-16} \times 1}$$

$$= 2\pi \times 9375 \times 10^6 \times 10^{-8}$$

$$= 2\pi \times 9375 \times 10^{-2}$$

$$= 2\pi \times 93.75$$

$$= 187.5\pi$$

23. **(d)**

Given : $E_x = 8\cos\omega t$

$E_y = 24\cos(\omega t + 90^\circ)$

$|E_x| = 8$

$|E_y| = 24$

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$\alpha = 90°$

If $|E_x| \neq |E_y|$

and $\alpha = \pm 90°$

Then it is elliptically polarised.

24. (a)

$E_i = 50$ V/m

So, $P_i = \frac{1}{2}\frac{E_i^2}{\eta_i}$

$P_i = \frac{1}{2} \times \frac{50^2 \times \sqrt{9}}{120\pi}$

$P_i = 9.947 \text{ W/m}^2$

Now, $\Gamma = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{3-1}{3+1} = \frac{1}{2}$

$P_r = \Gamma^2 \times P_i$

$P_r = \frac{1}{4} \times 9.947$

$P_r = 2.47 \text{ W/m}^2$

25. (d)

In free space $\sigma = 0, \mu = \mu_0, \varepsilon = \varepsilon_0, \beta = 250$

$\beta = \omega\sqrt{\mu_0\varepsilon_0} = \omega\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi} \times 10^{-9}}$

$= \omega\sqrt{\frac{1}{9} \times 10^{-16}}$

$\beta = \frac{\omega}{3} \times 10^{-8}$

$250 = \frac{\omega}{3} \times 10^{-8}$ (β = 250 given from equ.)

$\omega = 250 \times 3 \times 10^8 = 750 \times 10^8$

$\omega = 75 \times 10^9$ radian/s

26. (b)

From figure $V_{max} = 4$; $V_{min} = 1$

$\text{VSWR} = \frac{|V_{max}|}{|V_{min}|} = \frac{4}{1} = 4$

$|\Gamma| = \frac{1 + \text{VSWR}}{1 - \text{VSWR}} = \frac{1+4}{1-4}$

$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$

$|\Gamma| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = \frac{3}{5}$

$\Gamma = \pm 0.6$

Since there is minimum at the load (from figure),

$\Gamma = -0.6$

$\frac{z_L - z_0}{z_L + z_0} = -0.6$

$\frac{z_L - 50}{z_L + 50} = -0.6$

$1.6\ z_L = 20$

$z_L = \frac{20}{1.6} = 12.5\Omega$

27. (b)

For $\frac{\lambda}{2}$ line

$Z_{in} = Z_L = 200\Omega$

For $\frac{\lambda}{4}$ line

$Z_{in} = \frac{Z_0^2}{Z_C} = \frac{(100)^2}{50} = \frac{100 \times 100}{50} = 200\Omega$

$\lambda/2$ line and $\lambda/4$ line are parallel

$\therefore \quad Z_{in} = \frac{200 \times 200}{200 + 200} = 100\Omega$

for $\lambda/8$ line $Z_{in} = Z_0 = 100\Omega$

28. (b)

Loss tangent $= \frac{\sigma}{\omega \in} = \frac{5 \times 10^{-6}}{2\pi \times 10^9 \times 9 \times 8.85 \times 10^{-12}}$

$\frac{\sigma}{\omega \in} = 9.99 \times 10^{-6} \simeq 0$

As loss tangent is negligible, hence medium is lossless dielectric.

29. (a)

Given

$V_{in} = 15V, \Gamma = 0.5, P_{in} = ?\ Z_0 = 50\Omega$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

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$$\frac{1}{2} = \frac{Z_L - 50}{Z_L + 50}$$

$$Z_L + 50 = 2Z_L - 100$$

$$Z_L = 150\Omega$$

$$P_{in} = \frac{V_{in}^2}{Z_L} = \frac{(15)^2}{150}$$

$$= \frac{15 \times 15}{150} = \frac{15}{10} = 1.5 \text{ W}$$

30. (c)

$\sigma = 0$ (lossless medium)

$$\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \text{intrinsic impedance}$$

Given media 1 $\varepsilon_r = 16, \mu_r = 4, \sigma = 0$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}\frac{4}{16}} = \eta_0\sqrt{\frac{1}{4}}$$

$$= \frac{\eta_0}{2} = \frac{120\pi}{2} = 60\pi$$

Given media 2 = $\varepsilon_r = 8\ \mu_r = 2\ \sigma = 0$

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}\frac{\mu_r}{\varepsilon_r}} = \eta_0\sqrt{\frac{2}{8}} = \frac{\eta_0}{2} = \frac{120\pi}{2} = 60\pi$$

31. (b)

According to Smith chart, the given figure represent constant resistance circles. Because this represent a set of circles for a set of values of r and all circles pass through a common point for all value of r.

