[EE] CLASSROOM PRACTICE TEST-03
EMFT (SOLUTIONS)

ANSWER KEY

1.	(b)	**14.**	(b)	**27.**	(c)	**40.**	(b)	**53.**	(b)
2.	(b)	**15.**	(b)	**28.**	(a)	**41.**	(d)	**54.**	(b)
3.	(c)	**16.**	(a)	**29.**	(b)	**42.**	(c)	**55.**	(d)
4.	(b)	**17.**	(b)	**30.**	(c)	**43.**	(b)	**56.**	(d)
5.	(b)	**18.**	(d)	**31.**	(c)	**44.**	(a)	**57.**	(d)
6.	(d)	**19.**	(d)	**32.**	(c)	**45.**	(a)	**58.**	(d)
7.	(d)	**20.**	(d)	**33.**	(b)	**46.**	(d)	**59.**	(a)
8.	(d)	**21.**	(b)	**34.**	(d)	**47.**	(b)	**60.**	(b)
9.	(a)	**22.**	(d)	**35.**	(a)	**48.**	(c)	**61.**	(a)
10.	(a)	**23.**	(d)	**36.**	(a)	**49.**	(a)	**62.**	(d)
11.	(d)	**24.**	(a)	**37.**	(a)	**50.**	(a)	**63.**	(c)
12.	(a)	**25.**	(a)	**38.**	(b)	**51.**	(c)	**64.**	(a)
13.	(b)	**26.**	(a)	**39.**	(a)	**52.**	(c)	**65.**	(a)

1. (b)

- Gradient is always taken for a scalar quantity and it produces a vector.
- Divergence is always taken for a vector quantity and it results into a scalar.
- Curl is always taken for a vector quantity and it results into a vector.

So,

$\nabla(\nabla.\vec{A})$ -Gradient of a scalar- correct

$\nabla(\nabla\times\vec{A})$ - Gradient of a vector - Incorrect

$\nabla.(\nabla\times\vec{A})$ -Divergence of a vector - correct

$\nabla\times\nabla V$ - Curl of a vector - correct

2. (b)

Given, electric potential, $V = 3x^2y - yz$

So, Electric field intensity

$$\vec{E} = -\nabla V$$

$$= -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial x}\hat{a}_z\right]$$

$$= \left[6xy\,\hat{a}_x + (3x^2 - z)\hat{a}_y + (-y)\hat{a}_z\right]$$

$$\vec{E}\Big|_{(1,0,-1)} = \left[6\times1\times(0)\hat{a}_x + \left(3\times1^2 - (-1)\hat{a}_y + (0)\hat{a}_z\right)\right]$$

$$= -4\hat{a}_y \neq 0$$

$$\vec{E}\Big|_{(2,-1,4)} = -\left[6\times(2)\times(-1)\hat{a}_x + (30\times2^2 - 4)\hat{a}_y + (+1)\hat{a}_z\right]$$

$$= 12\hat{a}_x - 8\hat{a}_y - \hat{a}_z \text{ V/m}$$

Now, potential V at (2, –1, 4)

$$V_{(2,-1,4)} = 3\times(2)^2(-1) - (-1)(4)$$

$$= -12 + 4 = -8\text{V}$$

3. (c)

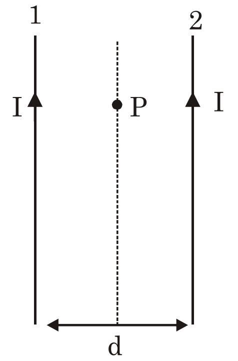
$$\oint_L \vec{H}\cdot\overline{dl} = \int_s (\nabla\times\vec{H})\cdot\overline{ds} \quad \text{(Stoke's Theorem)}$$

$$= \int_s J\cdot ds = I \quad \text{(current through the cross-section)}$$

$$= \psi \quad \text{(flux through the surface S)}$$

4. (b)

Magnetic field intensity (H) due to wire 1 at a distance d/2 (i.e at point P)



$$\overline{H}_1 = \frac{I}{\pi d} \text{ (downward)}$$

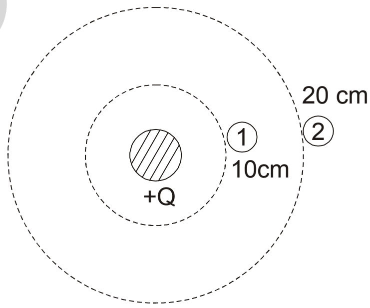
And due to wire 2 at P

$$\overline{H}_2 = \frac{I}{\pi d} \text{ (upward)}$$

The vector addition of these two fields

$$\overline{H} = \overline{H}_1 + \overline{H}_2 = \left(\frac{I}{\pi d} - \frac{I}{\pi d}\right) = 0$$

5. (b)



As the sphere of radius 10 cm encloses the total charge Q. So the concentric sphere of radius 20 cm will definitely enclose the total charge Q. According to Gauss's Law

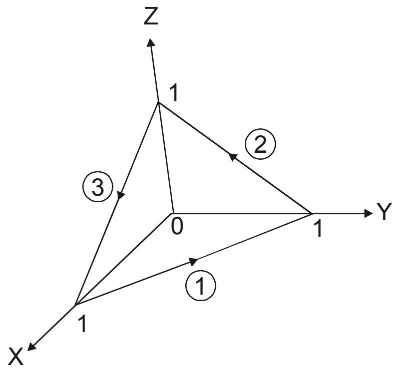
$(\text{flux})_2$ = total charges enclosed = $(\text{flux})_1$ = 20 V.m.

6. (d)

In electrostatics, a perfect conductor has following characteristics :

1. No electric field exist within a conductor i.e. electric field will be only external
2. Electric field will be normal to its surface.
3. Since $E = -\nabla V = 0$,there will be no potential difference between any points on its surface.

7. (d)



Circulation $\vec{F}$ around closed path

$= \oint \vec{F}.\vec{dl}$

$= \int_1 \vec{F}.\vec{dl} + \int_2 \vec{F}.\vec{dl} + \int_3 \vec{F}.\vec{dl}$

For curve (1), Z = 0

So, $\vec{F} = x\hat{a}_x + y\hat{a}_y$

Then, $\int_1 \vec{F}.\vec{dl} = \int (x\hat{a}_x + y\hat{a}_y).(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$

$= \int_1^0 x\,dx + \int_0^1 y\,dy$

$= \left.\frac{x^2}{2}\right|_1^0 + \left.\frac{y^2}{2}\right|_0^1 = -\frac{1}{2} + \frac{1}{2} = 0$

The value of $\int \vec{F}.\vec{dl}$ will be same for curve (2) and curve (3). So,

$\int \vec{F}.\vec{dl} = 0$

8. (d)

Ampere Law is a special case of Biot Savart Law. Using Ampere's Law, magnetic field intensity for symmetrical current distributor can be obtained easily.

[Note : Gauss Law is a special case of Coulomb's Law where charge distribution is symmetrical]

Ampere's Law states that the line integral of H around a closed path is the same as the net current enclosed by the path.

i.e. $\oint_L H \cdot dl = I_{enc.}$

9. (a)

For an electrostatic field $(\vec{E})$:

$$\oint \vec{E}.dl = \int_S (\nabla \times E)\,ds = 0$$

or $(\nabla \times E) = 0$

Any vector field that satisfies above equations are called conservative or irrotational field. In other words, vectors whose line integral does not depend on the path of integration are called conservative vector fields. This is true for electric fields as $\oint \vec{E}.dl$ represents potential difference in a closed loop which is definitely zero.

Hence, Both statements (i) and (ii) are correct and (ii) is the correct explanation of (i).

10. (a)

At dielectric-dielectric boundary, tangential components of electric field (E_t) are the same on two sides of the boundary i.e.

$$E_{t_1} = E_{t_2} \quad \text{... (i)}$$

We know, $D = \varepsilon E$

$\therefore \quad D_{t_1} = \varepsilon_1 E_{t_1}$ and $D_{t_2} = \varepsilon_2 E_{t_2}$

Using E_{t_1} and E_{t_2} in equation (i)

$$\frac{D_{t_1}}{\varepsilon_1} = \frac{D_{t_2}}{\varepsilon_2} \quad \text{... (ii)}$$

Therefore, tangential component of field density (D_t) undergoes some change across the interface.

