IES MASTER

Institute for Engineers (IES/GATE/PSUs)

UPSC ESE 2020

Mains Exam Solution

ELECTRICAL ENGINEERING

Paper-II

EXAM DATE: 18-10-2020

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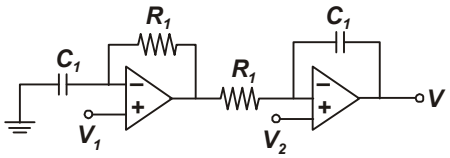
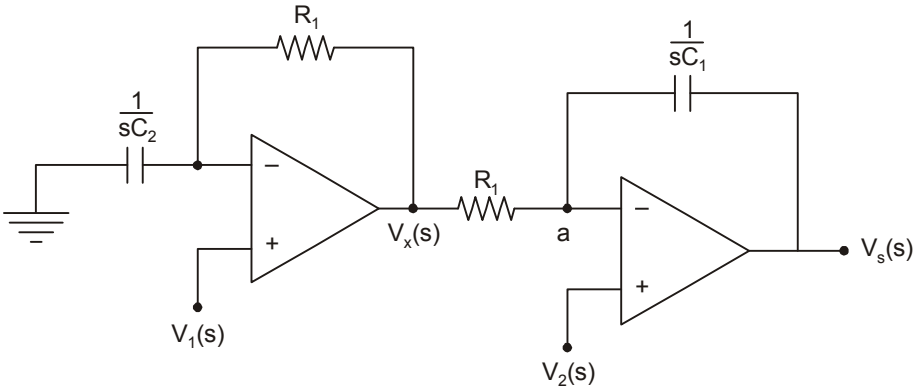
SECTION-A

Q–1(a) ***For the circuit shown in the figure below, derive the expression for output voltage and sketch the nature of the output when V_2 = 10V and V_1 = 5V.***



Sol: The given circuit in s-domain can be drawn as shown below.



Applying the concept of virtual ground

$$V_t = V_1(s) \text{ and } V_a = V_2(s)$$

So,
$$\frac{V_1(s)}{\frac{1}{sC_1}} + \frac{V_1(s) - V_x(s)}{R_1} = 0$$

or,
$$sC_1V_1(s) + \frac{V_1(s)}{R_1} = \frac{V_x(s)}{R_1}$$

or,
$$V_x(s) = (sC_1R_1 + 1)V_1(s) \quad ...(1)$$

Similarly,

$$\frac{V_2(s) - V_x(s)}{R_1} + \frac{V_2(s) - V_0(s)}{V / sC_1} = 0$$

or,
$$V_2(s)\left[\frac{1}{R_1} + sC_1\right] = \frac{V_x(s)}{R_1} + sC_1V_0(s)$$

or,
$$V_2(s)[1 + sR_1C_1] = [1 + sC_1R_1]V_1(s) + sC_1R_1(s)$$

or,
$$sC_1R_1V_0(s) = (1 + sC_1R_1)[V_2(s) - V_1(s)]$$

Given that
$$V_1 = 5V \text{ and } V_2 = 10V$$

So,
$$V_1(s) = \frac{5}{s} \text{ and } V_2 = \frac{10}{s}$$

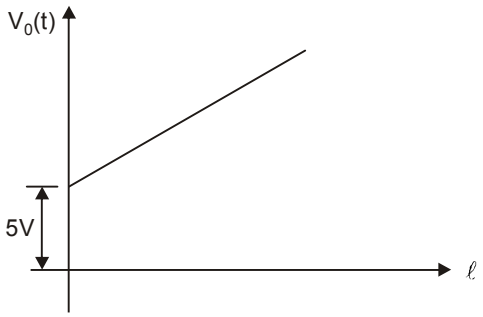
$\therefore$
$$sC_1R_1V_0(s) = (1 + sC_1R_1)\left(\frac{10}{s} - \frac{5}{s}\right)$$