32. (c)

Here, $R_{rad} = 73\Omega$

$I_o = 20\pi \times 10^{-3}$ A

for a half wave Dipole Antenna,

$$\therefore \quad R_{rad} = \frac{1}{2}I_o^2 R_{rad} = \frac{1}{2} \times (20\pi)^2 \times 10^{-6} \times 73$$

P_{rad} = 144 mW

33. (d)

As cavity resonates in the TE_{111} mode, the quality factor (Q) of the cavity will be

$$Q = \frac{f}{BW}$$

$$Q = \frac{9 \times 10^9}{2.4 \times 10^6}$$

$$\boxed{Q = \frac{9000}{2.4}}$$

34. (c)

In TEM mode both E and H fields are transverse to the direction of wave propagation. Since, the wave is propagating in +Z direction, the E and H components in Z direction will be zero.

35. (c)

For a Transverse magnetic wave propagating in a waveguide, the electric field component along the wave propagation is given by:

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

For $m = 0,\ n \neq 0,\ E_z = 0$

For $n = 0,\ m \neq 0,\ E_z = 0$

$\therefore$ TM_{01} and TM_{10} will not exist and TEM mode cannot propagate in a rectangular waveguide.

$\therefore$ Only TM_{11} can propagate.

36. (b)

$$\vec{D} = \varepsilon\vec{E}$$

$$\vec{D} = \left(\frac{10^{-5}}{\rho}\right)\cos(10^5 t)\hat{a}_\rho$$

$$\therefore \quad \vec{J}_D = \frac{\partial D}{\partial t}$$

$$= \frac{\partial}{\partial t}\left\{\left(\frac{10^{-5}}{\rho}\right)\cos(10^5 t)\right\}\hat{a}_\rho$$

$$\vec{J}_D = -\left\{\frac{10^{-5}}{\rho}\sin(10^5 t)\right\}(10^5)$$

$$\vec{J}_D = -\frac{1}{\rho}\sin(10^5 t)\ \hat{a}_\rho$$

The displacement current will flow through the surface area of the cylinderical capacitor, radially outward or inward.

$$I_D = 2\pi\rho l.J_D \Rightarrow 2\pi\rho l\left\{\frac{-1}{\rho}\sin(10^5 t)\right\}$$

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$I_D = -2\pi.I\sin(10^5 t) = -\ -2\pi(0.4)\sin(10^5 t)$

$\boxed{I_D = -0.8\pi\sin(10^5 t)}$ A .

37. (b)

Let the required point is $P_3(0, y, 0)$

Then, $\vec{R}_{13} = (0-4)\hat{a}_x + (y+2)\hat{a}_y + (0-7)\hat{a}_z$

$= -4\hat{a}_x + (y+2)\hat{a}_y - 7\hat{a}_z$

and $\vec{R}_{23} = (0+3)\hat{a}_x + (y-4)\hat{a}_y + (0+2)\hat{a}_z$

$= 3\hat{a}_x + (y-4)\hat{a}_y + 2\hat{a}_z$

Now, electric field at P_3 X-direction,

$$E_x = \frac{10^{-9}}{4\pi\varepsilon_0}\left[\frac{25\times(-4)}{\left(\sqrt{(-4)^2+(y+2)^2+(-7)^2}\right)^3} + \frac{60\times 3}{\left(\sqrt{(3)^2+(y-4)^2+(2^2)}\right)^3}\right]$$

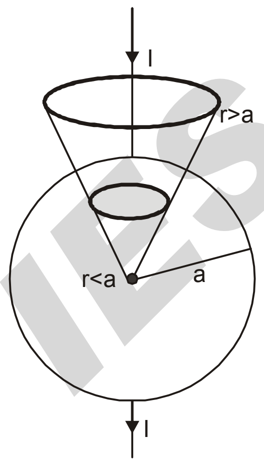
As $E_x = 0$, we get

$0.48y^2 + 13.9y + 73.12 = 0$

$\Rightarrow$ y = –6.89 or –22.11

i.e. P_3 = (0, –22.11, 0)

38. (d)



For r < a,

If we draw any circular contour, with its center at z-axis, the enclosed current by it will be zero. So, from ampere's law, the magnetic field intensity $\boxed{H = 0}$ for r < a.