11. (d)

Applying Gauss law on dielectric-dielectric boundary, we get

$D_{n_1} = D_{n_2}$ = normal components of D on both sides of interface

ρ_s = charge on interface

If no free charge exist at the interface then, $\rho_s = 0$

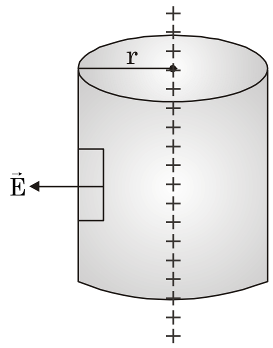
Therefore, from equation (i), we get

and $\boxed{D_{n_1} = D_{n_2}}$

$\boxed{\varepsilon_1 E_1 = \varepsilon_2 E_2}$

Normal component of D is continuous across the interface whereas normal component of E is discontinuous.

12. (a)

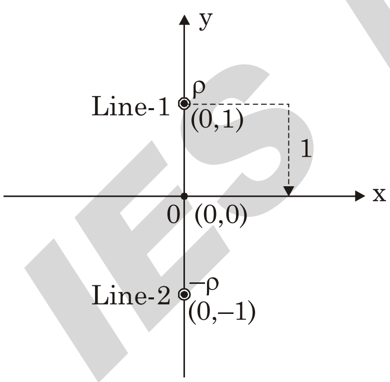


According to Gauss's Law,

$$\oint \vec{E} \cdot \overrightarrow{ds} = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \quad E \cdot 2\pi r \cdot l = \frac{(\rho l)}{\epsilon_0}$$

$$\therefore \quad \vec{E} = \frac{\rho}{2\pi \epsilon_0 r}\hat{a}_r$$



So, electric field at the origin by line-1,

$$\vec{E}_1 = \frac{\rho}{2\pi \epsilon_0 r}\left(-\hat{a}_y\right)$$

$$= \frac{\rho}{2\pi \epsilon_0}\left(-\hat{a}_y\right) \qquad [\because r = 1]$$

and electric field at the origin by line-2,

$$\vec{E}_2 = \frac{\rho}{2\pi \epsilon_0 r}\left(-\hat{a}_y\right)$$

$$= \frac{\rho}{2\pi \epsilon_0}\left(-\hat{a}_y\right) \qquad [\because r = 1]$$

so, resultant electric field intensity at the origin

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{2\rho}{2\pi \epsilon_0}\left(-\hat{a}_y\right) = \frac{\rho}{\pi \epsilon_0}\left(-\hat{a}_y\right)$$

13. (b)

$$E = -\frac{dV}{dx} = -\frac{d}{dx}(5x^2 + 10x - 9)$$

$= -[10x + 10]$

$= -[10(1) + 10]$

$= -20$ V/m

14. (b)

The electric field is given by—

$E = \nabla V$

$$= -80\frac{\partial}{\partial \rho}\rho^{0.6}\ \hat{a}_\rho$$

$$E = -48\rho^{0.4}\hat{a}_\rho \text{ V/m}$$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{D} = -48\ \varepsilon_0 \rho^{-0.4}\hat{a}_\rho \text{ C/m}^2$$

At $\rho = 0.6$m,

$$\vec{D} = -48\ \varepsilon_0 . (0.6)^{-0.4}\hat{a}_\rho$$

$$\vec{D} = -0.521 \times 10^{-9}\ \hat{a}_\rho \text{ C/m}^2$$

The flux density is constant.

So, the flux $\psi = \vec{D}.\vec{S}$

$= -0.521 \times 10^{-9} \times 2\pi\rho z$

$= -0.521 \times 10^{-9} \times 2\pi \times 0.6 \times 1$

$\psi = -1.96$ nC

From Gaussis law, $Q_{emc.} = \psi = -1.96$ nC.

15. (b)

The electric field intensity

$$\vec{E} = -\vec{\nabla}.V = \frac{\rho_0}{a\varepsilon_0}\alpha \,.\, e^{-\alpha x}\hat{a}_x$$

$$\vec{E} = \frac{\alpha\rho_0}{a\varepsilon_0} \cdot e^{-\alpha x}\,\hat{a}_x$$

Energy density, $W_E = \frac{1}{2}\varepsilon_0 E^2$

$$W_E = \frac{1}{2}\left(\frac{\alpha^2\rho_0^2}{a^2\varepsilon_0}\right) \cdot e^{-2\alpha x}$$

Energy stored in region,

$$W_E = \frac{1}{2}\left(\frac{\alpha^2\rho_0^2}{a^2\varepsilon_0}\right) \cdot \int_0^1\int_0^1\int_0^d e^{-2\alpha x}dx\,dy\,dz$$

$$W_E = \frac{1}{2}\frac{\alpha^2\rho_0^2}{a^2\varepsilon_0}(1)(1)\frac{(e^{-2\alpha d}-1)}{(-2\alpha)}$$

$$\boxed{W_E = \frac{\alpha\rho_0^2}{4a^2\varepsilon_0}\left(1-e^{-2\alpha d}\right)}$$

16. (a)

Since field $\vec{A}$ is irrotational

$\nabla \times \vec{A} = 0$

$\therefore$ $\vec{A}$ shold be gradient of some scalar field $A = \nabla V$, as curl of a gradient is always zero.

17. (b)

The magnetization $(\vec{M})$ is given by

$$\vec{M} = X_m\vec{H} = \frac{X_m\vec{H}}{\mu_0\mu_r} = \frac{X_m\vec{B}}{\mu_0(1+X_m)}$$

$$\vec{M} = \frac{3.1\times 0.4\hat{a}_y}{4\pi\times 10^{-7}\times(4.1)}$$

$\vec{M} = 2.41 \times 10^5\ \hat{a}_y$ A/m.

18. (d)

Ampere Law states that $\oint \vec{H}\cdot\vec{dl} = I_{enclosed}$.

$\therefore$ $\oint \vec{H}\cdot\vec{dl} = -30 + 10 = -20A$

–30A is taken because the path taken is opposite to the direction of magnetic field due to 30A current.

19. (d)

If the potential of cylinder at $\rho = a$ is given as V_0, and that of $\rho = b$ is given as 0 (b > a), then the potential of cylindrical surface at $\rho = \rho_1$ will be given by

$$V = V_o \cdot \frac{\ln(b/\rho_1)}{\ln(b/a)}$$

Here, V_0 = 100 V

b = 1.2 cm.

a = 0.5 cm.

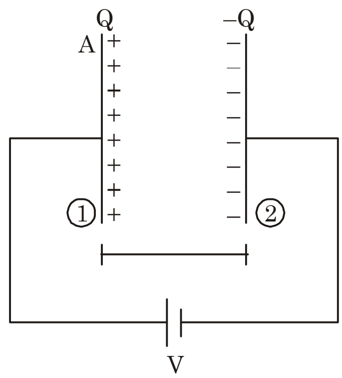
V = 20 V.

$$\therefore\quad 20 = 100\frac{\ln(1.2/\rho_1)}{\ln(1.2/0.5)}$$

$$\ln\left(\frac{1.2}{\rho_1}\right) = 0.175 \Rightarrow \frac{1.2}{\rho_1} = 1.19$$

$$\boxed{\rho_1 = 1.01\,\text{cm.}}$$

20. (d)



Electric field near plate 2 due to plate 1 will be :

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Therefore, force on plate Z due to electric field will be :

$$|\vec{F}| = QE = \frac{Q^2}{2A\varepsilon_0} = \frac{C^2V^2}{2A\varepsilon_0} \quad \because C = \frac{\varepsilon_0 A}{d}$$

$$\therefore\quad |\vec{F}| = \frac{(\varepsilon_0 A)^2.V^2}{2(\varepsilon_0 A).d^2}$$

$$\therefore\quad |\vec{F}| = \left(\frac{\varepsilon_0 A}{2d^2}\right)V^2$$

21. (b)