or, $$V_0(s) = \frac{5(1+sC_1R_1)}{s^2R_1C_1}$$

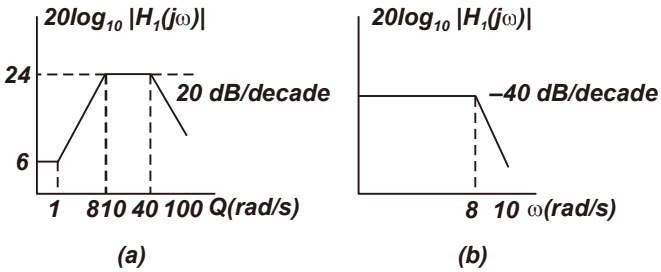
or, $$V_0(s) = \frac{5}{R_1C_1}\cdot\frac{1}{s^2}+\frac{5}{s}$$

Taking inverse laplace transform of above equation

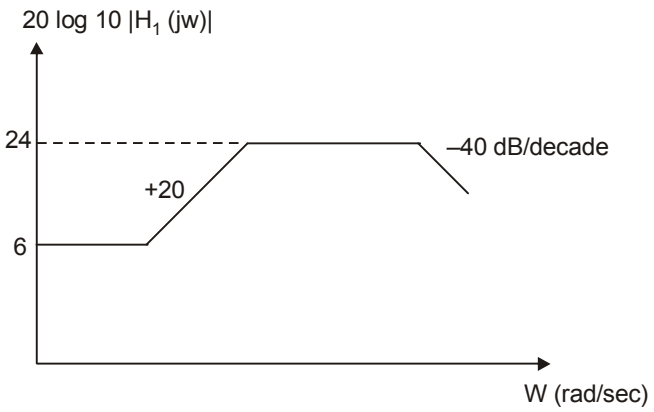
$$V_0(t) = 5\left(1+\frac{5}{R_1C_1}t\right)$$



Q–1(b) ***A continuous LTV system S with frequency response $H(j\omega)$ is constructed by cascading two continuous time LTI sytstems with frequency response $H_1(j\omega)$ and $H_2(j\omega)$ respsectively. Figures a and b show the straight line apporximations of bode magnitude plots of $H_1(j\omega)$ and $H(j\omega)$, respectively, Find $H_2(j\omega)$.***



Sol. Given Bode plot of H_1 $(j\Omega)$



Transfer function of H_1 (jw) is

$$H_1(s) = \frac{k\left(1+\frac{s}{1}\right)}{\left(1+\frac{s}{8}\right)\left(1+\frac{s}{40}\right)}$$

$$H_1(s) = \frac{320k(s+1)}{(s+8)(s+40)}$$

for k value use line equation

P {i.e. y = mx + c}

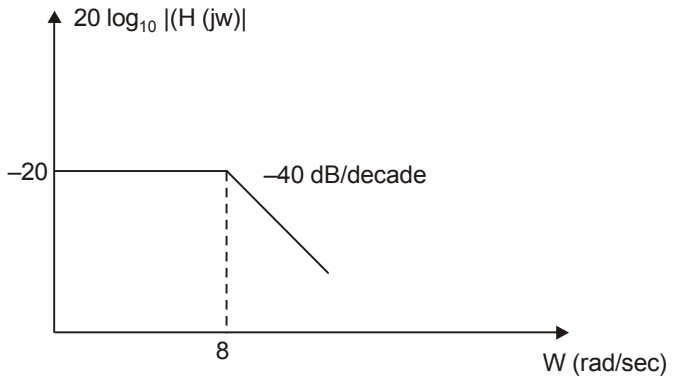
Here C = 20 log k

from graph 6 = 0 + 20 log k

k = 2

oo, $$H_1(s) = \frac{320\times 2(s+1)}{(s+8)(s+40)} = \frac{640(s+1)}{(s+8)(s+40)}$$