For r > a if we draw any circular contour of radius ρ, where $\rho = r\sin\theta$, then it will enclose a current equal to I. So the magnetic field intensity is given by—

$$\vec{H} = \frac{-I}{2\pi P}\hat{a}_\phi = \frac{-I}{2\pi r\sin\theta}\hat{a}_\phi$$

So, for r > a, $\boxed{H\alpha\frac{1}{r}}$.

39. (a)

If we draw a circular contour having a radius of 1.5 cm., then it will enclose both filament and current sheet. The net current enclosed,

$I = 20\pi\times 10^{-3} + 2\pi\times 0.01\times 400\times 10^{-3}$

$= 28\pi\times 10^{-3}$ A

From Ampere's law

$\oint H.dl = I_{enc}$.

$H.2\pi\rho = 28\pi\times 10^{-3}$

$$H = \frac{28\pi\times 10^{-3}}{2\pi 1.5\times 10^{-2}}$$

$\boxed{H = 0.933\ \text{A/m}}$

40. (d)

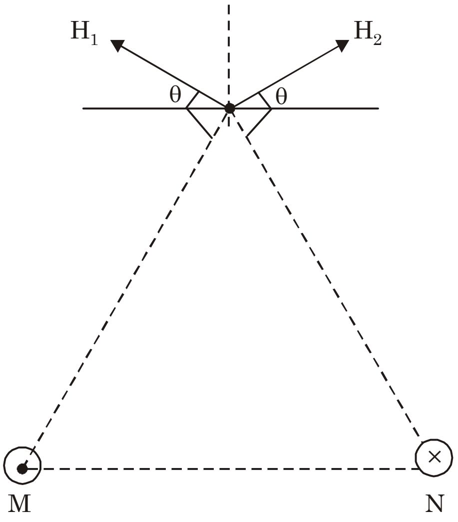
Magnetic field $\vec{B}$ due to infinitely long wire is given by:

$$\left|\vec{B}\right| = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = 90°$ and $\theta_2 = 0°$

$$\therefore \left|\vec{B}\right| = \frac{\mu_0 I}{4\pi a}(\sin 90° + \sin 0°) = \frac{\mu_0 I}{4\pi a}(-\hat{K})$$

41. (c)



Here, $\theta = 30°$

and $H_1 = H_2 = \frac{I}{2\pi a} = \frac{5}{2\pi}$

H_1 is due to 10 A_v and H_2 is due to 10A at N at M.

$H_1 \cos\theta$ and $H_2 \cos\theta$ will cancel each other.

Net field intensity = $H_1 \sin\theta + H_2 \sin\theta$

$= 2H_1 \sin 30°$

$= 2 \times \frac{5}{2\pi} \times \frac{1}{2}$

$H = \frac{5}{2\pi} A/m$.

42. (d)

Let the magnetic field $\vec{H} = H_0 \sin(\omega t + \beta y)\hat{a}_H$

here, $\frac{E_0}{H_0} = |\eta| \Rightarrow H_0 = \frac{E_0}{|\eta|} = \frac{10}{377}$

$\Rightarrow H_0 = 0.0265$

Also ω and β remains same for a wave.

Now, $\hat{a}_z \times \hat{a}_H = -\hat{a}_y$

$\Rightarrow \hat{a}_H = -\hat{a}_x$

So, $\vec{H} = -0.0265 \sin(2 \times 10^8 t + 3y)\hat{a}_x$ A/m

43. (a)

Displacement current density $J_d = \nabla \times H$

$$J_d = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(z) & 0 & 0 \\ \hat{a}_x & \hat{a}_y & \hat{a}_z \end{vmatrix}$$

$\Rightarrow J_d = \frac{\partial H_x}{\partial_z}\hat{a}_y$

$J_d = -15\cos(\omega t - 3z)\hat{a}_y \ mA/m^2$

44. (c)

Phase constant $\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

where $\beta' = \omega\sqrt{\varepsilon\mu} = 2\pi f \sqrt{4\varepsilon_0\mu_0}$

$\therefore \beta' = \frac{2\pi \times 15 \times 10^9 \times 2}{3 \times 10^8} = 628.31$