The current density is given by $\vec{J} = \rho_0 v$

as $v = \rho\Omega\hat{a}_\phi$

$$\boxed{\vec{J} = \rho_{0.}\ \rho\Omega\ \hat{a}_\phi\ \text{A/m}^2}.$$

22. (d)

The net capacitance can be considered as two capacitances in series.

$$C_1 = \frac{\varepsilon_1 A}{(d/2)} \text{ and } C_2 = \frac{\varepsilon_2 A}{(d/2)}$$

$$\therefore\ C = \frac{C_1C_2}{C_1+C_2} = \frac{4\varepsilon_1\varepsilon_2A^2}{d^2 \times \frac{2A}{d}(\varepsilon_1+\varepsilon_2)}$$

$$C = \frac{2A}{d}\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)$$

23. (d)

For $0<\rho<6 : 2\pi\rho H_\phi = 16, H = \frac{16}{2\pi\rho}\hat{a}_\phi$

For $6<\rho<10 : 2\pi\rho H_\phi = 16-12, H = \frac{4}{2\pi\rho}\hat{a}_\phi$

For $\rho>10 : 2\pi\rho H_\phi = 16-12-4 = 0, H = 0$

The best relation between H and P is described by (d).

24. (a)

Since both V_1 and V_2 satisfy the boundary condition and Laplace equation, hence both are correct and the solution is not unique.

25. (a)

$$\iint(\nabla\times\vec{P})ds$$

According to Stoke's theorem, which relates line integral to surface integral,

$$\oint \vec{H}.\vec{dl} = \iint(\nabla\times\vec{H}).ds$$

Thus, $\iint(\nabla\times\vec{P})ds = \oint\vec{P}.\vec{dl}$

26. (a)

The current density is given by

$$\vec{J} = \vec{\nabla}\times\vec{H}$$

$$\vec{J} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ 0 & \left(\frac{x+2y}{z^2}\right) & \frac{2}{z} \end{vmatrix}$$

$$= \hat{a}_x\left\{0-\frac{\partial}{\partial z}\left(\frac{x+2_y}{z^2}\right)\right\} - \hat{a}_y(0-0) + \hat{a}_z\left[\frac{\partial}{\partial z}\left(\frac{x+2_y}{z^2}\right)\right]$$

$$\vec{J} = \hat{a}_x\left(\frac{2}{z^3}\right)(x+2y) + \hat{a}_z\left(\frac{1}{z^2}\right)$$

at P (1,5,3)

$$\vec{J} = \hat{a}_x\left(\frac{2}{3^3}\right)(1+10) + \hat{a}_z\left(\frac{1}{3^2}\right)$$

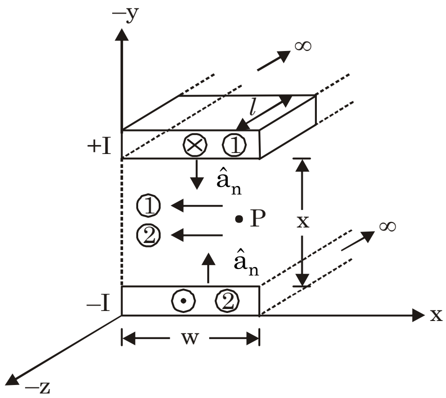
$$\boxed{\vec{J} = 0.81\hat{a}_x + 0.11\,\hat{a}_z}$$

27. (c)

The current density due to an infinite sheet is given by

$\bar{k} = \frac{I}{w}(\hat{a}_z)$ – for top strip

$\bar{k} = \frac{I}{w}(-\hat{a}_z)$ – for bottom strip



The magnetic field due to an infinite sheet of current is given by

$$\bar{H} = \frac{1}{2}\bar{k}\times\hat{a}_n$$

$\hat{a}_n$ = unit normal vector directed from the current sheet to point of interest.

For upper strip,

$$\bar{H} = \frac{1}{2}\cdot\frac{I}{w}(\hat{a}_z\times\hat{a}_y) = \frac{I}{2w}(-\hat{a}_x)$$

For lower strip,

$$\bar{H} = \frac{1}{2}\frac{I}{w}\left[(-\hat{a}_z)\times(-\hat{a}_y)\right] = \frac{I}{2w}(-\hat{a}_x)$$

$\therefore \bar{H}$ at point P due to both strips will be

$$\bar{H} = \frac{I}{2w}(-\hat{a}_x)\times 2 = -\frac{I}{w}\hat{a}_x$$

$$\therefore \quad \bar{B} = \mu_0\bar{H} \equiv -\frac{\mu_0 I}{w}\hat{a}_x$$

Now, differential area of the strip,

$$d\bar{S} = dydz(-\hat{a}_x)$$

$$\therefore \quad \psi_m = \int \bar{B}\cdot d\bar{S} = \int \frac{\mu_o I}{w}\hat{a}_x \cdot dydz\hat{a}_x$$

$$= \frac{\mu_o I}{w}\int_{y=0}^{x} dy \int_{z=0}^{l} dz$$

$$\Rightarrow \quad \psi_m = \frac{\mu_o I}{w}\cdot x \cdot l$$

$$\therefore \quad L = \frac{\psi_m}{I} = \frac{\mu_o x}{w}l$$

$\Rightarrow$ Inductance per unit length

$$\frac{L}{l} = L' = \mu_0 \frac{x}{w}$$

$$\therefore \quad L' \propto x$$

If $x \to \frac{x}{2}$

then, $L' \to \frac{L}{2}$ H/m.

Hence, option (c) is correct.

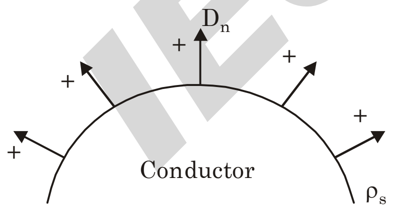
28. (a)

Image theory is applicable only in static fields.

(i) Charge distribution is replaced with the original charge distribution and its image charges.

(ii) The conducting surface is replaced by an equipotential surface.

29. (b)



- The tangential component of electric field lines does not exist on a conducting surface boundary.
- any conducting surface always supports the normal component of the electric field such that the normal component of electric flux density $\vec{D}$ is equal to the magnitude of the surface charge density ρ_S present on the conducting surface.

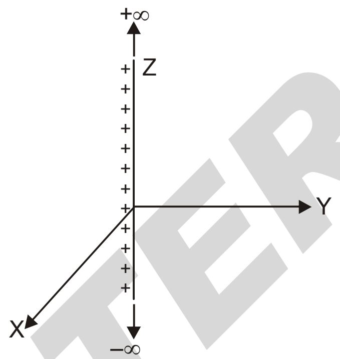
30. (c)

As we know that for an infinitely long current carrying conductor

$$H_\phi = \frac{I}{2\pi r}\hat{a}_\phi$$

$\therefore \quad H_\phi \alpha \frac{I}{r}$ Hence, Rectangular hyperbola.

31. (c)



According to Gauss's Law

$$\oint_s \vec{E}.\vec{ds} = \frac{\text{charge enclosed}}{\varepsilon_0}$$

$$E.2\pi R.l = \frac{\rho.l}{\varepsilon_0}$$

$$\therefore \quad E = \frac{\rho}{2\pi\varepsilon_0 R}$$

This is the electric field intensity by an infinitely long line charge at a distance R, and its direction will be radially outward.