Now Bode plot of H (jw) is given



Transfer function of above response is

$$H(s) = \frac{k}{\left(1+\frac{s}{8}\right)^2} = \frac{64k}{(s+8)^2}$$

for k use line equation

i.e. y = mx + c

–20 = 0 + 20 log k

k = 0.1

so, $$H(s) = \frac{6.4}{(s+8)^2}$$

In the quesion given that H(jw) cascaded by two (LTI system H, (jw_1) & H_2 (jw)

So, $$H(s) = H_1(s)\cdot H_2(s)$$

$$H_2(s) = \frac{H(s)}{H_1(s)} = \frac{6.4/\cancel{(s+8)^2}}{640(s+1)/\cancel{(s+8)}(s+40)}$$

$$H_s(s) = \frac{0.01(s+40)}{(s+1)(s+8)}$$ **Ans.**

Q–1(c) ***Consider a three phase induction motor with the following parameters:***

Number of poles : 4

Supply frequency : 50 Hz

Full load speed : 1470 rpm

Rotor resistance : 0.12 Ω

Standstill reactance : 1.12 Ω

Find the

(i) Slip for maximum torque

(ii) Ratio of maximum torque to full load torque.

Sol: Given, $P = 4$, $f = 50$ Hz, $N_r = 1470$ rpm

$$r_2 = 0.12\Omega,\ x_2 = 1.12\Omega$$

$$\text{Synchronous speed, } N_s = \frac{120f}{P} = \frac{120\times 50}{4}$$

or, $$N_s = 1500 \text{ rpm}$$

$$\text{Full load slip, } s_{f1} = \frac{N_s - N_r}{N_s} = \frac{1500-1470}{1500}$$

or, $$s_{f1} = \frac{30}{1500} = 0.02$$

(i) The slip at which maximum torque occurs is given by

$$s_{mr} = \frac{r_2}{x_2} = \frac{0.12}{1.12} = 0.107$$

(ii) We know that

$$\frac{T_{e.f\ell}}{T_{em}} = \frac{2}{\frac{s_{mr}}{s_{f\ell}} - \frac{s_{f\ell}}{s_{mr}}} = \frac{2}{\frac{0.107}{0.02} + \frac{0.02}{0.107}}$$

or $$\frac{T_{e.f\ell}}{T_{em}} = \frac{2}{5.357 + 0.1867} = \frac{2}{5.544}$$

$\therefore$ required ratio,

$$\frac{T_{em}}{T_{ef\ell}} = \frac{5.544}{2} = 2.772$$

Q–1(d) ***(i) What is smart grid?***

(ii) Compared to supervisory control and data acquisition (SCADA) system. What are the advantages of phasor measurement unit (PMU)?

(iii) Explain operation of PMU with a neat diagram.

Sol. **(i)**

- A smart grid is an electricity network that uses digital and other advanced technology to monitor and manage the transport of electricity from all generation sources to meet the varying electricity demand of end users.
- Smart grid system is automated for tracking the electricity consumption at all the location.

Smart grid has following capabilities:-

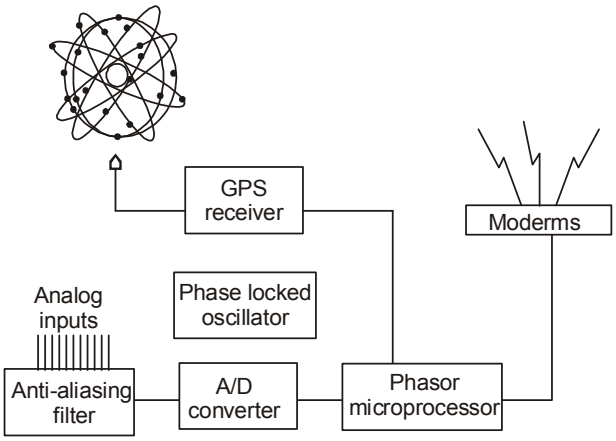
- It can repair itself
- It encourages consumer participation in grid operation.
- It allow electricity market to grow
- It can be operated more efficiently.