Now, $fc = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} = \frac{1}{2\sqrt{4 \in_0 \mu_0}}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

$\therefore f_c = \frac{3 \times 10^8}{2 \times 2} \times \frac{\sqrt{(2.5)^2 + (1)^2}}{2.5 \times 1 \times 10^{-2}}$

$f_c = 8.07$ GHz

$\therefore \beta = 628.31 \times \sqrt{1 - \left(\frac{8}{15}\right)^2} = 531.49$

wave impedance in TM mode $\eta = \eta^1 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$\eta^1 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{377}{2} = 188.5\Omega$

$\therefore \eta = 188.5\sqrt{1 - \left(\frac{8.07}{15}\right)^2} = 159\Omega$

45. (a)

Since, potential due to dipole

$V = \frac{1}{4\pi \in_0} \cdot \frac{\vec{p}.\vec{r}}{r^3}$

Hence, $V = \frac{1}{4\pi \in_0}\left[\frac{\vec{p}_1.\vec{r}_1}{r_1^3} + \frac{\vec{p}_2.\vec{r}_2}{r_2^3}\right]$

where, $\vec{r}_1 = (0-2)\hat{a}_x + (0-0)\hat{a}_y + (0-0)\hat{a}_z$

$= -2\hat{a}_x$

and, $\vec{r}_2 = (0-0)\hat{a}_x + (0-0)\hat{a}_y + [0-(-2)]\hat{a}_z$

$= 2\hat{a}_z$

$\therefore V = \frac{1}{4\pi \in_0}\left[\frac{(10\hat{a}_x).(-2\hat{a}_x)}{(2)^3} + \frac{(-5\hat{a}_x).(2\hat{a}_z)}{(2)^3}\right] \times 10^{-9}$

$= \frac{1}{4\pi \in_0} \cdot \frac{(-20)}{8} \times 10^{-9}$

$= 9 \times 10^9 \times \frac{(-20)}{8} \times 10^{-9}$

$= \frac{-45}{2} = -22.5V$

i.e. option (a).

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46. (d)

Electric flux density due to point charge

$$\vec{D}_Q = \frac{Q}{4\pi r^2}\hat{a}_r = \frac{30\times10^{-9}}{4\pi\left(\sqrt{0^2+4^2+3^2}\right)}\cdot\frac{0\hat{a}_x+4\hat{a}_y+3\hat{a}_z}{\sqrt{0^2+4^2+3^2}}$$

$$= \frac{30\times10^{-9}}{500\pi}\left(4\hat{a}_y+3\hat{a}_z\right)$$

$$= \left(0.076\hat{a}_y+0.057\hat{a}_z\right)\text{nC/m}^2$$

Electric flux density due to surface charge

$$\vec{D}_{\rho_s} = \frac{\rho_s}{2}\hat{a}_n = \frac{10\times10^9}{2}\hat{a}_y = 5\hat{a}_y \ \text{nC/m}^2$$

$$\therefore \quad \vec{D} = \vec{D}_Q+\vec{D}_{\rho_s} = \left(5.076\hat{a}_y+0.057\hat{a}_z\right)$$

47. (a)

$$\vec{E} = \vec{E}_1+\vec{E}_2+\vec{E}_3$$

$$= \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r+\frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho+\frac{\rho_s}{2\epsilon_0}\hat{a}_n$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r^2}\hat{a}_r+\frac{2\rho_L}{\rho}\hat{a}_\rho+\frac{2\pi\rho_s}{1}\hat{a}_n\right]$$

$$= 9\times10^9\left[\frac{100\times10^{-12}}{\left(\sqrt{(1-4)^2+(1-1)^2+(1+3)^2}\right)^3}\left(-3\hat{a}_x+4\hat{a}_z\right)\right.$$

$$\left.+\frac{2\times2\times10^{-9}}{\left[(1-1)^2+(1-0)^2+(1-0)^2\right]}\left(\hat{a}_y+\hat{a}_z\right)+2\pi\times5\times10^{-9}\hat{a}_z\right]$$

$$= 9\left[\frac{0.1}{(5)^3}\left(-3\hat{a}_x+4\hat{a}_z\right)+\frac{4}{2}\left(\hat{a}_y+\hat{a}_z\right)+10\pi\hat{a}_z\right]$$

$$\therefore \quad E_y = 18$$

48. (c)