So, $\bar{E}$ at (R, 0, 0) $= \frac{\rho}{2\pi\varepsilon_0 R}\hat{a}_x$

32. (c)

$$\vec{E} = \rho\cos 2\phi\,\widehat{a_\rho} - \rho\sin 2\phi\,\widehat{a_\phi}$$

$$\frac{d\rho}{\rho d\phi} = \frac{E_\rho}{E\phi}$$

$$\frac{d\rho}{\rho} = \frac{\rho\cos 2\phi}{\rho\sin 2\phi}\, d\phi$$

Integrating $\Rightarrow \ln\rho = \frac{-1}{2}\ln\{\sin 2\phi\} + C$

The line passes through P(2, 30°, 0),

$\Rightarrow \ln 2 = \frac{-1}{2}\ln\{\sin 60°\} + C$

C = 0.62

$\therefore \ln\rho = \frac{-1}{2}\ln\{\sin 2\phi\} + 0.62$

$2\ln\rho + \ln\{\sin 2\phi\} = 0.62 \times 2$

$\boxed{\rho^2 \sin^2\phi = 3.46}$

33. (b)

The electric flux leaving the surface, will be equal to the charge contained within it.

$$\therefore \psi = Q = \int_{30}^{90^\circ}\int_{.01}^{0.05} \rho_s.\rho\, d\phi\, dz$$

$$\therefore \psi = \rho.\int_{\pi/6}^{\pi/2} d\phi.\int_{0.01}^{0.05} 5e^{-20z}\, dz$$

$$= (0.08)\left(\frac{\pi}{2}-\frac{\pi}{6}\right).\left(\frac{5}{-20}\right)\left|\rho^{-20z}\right|_{0.01}^{0.05}$$

$= -0.0209 \times (-0.45)$

$\therefore \psi = 9.41\times 10^{-3}$ nC

$\boxed{\psi = 9.41\text{pC}}$

34. (d)

Magnetic boundary conditions are

$\left[B_{n_1} = B_{n_2}\right]$ or $\left[\mu_1 H_{n_1} = \mu_2 H_{n_2}\right]$

Therefore, B_n is continuous ; H_n is discontinuous at boundary.

Here, B_n, H_n = normal components of B and H.

Also, $H_{1t} = H_{2t} = k$

$$\therefore \quad \frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = k$$

Therefore, B_t is discontinuous at boundary

Here, B_t, H_t are tangential components

K = free current density at interface

35. (a)

$\vec{E} = 20\, e^{-5y}\left(\cos 5x\,\widehat{a_x} - \sin 5x\,\widehat{a_y}\right)$

$= 20\, e^{-5y}\cos 5x\,\widehat{a_x} - 20\, e^{-5y}\sin 5x\,\widehat{a_x}$

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{-\sin 5x}{\cos 5x}$$

$\therefore dy = -\tan 5x\, dx$

Integrating $\Rightarrow \int dy = -\int \tan 5x dx$

$y = \frac{1}{5}\ln\{\cos 5x\} + C \quad$ –(i)

As the line represented by eq. (i) passes through $\left(\frac{\pi}{3}, 0.1, 2\right)$, so $\rightarrow$

$$0.1 = \frac{1}{5}\ln\cos\left(\frac{5\pi}{3}\right) + C$$

$\boxed{C = 0.24}$

$\therefore$ The equation of stream line is

$$\boxed{y = \frac{1}{5}\ln(\cos 5x) + 0.24}$$

36. (a)

The differential area in $\hat{a}_z$ direction is given by $d\vec{s} = \rho d\rho d\phi \hat{a}_z$

$\therefore \quad I = \int \vec{J}.\,d\vec{s}$

$$I = \int_0^{2\pi}\int_0^{0.4}\frac{-20}{\rho^2+0.01}\rho d\rho\, d\phi$$

$$= -20\int_0^{2\pi} d\phi \int_0^{0.4}\frac{\rho d\rho}{(\rho^2+0.01)}$$

$$= -20\,(2\pi)\frac{1}{2}\left|ln(\rho^2+0.01)\right|_0^{0.4}$$

$= -20\pi\,[ln\,(0.16 + 0.01) - ln\,(0.01)]$

$$I = -20\pi\, ln\left(\frac{0.17}{0.01}\right) = -20\pi\, ln(17)$$

$\boxed{I = -178\text{ A}}$.

37. (a) Since, $\chi_m = \mu_r - 1 = 6.5 - 1 = 5.5$

then, magnetisation

$\vec{M} = \chi_m \vec{H}$

$\therefore \quad \vec{M} = 5.5\times\left(10\hat{a}_x + 25\hat{a}_y - 40\hat{a}_z\right)$

$= \left(55\hat{a}_x + 137.5\hat{a}_y - 220\hat{a}_z\right)$ A/m^2

38. (b)

$\vec{D} = \varepsilon\vec{E}$

$$\vec{D} = \left(\frac{10^{-5}}{\rho}\right)\cos(10^5 t)\hat{a}_\rho$$

$\therefore \quad \vec{J}_D = \frac{\partial D}{\partial t}$

$= \frac{\partial}{\partial t}\left\{\left(\frac{10^{-5}}{\rho}\right)\cos(10^5 t)\right\}\hat{a}_\rho$

$\vec{J}_D = -\left\{\frac{10^{-5}}{\rho}\sin(10^5 t)\right\}(10^5)$

$\vec{J}_D = -\frac{1}{\rho}\sin(10^5 t)\,\hat{a}_\rho$

The displacement current will flow through the surface area of the cylinderical capacitor, radially outward or inward.

$I_D = 2\pi\rho l.J_D \Rightarrow 2\pi\rho l\left\{\frac{-1}{\rho}\sin(10^5 t)\right\}$

$I_D = -2\pi.l\sin(10^5 t)$

$= - \; -2\pi(0.4)\sin(10^5 t)$

$\boxed{I_D = -0.8\pi\sin(10^5 t)}$ A .

39. **(a)**

The two planes are →

2x + 3y – 12 = 0

and 2x + 3y – 18 = 0

Using two boundary conditons, we can write the general potential function as ⇒

V (x_1 y) = A (2x + 3y – 12) +100

V (x_1 y) = A (2x + 3y – 18) + 0

A (2x + 3y –12) + 100 = A (2x + 3y –18)

100 = –6A

$\boxed{A = \frac{-100}{6}}$

$V(x, y) = \frac{-100}{6}(2x + 3y - 18)$

= –33.33x –50y + 300

$E = -\nabla V$

$\boxed{\vec{E} = 33.33\hat{a}_x + 50\hat{a}_y}$

The electric field function is independent of position.

40. **(b)**

Let the required point is $P_3(0, y, 0)$

Then,

$\vec{R}_{13} = (0-4)\hat{a}_x + (y+2)\hat{a}_y + (0-7)\hat{a}_z$

$= -4\hat{a}_x + (y+2)\hat{a}_y - 7\hat{a}_z$

and $\vec{R}_{23} = (0+3)\hat{a}_x + (y-4)\hat{a}_y + (0+2)\hat{a}_z$

$= 3\hat{a}_x + (y-4)\hat{a}_y + 2\hat{a}_z$

Now, electric field at P_3 X-direction,

$$E_x = \frac{10^{-9}}{4\pi\varepsilon_0}\left[\frac{25\times(-4)}{\left(\sqrt{(-4)^2 + (y+2)^2 + (-7)^2}\right)^3} + \frac{60\times 3}{\left(\sqrt{(3)^2 + (y-4)^2 + (2^2)}\right)^3}\right]$$

As $E_x = 0$, we get

$0.48y^2 + 13.9y + 73.12 = 0$

$\Rightarrow$ y = –6.89 or –22.11

i.e. P_3 = (0, –22.11, 0)

41. **(d)**

The potential at any point due to the combination of these three charge distributions is

$$V = \frac{2\times 10^{-6}}{4\pi\epsilon_0 r} - \frac{8\times 10^{-9}}{2\pi\epsilon_0}ln(\rho) - \frac{5\times 10^{-9}}{2\varepsilon_0}(z) + C$$

Where, r = distance of the point from (2, 0, 0)

ρ = distance of the point from line x = 0, z = 4.

z = z co-ordinate of the point.

at m (0, 0, 5) —

$r = \sqrt{2^2 + 5^2} = \sqrt{29}$

ρ = 5 – 4 = 1

z = 5

V = 0.

$$0 = \frac{2\times 10^{-6}}{4\pi\varepsilon_0\sqrt{29}} - \frac{8\times 10^{-9}}{2\pi\varepsilon_0}ln(1) - \frac{-5\times 10^{-9}}{2\varepsilon_0}(z) + C$$

$$= \frac{2\times 10^{-6}\times 9\times 10^9}{\sqrt{29}} - 0 - \frac{5\times 10^{-9}\times 5}{2\times 8.85\times 10^{-12}} + C$$