(ii) Comparison between SCADA AND PMUs.

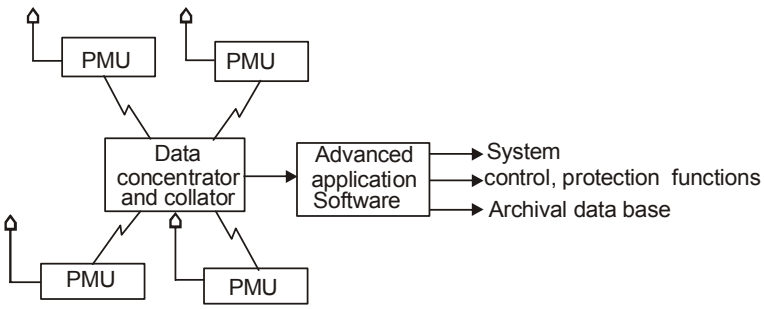
Attribute	SCADA	PMU
Measurment	Analogue	Digital
Resolution	2-4 samples per cycle	up to 60 samples per cycles
Observability	Steady state	Dynamic/Transient
Monitoring	Local	Wide-Area
Phasor angle measurement	No	Yes

From above table it is concluded that the PMU_s are given more accurate & efficient results as comparisons to SCADA systems.

(iii) The GPS system consists of 24 satellites in six orbits at an approximate altitude of 10,000 miles above the surface of the earth. They are thus approximately at one half the altitudes corresponding to a geo-synchronous orbit. The positioning of the orbital plane and the positioning of the satellites in the orbits is such that at any given instant at least four satellites are in view from any point on the surface of the earth. Often, more than six satellites are visible.



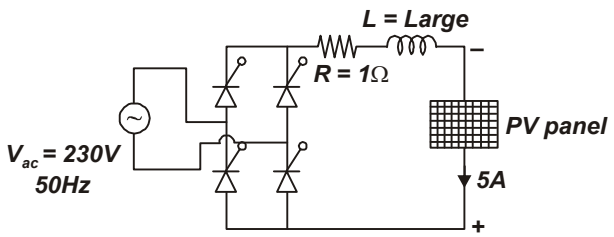
Block diagram of the phasor measurment unit



PMU utilization in a power system

The civilian-use channel of the GPS system transmits positional coordinates of the satellites from which the location of the receiver station on earth could be determined. The Satellites transmit a one-pulse per-second signal, along with an identifier for the signal that can be interpreted by the earth station receivers. The civilian-use transmission of the time signal is precise to within 1 microsecond, and often in practice is found to be much more accurate. The normal practice is to phaselock a sampling clock to this pulse. The sampling instant would be identified as the pulse number within a one-second interval identified by the GPS time-tag. The exact format for time-tagging is defined in IEEE standard 1344. It should be mentioned that a time standard known as the IRIG-B standard is currently being used by the power industry for time-tagging digital fault recorders and other substation event monitoring systems. However, with standard IRIG-B receivers the synchronization accuracy is of the order of 1 millisecond, which is not enough for precise power measurement (a tolerance of 1 millisecond corresponds to an uncertainity of about 20°). The complete block diagram of PMUs and PMUs utilization in power systems are shown in figure 3.11 and 3.12.

Q–1(e) ***A PV pane is connected with a single phase fully controlled converter as shown in the circuit below. The panel is supplying a current of 5A and generated power is 1000W. The series inductance in the circuit is large to make the current flat and continuous. Find (i) the triggering angle of the thyristor birdge, (ii) output voltage at rectifier terminal, and (iii) input power factor.***



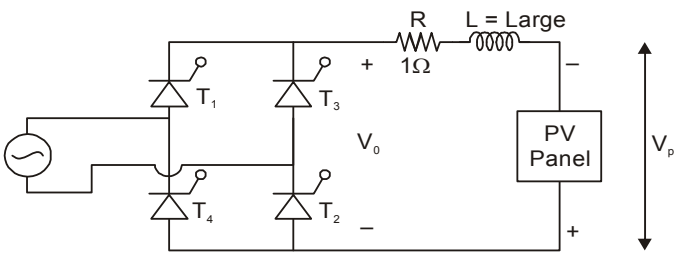
Sol.