$$\frac{\sigma}{\omega\epsilon} = \frac{4\times36\pi}{3\times3\times10^8\times80\times10^{-9}}$$

$$\frac{\sigma}{\omega\epsilon} = 2\pi = 6.288 = \tan^2\omega_\eta$$

$$\Rightarrow \quad \theta_\eta = 40.47^\circ$$

$$|\eta_2| = \frac{\sqrt{\mu_2/\epsilon_2}}{\sqrt{1+\left(\dfrac{\sigma_2}{\omega\epsilon_2}\right)^2}}$$

$$|\eta_2| = 16.71$$

$$\Rightarrow \quad \eta_2 = 16.71\angle40.47^\circ$$

Now, $\Gamma = \dfrac{\eta_2-\eta_1}{\eta_2+\eta_1}$

$$\Gamma = \frac{16.71\angle40.47^\circ-377}{16.71\angle40.47^\circ+377}$$

$$\Rightarrow \quad \Gamma = 0.935\angle179.7^\circ$$

So, $E_r = \Gamma E_i = 18.7\angle179.7^\circ$

So, $\vec{E}_r = 18.7\sin(\omega t-3z+179.7)\hat{a}_x$ V/m

49. (b)

Given,

For Copper,

Skin depth, (δ_{cu}) = 1 micron $= 10^{-6}$ m

Frequency, (f_{cu}) = 3 GHz

For non-magnetic conductor,

Frequency (f_{non}) = 12 GHz

Conductivity of nonmagnetic conductor

$= \dfrac{1}{9}$ conductivity of copper

$$(\sigma_{non}) = \frac{1}{9}(\sigma_{cu})$$

$$\because \quad \text{Skin depth } (\delta) = \frac{1}{\sqrt{\pi f\mu\sigma}} \propto \frac{1}{\sqrt{f\sigma}}$$

$$\Rightarrow \quad \delta_{cu} = \frac{1}{\sqrt{\pi f_{cu}\mu_0\sigma_{cu}}} \qquad \ldots(1)$$

$$\delta_{non} = \frac{1}{\sqrt{\pi f_{non}\mu_0\sigma_{non}}} \qquad \ldots(2)$$

From equation (1) and (2)

$$\frac{\delta_{non}}{\delta_{cu}} = \sqrt{\frac{f_{cu}\sigma_{cu}}{f_{non}\sigma_{non}}} = \sqrt{\frac{3}{12}\frac{\sigma_{cu}}{\frac{1}{9}\sigma_{cu}}}$$

$$\therefore \quad \delta_{non} = \sqrt{\frac{9}{4}}\delta_{Cu} = \frac{3}{2}\text{ micron}$$

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50. (c)

$|E| = E_0 e^{-\alpha z}$

After travelling 1 meter

$$E_0 e^{-\alpha(1)} = (1-0.18)E_0$$

$\Rightarrow\ e^{-\alpha} = 0.82$

$\alpha = ln\left(\frac{1}{0.82}\right) = 0.1984$

Now, $\theta_n = 24°$

also, $\tan 2\theta_n = \frac{\sigma}{\omega \in} = 1.111$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}-1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}+1}} = \frac{\sqrt{\sqrt{2.233}-1}}{\sqrt{\sqrt{2.233}+1}} = 2.247$$

$\Rightarrow\ \beta = 0.4458$

So,

$\gamma = \alpha + j\beta = 0.1984 + 0.4458j$

51. (b)

Let

$\vec{E} = E_0 \sin(\omega t - \beta x + \phi)\hat{a}_z$

$\frac{\lambda}{2} = 200 - 50 = 150\text{m} \ \Rightarrow\ \lambda = 300\text{m}$

Now, $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{300}$

$\omega = \beta \times c$

$\omega = \frac{2\pi}{\lambda} \times 3 \times 10^8$

$\omega = \frac{2\pi}{300} \times 3 \times 10^8 = 2\pi \times 10^6$

If $E(0,0) = 0 \Rightarrow \phi = 0$

So,

$\vec{E} = 2\sin\left(2\pi \times 10^6 t - \frac{\pi}{150}x\right)\text{V/m}$

Now, at x = 100 and t = 3×10^{-6}

$\vec{E} = 2\sin\left(2\pi \times 10^6 \times 3 \times 10^{-6} - \frac{\pi}{150} \times 100\right)$

$\vec{E} = -1.732\,\text{V/m}$

52. (c)

A uniform plane electromagnetic wave propagating in the y direction having $\vec{E}$ and $\vec{H}$ should be function of y and t. Also the components of $\vec{E}$ and $\vec{H}$ in the direction of propagation are zero.