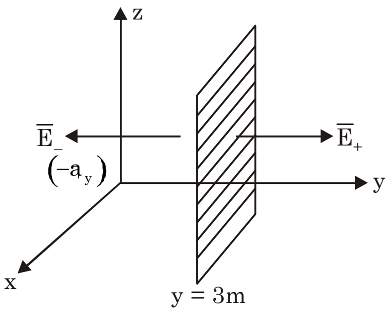
$C = -3.34 \times 10^3 + 1.414 \times 10^3$

$= -1.93 \times 10^3$

$\therefore$ The expression for potential is given by—

$$V = \frac{2\times10^{-6}}{4\pi\varepsilon_0 r} - \frac{8\times10^{-9}}{2\pi\varepsilon_0} ln(\rho) - \frac{5\times10^{-9}}{2\varepsilon_0}(z) - 1.93\times10^3.$$

42. (c)



The electric field intensity at a point due to plane

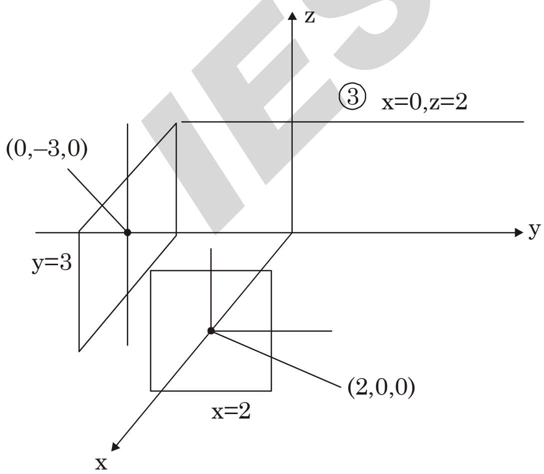
$$E = \frac{\rho_s}{2\varepsilon_0}\hat{a}_n$$

$$E_{(3,-3,3)} = \frac{10^{-8}\times36\pi\times10^9}{6\pi\times2}(-\hat{a}_y)$$

$= -30\hat{a}_y$ V/m

43. (b)

Contribution to $\vec{E}$ at point (1, 1, –1) due to the infinite sheet 1, infinite sheet 2, and infinite line 3 as shown in figure. Let these are E_1, E_2 and E_3 respectively.



$$E_1 = \frac{\rho_{s1}}{2\in_0}(-\hat{a}_x) = \frac{-10\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}a_x = -180\pi\hat{a}_x$$

$$E_2 = \frac{\rho_{s2}}{2\in_0}\hat{a}_y = \frac{15\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}\hat{a}_y = 270\pi\,\hat{a}_y$$

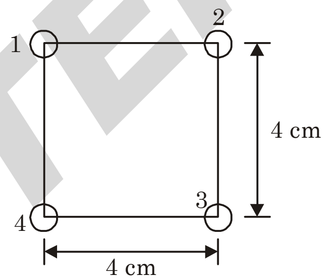
and

$$E_3 = \frac{\rho_L}{2\pi\in_0\rho}\hat{a}_\rho = \frac{\rho_L}{2\pi\in_0\rho^2}\cdot\vec{a}_\rho = \frac{10\pi\cdot10^{-9}}{2\pi\cdot\frac{10^{-9}}{36\pi}\cdot10}\cdot(\hat{a}_x - 3\hat{a}_z)$$

$= +18\pi(\hat{a}_x - 3\hat{a}_z)$

$\therefore \quad \vec{E} = (-162\pi\hat{a}_x + 270\pi\hat{a}_y - 54\pi\hat{a}_z)$ V/m

44. (a)



Total potential energy stored,

$$W = \frac{1}{2}\sum_{n=1}^{4} q_n V_n$$

$$= \frac{1}{2}q_1V_1 + \frac{1}{2}q_1V_2 + \frac{1}{2}q_3V_3 + \frac{1}{2}q_4V_4$$

Here, potential at charge '1' is,

$V_1 = V_{21} + V_{31} + V_{41}$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{0.04} + \frac{1}{0.04} + \frac{1}{0.04\sqrt{2}}\right]$$

$$= 1.2\times10^{-9}\times9\times10^9\left[\frac{1}{0.04}\left(2+\frac{1}{\sqrt{2}}\right)\right] = 730.89$$

Here, $V_1 = V_2 = V_3 = V_4 = V$

So, total potential energy stored.

$$= 4\times\frac{1}{2}qV = 2\times1.2\times10^{-9}\times730.89$$

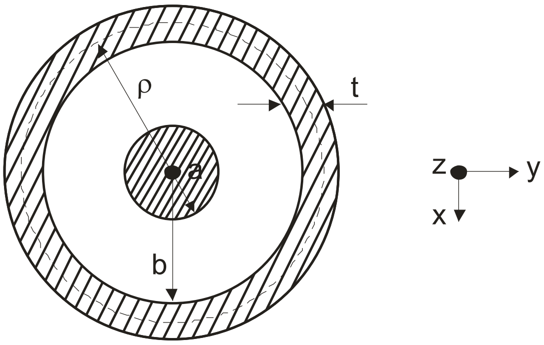
$= 1.75\ \mu J$

45. (a)

Step 1: Since we wish to determine $|\vec{H}|$ at distance ρ such that

$b < \rho < (b + t)$

$\therefore$ We first draw an amperian loop (L) of radius ρ



Step 2: Using Ampere's Law

$$\oint_L \vec{H}\cdot\vec{dl} = H_\phi \cdot 2\pi\rho = I_{enc} = \underset{\substack{\text{due to}\\ \text{inner}\\ \text{conductor}}}{I} + \underset{\substack{\text{due to outer}\\ \text{conductor}}}{\int \vec{J}\cdot\vec{dl}} \quad \ldots(i)$$

Now,

$\vec{J}$ is the current density of the outer conductor and is along $-\hat{a}_z$

$$\therefore \quad \vec{J} = \frac{I}{\pi\left[(b+t)^2 - b^2\right]}(-\hat{a}_z) \quad \ldots(ii)$$

$$\therefore \quad I_{enc} = I(\hat{a}_z) - \left[\frac{I}{\pi\left[(b+t)^2 - b^2\right]}\int_{\phi=0}^{2\pi}\int_{\rho=b}^{\rho} \rho\, d\rho\, d\phi\right](\hat{a}_z) \quad \ldots(iii)$$

$$\text{or} \quad I_{enc} = I\left[1 - \frac{1}{\pi\left[(b+t)^2 - b^2\right]}\left(\frac{\rho^2}{2} - \frac{b^2}{2}\right)(\phi)\Big|_0^{2\pi}\right]$$

$$\text{or} \quad I_{enc} = I\left[1 - \frac{(\rho^2 - b^2)}{t^2 + 2bt}\right]$$

Step 3: $\therefore$ using equation (iv) in equation (i)

$$H_\phi \cdot 2\pi\rho = I\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2bt)}\right]$$

$$\therefore \quad H_\phi = \frac{I}{2\pi\rho}\left[\frac{1 - (\rho^2 - b^2)}{(t^2 + 2bt)}\right] \quad \ldots(v)$$

Which we wish to find.

Step 4: Substituting I = 5mA

$\rho = 16$ mm ; $b = 15$ mm ; $t = 2$ mm

In equation (v); we get :

$$\left|\vec{H}_\phi\right| = \frac{5\times10^3}{2\pi(16)}\left[1 - \frac{(16^2 - 15^2)}{(4+60)}\right] (\text{mA/m})$$

$$\text{or} \quad \left|\vec{H}_\phi\right| = 25.64 \text{ (mA/m) Ans.}$$

46. (d)

Ampere Law states that $\oint \vec{H}\cdot\vec{dl} = I_{\text{enclosed}}$.

$$\oint \vec{H}\cdot\vec{dl} = -30 + 10 = -20\text{A}$$

–30A is taken because the path taken is opposite to the direction of magnetic field due to 30A current.