$$V_{AC} = 230 \text{ Volt}$$

$$f = 50 \text{ Hz}$$

$$I_o = 5 \text{ A}$$

$$P_0 = 1000 \text{ watt}$$



(a) Single phase fully controlled converter
(in inversion mode $\alpha > 90°$)

$$\text{Panel voltage} = \frac{1000}{5} = 200 \text{ Volt}$$

(i) $$V_o = Io \times R - V_p = (5 \times 1 - 200)$$

$$\boxed{V_o = -195 \text{ volt}}$$

$$V_o = \frac{2V_m}{\pi}\cos\alpha$$

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$$\alpha = \text{triggering angle of the thyristor}$$

$$= \cos^{-1}\left(\frac{-195\times\pi}{2\times\sqrt{2}\times 230}\right) = 160.34°$$

(ii) Output voltage at rectifier terminal= – 195 volt

(iii) IPF (input power factor)

$$P_o = P_{in}$$

$$\Rightarrow \quad \left(1000 + I_o^2 \times R\right) = V_s I_{sr}\,\text{IPF}$$

$$\Rightarrow \quad \text{IPF} = \left(\frac{1000+25}{230\times 5}\right)$$

$$\boxed{\text{IPF} = \frac{1025}{230\times 5} = 0.891}$$

Q–2(a) ***The DC-DC converter given below is operating at 30 kHz and drawing an input current of 25 A at 48 V DC.***

(i) ***For a load current of 10A, find***

(i) the duty ratio of the switch,

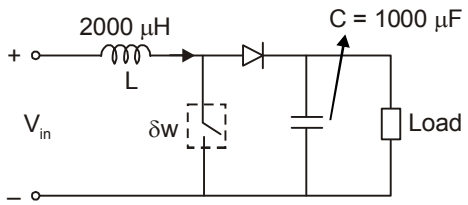
(ii) output voltage

(iii) peak inductor current,

(iv) output voltage ripple, and

(v) the load current where the inductor just becomes discontinuous.

Sol.



(a) Boost converter

Given: DC to DC converter

$$f = 30\text{ kHz}$$

$$\text{Input current} = 25\text{ A}$$

$$\text{Input voltage} = 48\text{ V DC}$$

(i) $\text{load current} = 10\text{A}$

(I) For lossless system

$$P_o = P_{in}$$

$$\Rightarrow \quad V_o I_o = V_{in} I_{in}$$

$$\Rightarrow \quad V_o = \frac{48\times 25}{10} = 120\text{ volt}$$

$$V_o = \frac{V_{in}}{1-\alpha}$$

$$\Rightarrow \quad (1-\alpha) = \frac{48}{120}$$

$$\Rightarrow \quad \alpha = \left(1-\frac{48}{120}\right) = \frac{3}{5} = 0.6$$

$$\boxed{\text{Duty ratio of the switch} = 0.6}$$

(II) Output voltage = 120 volt

(III) Peak inductor current

$$I_{max} = I_{in} + \frac{\Delta I_L}{2}$$

$$= 25 + \frac{\alpha v_{in}}{2fL}$$

$$= 25 + \frac{0.6 \times 48}{2 \times 30 \times 10^3 \times 200 \times 10^{-6}}$$