$\therefore\ E_y = H_y = 0$

So, wave equations are

$$\frac{\partial^2 E_x}{\partial y^2} = \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} \text{ or } \frac{\partial^2 H_x}{\partial y^2} = \mu\varepsilon\frac{\partial^2 H_x}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial y^2} = \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} \text{ or } \frac{\partial^2 H_z}{\partial y^2} = \mu\varepsilon\frac{\partial^2 H_z}{\partial t^2}$$

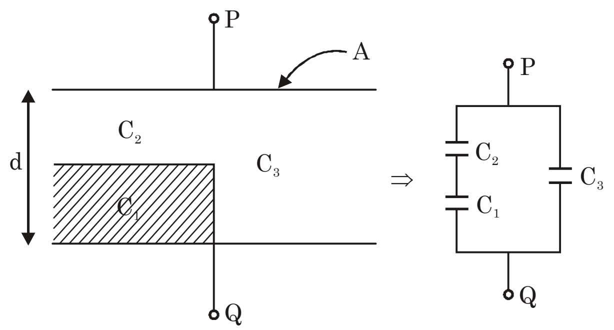
$\because\ \frac{E}{H} = \eta$

$\Rightarrow\ \frac{E_z}{H_x} = \eta,\ \frac{E_x}{H_z} = -\eta$

$$\frac{E}{H} = \frac{\sqrt{E_x^2 + E_z^2}}{\sqrt{H_x^2 + H_z^2}} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

= Intrinsic impedance of the medium

53. (a)



$C_{PQ} = \frac{C_1 C_2}{C_1 + C_2} + C_3$

where, $C_2 = \frac{\in_0 A/2}{d/2} = \frac{\in_0 A}{d} = C$ (say)

$C_1 = \frac{\in A/2}{d/2} = \frac{\in A}{d} = \frac{\in_0 \in_r A}{d} = \in_r C$

and $C_3 = \frac{\in_0 A/2}{d} = \frac{\in_0 A}{2d} = \frac{C}{2}$

$\therefore \quad C_{PQ} = \dfrac{\epsilon_r\, C.C}{C(1+\epsilon_r)} + \dfrac{C}{2}$

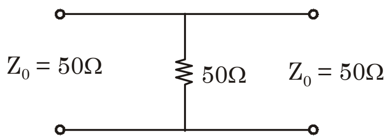
$= C\left[\dfrac{\epsilon_r}{\epsilon_r+1} + \dfrac{1}{2}\right] = C.\left[\dfrac{6}{6+1} + \dfrac{1}{2}\right] = \dfrac{19}{14}C$

where, $\quad C =$

$\dfrac{8.854\times10^{-12}\times10\times10^{-4}}{2\times10^{-3}} = 4.427\text{pF}$

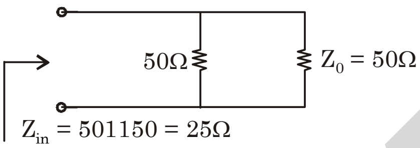
$\therefore \quad C_{PQ} = \dfrac{19}{14}\times 4.427 = 6\text{pF}.$

54. (c)



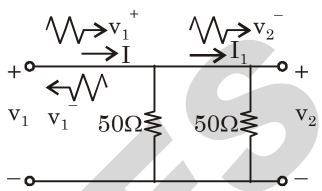
(1) Calculation of S_{11} and S_{21}:

$S_{11} = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0}$



$S_{11} = \dfrac{25-50}{25+50} = \dfrac{-25}{75} = \dfrac{-1}{3}$

From the figure



$V_2 = I_1 \times 50\Omega$

$\Rightarrow V_2 = \dfrac{50}{100}\times I \times 50 = 25\times I$

$= 25\times\dfrac{V_1}{Z_{in}} = 25\times\dfrac{V_1}{25} = V_1$

$\Rightarrow V_2^+ + V_2^- = V_1^+ + V_1^- = (V_1^+ + V_1^-)$

$= V_1^+\left[1 + \dfrac{V_1^-}{V_1^+}\right]$

$\Rightarrow V_2^+ + V_2^- = V_1^+\left[1 + S_{11}\right]$

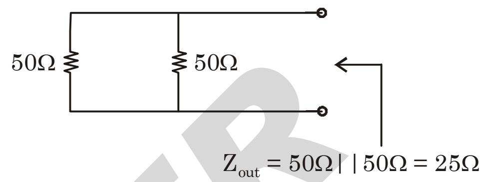
For $S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+=0}$