47. (b)

$$D_2 = ? \text{ Region} - 2\ (\varepsilon_2 = 1)$$
$$z = 0$$
$$D_1 = 10a_x + 7.5a_y + 6a_z$$
$$\text{Region} - 1\ (\varepsilon_1 = 2.5)$$

According to boundary conditions,

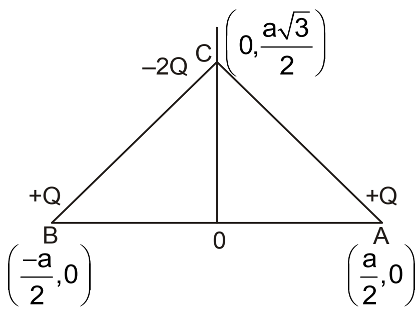
$$D_{n_1} = D_{n_2} \qquad E_{t_1} = E_{t_2}$$

$$\boxed{6a_z = D_{n_2}} \quad \Rightarrow \quad \frac{D_{t_2}}{\varepsilon_2} = \frac{D_{t_1}}{\varepsilon_1}$$

$$\Rightarrow \quad D_{t_2} = \frac{10a_x + 7.5a_y}{2.5} = 4a_x + 3a_y$$

$$\therefore \quad D_2 = D_{t_2} + D_{n_2} = 4a_x + 3a_y + 6a_z$$

48. (c)



Let the point P(0, y)

Potential at point P(0, y) due to all three charges

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{AP} = \frac{KQ}{AP}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{BP} = \frac{KQ}{BP}$$

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{CP} = -\frac{2KQ}{CP}$$

$$= V_1 + V_2 + V_3 = 0 = \frac{KQ}{AP} + \frac{KQ}{BP} - \frac{2KQ}{CP}$$

$[\because\ AP = BP]$

or, $$\frac{2KQ}{AP} = \frac{2KQ}{CP}$$

or, $$AP = CP$$

$$\sqrt{\frac{a^2}{4} + y^2} = \left(y - \frac{a\sqrt{3}}{2}\right)$$

or, $$\frac{a^2}{4} + y^2 = y^2 + \frac{a^2 3}{4} - ay\sqrt{3}$$

$$y\left(a\sqrt{3}\right) = \frac{a^2}{2}$$

$$y = \frac{a}{2\sqrt{3}}$$

Therefore, the coordinates of the point P is $P\left(0, \frac{a}{2\sqrt{3}}\right)$.

49. **(a)**

The general point in z = 0 plane is P (x,y,0)

The vector directed from the line towards this point is

$$\vec{R} = (x-x)\hat{a}_x + (y+2)\hat{a}_y + (0-5)\hat{a}_z$$

$$= (y+2)\hat{a}_y - 5\hat{a}_z$$

The direction of electric field will be same as that of the vector.

$$\therefore \frac{(y+2)\hat{a}_y - 5\hat{a}_z}{\sqrt{(y+2)^2 + (5)^2}} = \frac{1}{3}\hat{a}_y - \frac{2}{3}\hat{a}_z$$

$$\therefore \frac{y+2}{5} = \frac{1}{2}$$

$$y = 0.5$$

$$\therefore \vec{R} = 2.5\hat{a}_y - 5\hat{a}_z$$

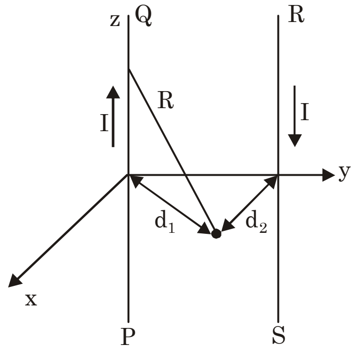
$$\therefore \vec{E} = \frac{l}{2\pi\epsilon_0}\cdot\frac{\vec{R}}{|R|^2}$$

$$= \frac{16\times10^{-9}\times9\times10^{9}}{2}\times\frac{\left(2.5\hat{a}_y - 5\hat{a}_z\right)}{31.25}$$

$$= 23\hat{a}_y - 46\hat{a}_z$$

50. **(a)**

Taking an element current $\overrightarrow{Idz}$



Magnetic vector potential A_1 due to wire PQ

$$A_1 = \frac{\mu_o}{4\pi} I\int_{-L}^{L}\frac{dz}{R}$$

$$A_1 = \frac{\mu_o I}{2\pi}\int_0^L \frac{dz}{\sqrt{d_1^2 + z^2}}$$

$$A_1 = \frac{\mu_o z}{2\pi}\ln\left[z + \sqrt{z^2 + d_1^2}\right]_0^L$$

$$A_1 = \frac{\mu_o I}{2\pi}\ln\left[L + \sqrt{L^2 + d_1^2}\right] - \ln d_1$$

$$A_1 = \frac{\mu_o I}{2\pi}\left[\ln(2L) - \ln d_1\right] \quad ...(1)$$

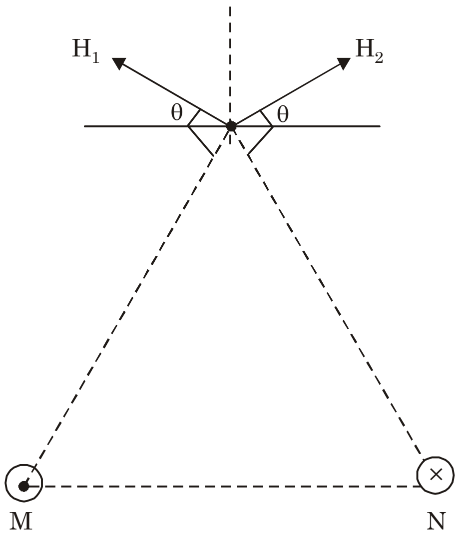
Similarly, for wire RS

$$A_2 = -\frac{\mu_o}{2\pi}\left[\ln(2L) - \ln d_2\right]$$

Resultant $A = A_1 + A_2$

$$A = \frac{\mu_o I}{2\pi}\left[\ln d_2 - \ln d_1\right] = \frac{\mu_o I}{4\pi}\ln\left(\frac{d_2^2}{d_1^2}\right)$$

51. **(c)**



Here, $\theta = 30°$

and $H_1 = H_2 = \frac{I}{2\pi a} = \frac{5}{2\pi}$

H_1 is due to 10 A_v and H_2 is due to 10A at N at M.

$H_1 \cos\theta$ and $H_2 \cos\theta$ will cancel each other.

Net field intensity = $H_1 \sin\theta + H_2 \sin\theta$

$= 2H_1 \sin 30° = 2\times\frac{5}{2\pi}\times\frac{1}{2}$

$H = \frac{5}{2\pi} A/m$.

52. (c)

From the right hand rule, we can say, that magnetic field due to current flowing is in $\hat{a}_\phi$ direction.

So, using Ampere's law—

For $0 < \rho < 6$:

$2\pi\rho . H_\phi = 16$

$H_\phi = \frac{16}{2\pi\rho}$

For $6 < \rho < 10$:

$2\pi\rho H_\phi = 16 - 12 = 4$

$H_\phi = \frac{4}{2\pi\rho}$

So, $\vec{H} = \frac{16}{2\pi\rho}\hat{a}_\phi \quad 0 < \rho < 6$

$= \frac{4}{2\pi\rho}\hat{a}_\phi \quad 0 < \rho < 10$

$\vec{B} = \frac{16\mu_0}{2\pi\rho}\hat{a}_\phi \quad 0 < \rho < 6$

$= \frac{4\mu_0}{2\pi\rho}\hat{a}_\phi \quad 6 < \rho < 10$

Total flux,

$$\psi = \int B.ds = \int_0^{0.01}\int_{0.01}^{0.06}\frac{16\mu_0}{2\pi\rho}\hat{a}_\phi.(d\rho dz\hat{a}_\phi) + \int_0^{0.01}\int_{0.06}^{0.07}\frac{4\mu_0}{2\pi\rho}\hat{a}_\phi(d\rho d_z\hat{a}_\phi)$$

$$= \frac{4\mu_0}{2\pi}\left[4\int_0^{0.01}d_z\int_{0.01}^{0.06}\frac{d\rho}{\rho} + \int_0^{0.01}dz\int_{0.06}^{0.07}\frac{d\rho}{\rho}\right]$$

$$= \frac{4\mu_0}{2\pi}[4(0.01)\ln(6) + (0.01)\ln(7/6)]$$

$\psi = 58.4$ nWb.