$\Rightarrow V_2^- = V_1^+\left[1 + S_{11}\right]$

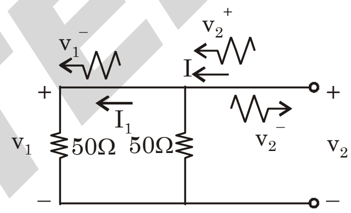
$\Rightarrow \dfrac{V_2^-}{V_1^+} = S_{21} = (1+S_{11}) = 2\left(1 - \dfrac{1}{3}\right) = \dfrac{2}{3}$

(2) Calculation of S_{22} and S_{12}:

$S_{22} = \dfrac{Z_{out} - Z_0}{Z_{out} + Z_0} = \dfrac{25-50}{25+50} = -\dfrac{1}{3}$



From the circuit;



$V_1 = I_1 \times 50\Omega$

$\Rightarrow V_1 = V_2 \qquad$ (From the figure)

$\Rightarrow V_1^+ + V_1^- = V_2^+ + V_2^- = V_2^+\left[1 + \dfrac{V_2^-}{V_2^+}\right]$

$= V_2^+\left[1 + S_{22}\right]$

$\therefore \quad S_{12} = \left.\dfrac{V_1^-}{V_2^+}\right|_{V_1^+=0}$

$\Rightarrow S_{12} = V_1^- = V_2^+\left[1 + \left(\dfrac{-1}{3}\right)\right]$

$\Rightarrow S_{12} = \dfrac{V_1^-}{V_2^+} = \dfrac{2}{3}$

Hence,

$[S] = \begin{bmatrix} \dfrac{-1}{3} & \dfrac{2}{3} \\ \dfrac{2}{3} & \dfrac{-1}{3} \end{bmatrix}$

55. (a)

In Smith chart, any point moving along a constant resistance circle in the clockwise direction the value of reactance (X) will increase which can be obtained by adding an inductance in series with Z = R + jX.

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56. (c)

$$R_{dc} = \frac{l}{\sigma \times \text{area}} = \frac{l}{\sigma\pi a^2}$$

$$R_{ac} = \frac{l}{\sigma 2\pi a\delta}$$

So, $\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = 1$

So, $\delta = \frac{a}{2} = \frac{66.1\times10^{-3}}{\sqrt{f}}$

$$\sqrt{f} = \frac{66.1\times10^{-3}\times2}{a}$$

$$\sqrt{f} = \frac{66.1\times2}{1.2}$$

$\Rightarrow$ f = 12.137 KHz.

57. (d)

$\vec{E}$ can be written as

$$\vec{E} = 40\frac{\left(e^{jax}-e^{-jax}\right)}{2j}\times\frac{\left(e^{jby}+e^{-jby}\right)}{2}$$

$$\vec{E} = -j10\left[e^{j(ax+by)}+e^{j(ax-by)}-e^{-j(ax-by)}-e^{-j(ax+by)}\right]$$

Hence it is a superposition of 4 plane waves.

58. (c)

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$2 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{1}{3}$$

$\because$ Voltage minimum point is found to be at the load. $l = 0$ and $n = 0$

$$\theta - 2\beta l = (2n+1)\pi$$

$$\theta = \pi$$

$$\therefore \quad \Gamma = |\Gamma|e^{j\pi}$$

$$\Gamma = -1/3$$

$$\Rightarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\frac{-1}{3} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Rightarrow Z_L = \frac{Z_0}{2} = \frac{R_0}{2}$$

[$\because$ Characteristic impedance (Z_0) is purely resistive.]

$\therefore$ It is clear that Z_L is also purely resistive.

59. (a)

Average power density = $\frac{1}{2}R_e(E\times H)$

$$\vec{P}_{avg} = \frac{1}{2}\frac{10}{\rho \ln(b/a)}\times\frac{5}{2\pi\rho}\hat{a}_z$$

$$\vec{P}_{avg} = \frac{50}{4\pi\rho^2 \ln(b/a)}\hat{a}_z \ \text{w/m}^2$$

Average power (P) $= \iint \vec{P}_{avg}.ds$

$$P = \int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{50}{4\pi\rho^2 \ln(b/a)}\hat{a}_z\ \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$P = 2\pi\times\frac{50}{4\pi \ln(b/a)}(\ln\rho)_a^b$$

P = 25 watts

60. (d)

Comparing with standard equation

$$E_z = E_o \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\omega t - \beta_g z\right) \ \text{V/m}$$

So, m = 3 , n = 2 , $\beta_g = 50$

As, $\vec{E}$ has a component in direction of wave propagation i.e., Z-direction hence mode is TM mode.

So, the mode is TM_{32} mode.

Now, β (in free space) $= \frac{2\pi\times6\times10^9}{3\times10^8} = 40\pi$

$$\beta_g = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$50 = 40\pi\sqrt{1-\left(\frac{f_c}{6}\right)^2}$$

$$\Rightarrow \boxed{f_c = 5.5\text{GHz}}$$

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61. (a)

For a Hertgian dipole, magnetic field strength is given by

$$H = \frac{I_o B dl \sin\theta}{4\pi r}$$

Here, $dl = \frac{\lambda}{25}$ and $Bdl = \frac{2\pi}{\lambda} \times \frac{\lambda}{25} = \frac{2\pi}{25}$

$$\therefore \; 5 \times 10^{-6} = \frac{I_o \times \left(\frac{2\pi}{25}\right) \times \sin 90°}{4\pi \times 2 \times 10^3} = \frac{I_o}{10^5}$$

$\therefore$ $I_o = 0.5A$

Radiated power is given by:

$$P_{rad} = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_o^2 = \frac{40\pi^2 (0.5)^2}{(25)^2}$$

$\therefore$ $P_{rad} = 158$ mW

62. (d)

$$\because \; \lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m}$$

$P_{rad} = 100$ kW, $A_{et} = 9$ m^2, $\sigma = 10$ m^2, r = 200 nm

Power of reflected signal at the radar is given by:

$$P_r = \frac{A_e \sigma G_d P_{rad}}{[4\pi r^2]^2}$$

Here, $G_d = \frac{4\pi}{\lambda^2} A_e = \frac{4\pi}{(0.05)^2} \times 9 = 14,400\pi$

$$\therefore \; P_r = \frac{9 \times 10 \times 14400 \times \pi \times 100 \times 10^3}{16 \times \pi^2 \times (200 \times 1852)^4}$$

$\therefore$ $P_r = 1.37 \times 10^{-13}$W

63. (c)

For TM mode of operation, if:

$$K^2 = \omega^2 \mu \in < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

then $\gamma = \alpha$ and $\beta = 0$

where γ = Propagation constant = $\alpha + j\beta$

and α = attenuation factor

In this case, there is no propagation at all. These non-propagating a attenuating modes are said to be evancscent.

64. (a)

Given : a = 6 cm   b = 3 cm

$\in_r = 2.25$

$\alpha_g = 5\pi$ Np/m

$\alpha_g = \sqrt{K_c^2 - \omega^2 \mu \in}$

Here, $K_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$$K_c = \sqrt{\left(\frac{\pi}{6}\right)^2 + \left(\frac{\pi}{3}\right)^2} \times \frac{1}{10^{-2}}$$

$K_c = 1.17 \times 10^2$ rad/m

Now, $\alpha_g = 5\pi = \sqrt{K_c^2 - \omega^2 \mu \in}$

$$25\pi^2 - (1.17 \times 10^2)^2 = -\omega^2 \mu \in$$

$$25\pi^2 - (117)^2 = -\left(\frac{2\pi f}{3 \times 10^8} \times \sqrt{2.25}\right)^2$$

$$f = \frac{116 \times 3 \times 10^8}{2\pi \times 1.5}$$

$\boxed{f = 3.69 \text{ GHz}}$

65. (d)

Electric field at a distance 'r' from the half wave dipole antenna is given by:

$$E = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

when, η_o = Free space Impedance = 120π

I_o = current

$$\therefore \; I_o = \frac{20 \times 10^{-6} \times 2\pi \times 500 \times 10^3 \times \sin 90°}{120\pi \cos\left(\frac{\pi}{2}\cos 90°\right)}$$

$\therefore$ $I_o = 166.66$ mA

$$\therefore \; P_{rad} = \frac{1}{2} I_o^2 R_{rad} = \frac{1}{2} \times (166.66 \times 10^{-3})^2 \times 73$$

$\therefore$ $P_{rad} = 1013.8$ mW

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