53. (b)

Since, Magnetic field intensity,

$\vec{H} = \frac{1}{2}\vec{J}\times\hat{a}_n$

$= \frac{1}{2}[(30-40)\hat{a}_x \times (-\hat{a}_z)]$

$= (-5\hat{a}_x)\times(-\hat{a}_z)$

$= 5(\hat{a}_x \times \hat{a}_z)$

$= -5\hat{a}_y$ A/m

54. (b)

Current density $\vec{J} = \nabla\times\vec{H}$

$$\vec{J} = \frac{1}{\rho}\begin{vmatrix}\hat{a}\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & \rho.(10^5\rho^2) & 0\end{vmatrix}$$

$$\vec{J} = \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}(10^5\rho^3)\,\hat{a}_z = 3\times10^5\rho\hat{a}_z$$

The total current flowing in the wire is—

$$I = \int_0^{0.005} J.ds = \int_0^{2\pi}\int_0^{0.005}(3\times10^5\ \rho\hat{a}_z).\rho d\rho\, d\phi\hat{a}_z$$

$$= 3\times10^5\int_0^{2\pi}d\phi\int_0^{0.005}\rho^2 d\rho$$

$$= 3\times10^5 \times 2\pi\times\frac{(0.005)^3}{3} = 0.079A$$

$\therefore \quad R = \frac{V}{I} = \frac{0.1}{0.079} = 1.26\,\Omega$.

55. (d)

Magnetic field due to a solenoid with 'N' turns per unit length and current I will be:

$B = \mu_0 NI$

$= 4\pi\times10^{-7}\times\frac{100}{10\times10^{-2}}\times5$

$B = 2\pi \times 10^{-3}\ W_b/m^2$

56. (d)

Given,

$\vec{F} = (K_1xy + K_2z^3)\vec{a}_x + (3x^2 - K_3z)\vec{a}_y + (3xz^2 - y)\vec{a}_z$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (K_1xy + K_2z^3) & (3x^2 - K_3z) & (3xz^2 - y) \end{vmatrix}$$

$= \vec{a}_x[-1 + K_3] + \vec{a}_y|3z^2K_2 - 3z^2| + \vec{a}_z(6x - K_1x)$

$= (K_3 - 1)\vec{a}_x + 3z^2(K_2 - 1)\vec{a}_y + x(6 - K_1)\vec{a}_z$

Given vector field $\vec{F}$ is irrotational i.e.

$\nabla \times \vec{F} = 0$

$\Rightarrow (K_3 - 1)\vec{a}_x + 3z^2(K_2 - 1)\vec{a}_y + x(6 - K_1)\vec{a}_z = 0$

if $K_3 - 1 = 0, K_2 - 1 = 0, 6 - K_1 = 0$

$K_3 = 1, K_2 = 1, K_1 = 6$

$$\nabla.\vec{F} = \left(\frac{\partial}{\partial x}\vec{a}_x + \frac{\partial}{\partial y}\vec{a}_y + \frac{\partial}{\partial z}\vec{a}_z\right).\begin{bmatrix}(K_1xy + K_2z^3)\vec{a}_x \\ +(3x^2 - K_3z)\vec{a}_y \\ +(3xz^2 - y)\vec{a}_z\end{bmatrix}$$

$$= \frac{\partial}{\partial x}(K_1xy + K_2z^3) + \frac{\partial}{\partial y}(3x^2 - K_3z) + \frac{\partial}{\partial z}(3xz^2 - y)$$

$= K_1y + 0 + 6xz$

At point (1, 1, –2)

$\nabla.\vec{F} = K_1(1) + 6(1)(-2)$

$= -12 + K_1$

$= -12 + 6 = -6 \quad [\because K_1 = 6]$

57. (d)

$\vec{V} = (2\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z) \times 10^5$ m/sec

$\vec{B} = (-2\hat{a}_x + 2\hat{a}_y - \hat{a}_z)$ mT

$$\therefore \quad \vec{V} \times \vec{B} = 10^2 \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -3 & -4 \\ -3 & 2 & -1 \end{vmatrix}$$

$= 10^2\{\hat{a}_x(3+8) - \hat{a}_y(-2-12) + \hat{a}_z(4-9)\}$

$= 1100\hat{a}_x + 1400\hat{a}_y - 500\hat{a}_z$

$\therefore$ Total force, $\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$

$= 2 \times 10^{-16}\ [100\hat{a}_x - 200\hat{a}_y + 300\hat{a}_z + 1100\hat{a}_x$
$1100\hat{a}_x + 1400\hat{a}_y - 500\hat{a}_z]$

$= 2 \times 10^{-16}\left[1200\hat{a}_x + 1200\hat{a}_y - 200\hat{a}_z\right]$

$\vec{F} = 4 \times 10^{-14}(6\hat{a}_x + 6\hat{a}_y - \hat{a}_z)$

$\therefore$ The acceleration,

$$\vec{a} = \frac{\vec{F}}{m} = \frac{4 \times 10^{-14}(6\hat{a}_x + 6\hat{a}_y - \hat{a}_z)}{5 \times 10^{-26}}$$

$\vec{a} = 8 \times 10^{11}\ (6\hat{a}_x + 6\hat{a}_y - \hat{a}_z).$

58. (d)

Given, $H_1 = 4a_x + 6a_y + 8a_z$

Therefore, $H_{1t} = 6a_y + 8a_z$

$H_{1n} = 4a_x$

According to magnetic boundary conditions,

$\mu_1H_{1n} = H_{2n}\mu_2$	$H_{1t} = H_{2t}$
$\Rightarrow 2\mu_o 4a_x = \mu_o H_{2n}$	
$\Rightarrow \boxed{H_{2n} = 8a_x}$	$\boxed{6a_y + 8a_z = H_{2t}}$

Therefore, $H_2 = H_{2n} + H_{2t} = 8a_z + 6a_y + 8a_z$

Therefore comparing with $H_2 = pa_x + qa_y + ra_z$

$p = 8,\ q = 6,\ r = 8$

59. (a)

The amount of energy required or, work done does not depend on the path followed, but depends on the initial and final position i.e. displacement.

So,

$E = W = -\int q\vec{E}.\vec{dl}$

$= -q\int \vec{E}.\vec{dl}$

$= -10\int(2x\hat{a}_x - 3y^2\hat{a}_y + 4\hat{a}_z)(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$

$$= -10\left[\int_0^3 2x.dx - \int_0^1 3y^2dy + 4\int_0^{-1} dz\right]$$

$= -10[(3^2 - 0^2) - (1^3 - 0^3) + 4(-1 - 0)]$

$= -10 \times 4$

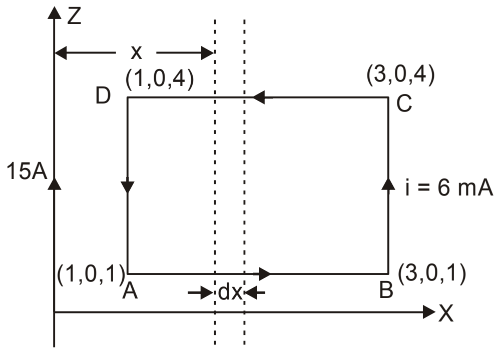
$= -40$ J

60. (b)

The magnetic flux density at a distance 'x' from the wire, is

$$\vec{B} = \frac{\mu_0 I}{2\pi x}\hat{a}_y$$

$$\vec{B} = \frac{3\times10^{-6}}{x}\hat{a}_y$$



The force on differential wire of length dx is—

$$\vec{dF} = i(\vec{dl}\times\vec{B})$$

$$= 6\times10^{-3}\left(dx\,\hat{a}_x \times \frac{3\times10^{-6}}{x}\hat{a}_y\right)$$

$$\vec{dF} = 18\times10^{-19}\frac{dx}{x}\hat{a}_z$$

$\therefore$ Total force $\vec{F} = \int_1^3 dF = 18\times10^{-9}\int_1^3 \frac{dx}{x}\hat{a}_z$

$= 18\times10^{-9}\,|\ln(x)|_1^3$

$= 18\times10^{-9}\ \ln(3)\ \hat{a}_z$

$\vec{F} = 19.8\ \hat{a}_z$ nN.

61. (a)

The conduction current density is given by,

$$\vec{J}_c = \sigma\vec{E}$$

$$\vec{J}_c = 10^{-5}\times\frac{10^6}{\rho}\cos(10^5 t)\ \hat{a}_\rho$$

$$\vec{J}_c = \frac{10}{\rho}\cos(10^5 t)\ \hat{a}_\rho$$

The conduction current will flow radially along the length of capacitor.

so, The total conduction current,

$$I_c = \vec{J}_c\times\vec{dS}$$

$$= \frac{10}{\rho}\cos(10^5 t)\hat{a}_\rho \times 2\pi\rho\ell\hat{a}_\rho$$

$$I_c = 20\pi\ell\cos(10^5 t)$$

$$\therefore \quad \boxed{I_c = 8\pi\cos(10^5 t)\ A}$$

62. (d)

The magnetic field intensity at x = 0, y = 0.5 due to current shee4t at y = 0, will be—

$$\vec{H} = \frac{1}{2}\vec{k}\times\vec{a_n} = \frac{1}{2}(8\hat{a}_z)\times(\hat{a}_y) = -4\hat{a}_x$$

The magnetic field intensity at x = 0 y = 0.5 m. due to current sheeet at y = 1 m. will be—

$$\vec{H}_2 = \frac{1}{2}\vec{k}\times\hat{a}_n = \frac{1}{2}(-4\hat{a}_z)\times(-\hat{a}_y)$$

$$\vec{H}_2 = -2\hat{a}_x$$

$\therefore$ The net magnetic field intensity at x = 0, y = 0.5 m. will be—

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

$$\vec{H} = -6\hat{a}_x$$

$\therefore$ The magnetic flux density

$$\vec{B} = \mu_0\vec{H} = -6\mu_0\hat{a}_x$$

$\therefore$ Force per unit length on wire is—

$$\vec{F} = i(\vec{d\ell}\times\vec{B})$$

$$= 7\times10^{-3}\left(\hat{a}_z\times(-6\mu_0)\hat{a}_x\right)$$

$$= -42\mu_0\times10^{-3}\hat{a}_y$$

$$\vec{F} = -52.8\ \hat{a}_y\ \text{nN}.$$

Consider magnitude only, F = 52.8 nN

63. (c)

Magnetic field in air is given y

$$\vec{B} = B_0\left(\frac{x}{x^2+y^2}\hat{a}_y - \frac{y}{x^2+y^2}\hat{a}_x\right)$$

Converting trivial to cylindrical coordinates

$$x = r\cos\phi,\quad y = r\sin\phi$$

Relation between cartesian and cylindrical co-ordinates

$$\hat{a}_x = \cos\phi\hat{a}_r - \sin\phi a_\phi,\ \hat{a}_y = \sin\theta\hat{a}_r + \cos\phi\hat{a}_\phi$$

Magnetic field in cylindrical coordinate is

$$\vec{B} = B_0\left[\frac{r\cos\phi(\sin\phi\hat{a}_r + \cos\phi\hat{a}_\phi)}{r^2(\cos^2\phi + \sin^2\phi)}\right.$$

$$\left. -\frac{r\sin\phi\left(\cos\phi\hat{a}_r - \sin\phi\hat{a}_\phi\right)}{r^2\left(\cos^2\phi + \sin^2\phi\right)}\right]$$

$$= B_0\left[\frac{\sin\phi\cos\phi\hat{a}_r + \cos^2\hat{a}_\phi - \sin\phi\cos\phi\hat{a}_r + \sin^2\phi\hat{a}_\phi}{r}\right]$$

$$\left[\because \cos^2\phi + \sin^2\phi = 1\right]$$

$$\vec{B} = \frac{B_0}{r}\hat{a}_\phi;\ \ r \neq 0$$

$$\because\ B = \mu H \Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0} = \frac{B_0}{\mu_0}\frac{\hat{a}_\phi}{r}$$

By Ampere's Law, $\oint \vec{H}\,\vec{dl} = I$

$\nabla \times \vec{H} = \vec{J}$ [By Stoke's theorem]

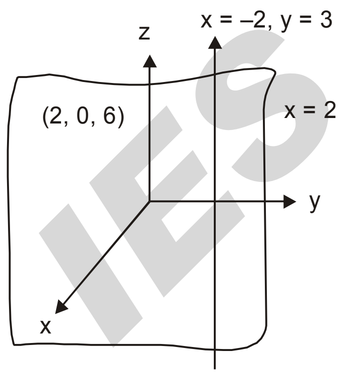
$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_r & rH\phi & H_z \end{vmatrix} = \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & r\frac{B_o}{r} & 0 \end{vmatrix}$$

$$= \hat{a}_r(0) + r\hat{a}_\phi(0) + \hat{a}_z(0)$$

$$\nabla \times \vec{H} = 0$$

$$\therefore\ \vec{J} = \nabla \times \vec{H} = 0$$

64. (a)



The electric field at origin due to point charge,

$$\vec{E}_1 = \frac{12\times10^{-9}\left((0-2)\hat{a}_x + 0.\hat{a}_y + (0-6)\hat{a}_z\right)}{4\pi\varepsilon_0(2^2+6^2)^{\frac{3}{2}}}$$

$$= \frac{12\times10^{-9}\times9\times10^{9}}{253}\left(-2\hat{a}_x - 6\hat{a}_z\right)$$

$$= \vec{E}_1 = -0.84\hat{a}_x - 2.52\hat{a}_z$$

The electric field due to line charge,

$$\vec{E}_2 = \frac{3\times10^{-9}}{2\pi\varepsilon_0(2^2+3^2)}\left((0+2)\hat{a}_x + (0-3)\hat{a}_y\right)$$

$$= \frac{18\times10^{9}\times3\times10^{-9}}{13}\left(2\hat{a}_x - 3\hat{a}_y\right) = 8.31\hat{a}_x - 12.46\hat{a}_y$$

The electric field due to sheet charge,

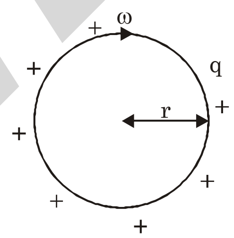
$$E_3 \Rightarrow \frac{0.2\times10^{-9}}{2\varepsilon_0}\left(-\hat{a}_x\right) = -11.3\hat{a}_x$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= -0.84\,\hat{a}_x - 2.52\,\hat{a}_z + 8.31\,\hat{a}_x - 12.46\,\hat{a}_y - 11.3\,\hat{a}_x$$

$$\vec{E} = -3.84\,\hat{a}_x - 12.4\,\hat{a}_y - 2.52\,\hat{a}_z$$

65. (a)



For a uniformly charged ring rotating current of frequency 'ω' current is given by

$$I = \text{charge}\times\text{revolution freq.}$$

$$= q\times\frac{\omega}{2\pi} = \frac{q\omega}{2\pi}$$

Magnetic moment of this equivalent current

$$m = IA = \frac{q\omega}{2\pi}.\pi r^2$$

$$= \frac{1}{2}.q\omega r^2$$

Magnetic flux density (B) : $B = \frac{\mu_0 I}{2r}$

$$q = \rho.(2\pi r)$$

ρ is charge density (c/m)

$$B = \frac{\mu_0}{2r}\left(\frac{q\omega}{2\pi}\right)$$

$$= \frac{\mu_0(2\pi r)\rho\omega}{4\pi r} = \frac{\mu_0\rho\omega}{2}$$

Given $\rho_1 = \rho_2$, $\omega_1 = \omega_2$, $r_2 = 2r_1$

By B is independent of r.

$$\frac{B_1}{B_2} = \frac{\rho_1\omega_1}{\rho_2\omega_2} = 1:1$$