$$\boxed{I_{max} = 27.4\text{Amp}}$$

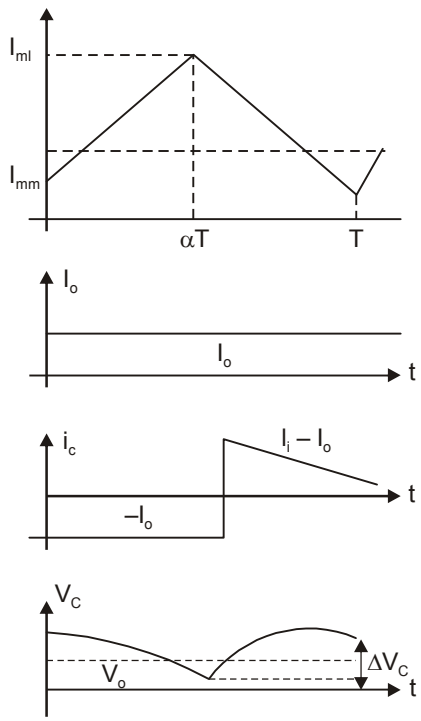
(iv) Output voltage ripple (ΔV_C)

$$\Delta V_C = \frac{\alpha I_o}{fc}$$

$$\Delta V_C = \frac{0.6 \times 10}{30 \times 10^3 \times 1000 \times 10^{-6}}$$

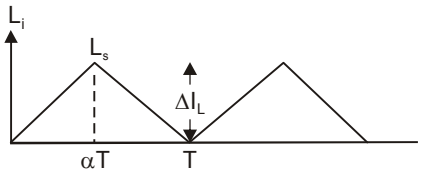
$$= \frac{6}{30 \times 1000 \times 10^{-3}}$$

$$\boxed{\Delta V_C = 0.2 \text{ volt}}$$



(a) Wave form of current and voltage

(v) At the boundary :

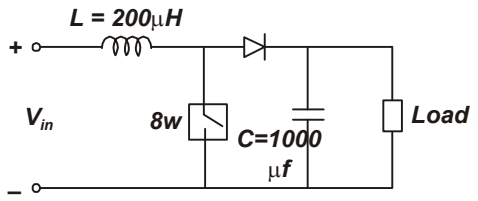


$$(i_L)_{ay} = I_s = \frac{I_o}{1-\alpha} = \frac{\Delta I_L}{2}$$

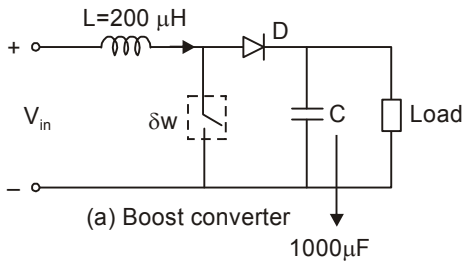
$$\Rightarrow \quad \frac{I_o B}{1-\alpha} = \frac{\alpha V_{in}}{2fL} = \frac{0.6 \times 48}{2 \times 30 \times 10^3 \times 200 \times 10^{-6}}$$

$$\Rightarrow \quad I_{OB} = (1 - 0.6) \times 2.4 = 0.96 \text{ Amp}$$

Q–2(a) (ii) Also find the critical value of L to keep the inductor current just continuous when the input voltage changes to 60 V with output remaining same. (Assume lossless operation of converter components)



Sol.

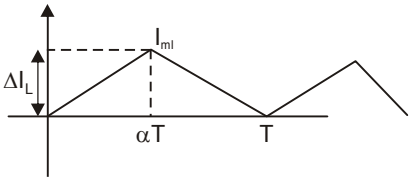


(a) Boost converter

Given: $V_n = 60$ volt

Output remain same P_o = constant

$$\boxed{\begin{aligned} V_o &= 120\,\text{volt} \\ I_o &= 10\text{A} \end{aligned}}$$



Critical inductance

$$I_{mn} = 0$$

$$i_{lag} = \frac{\Delta I_L}{2} = I_{in} = \frac{I_o}{(1-\alpha)}$$

$$\Rightarrow \quad \Delta I_L = \frac{2I_o}{(1-\alpha)}$$

$$\Rightarrow \quad \boxed{L_C = \frac{\alpha(1-\alpha)V_{in}}{2I_o f}} \qquad \text{...(i)}$$

Duty ratio

$$V_o = \frac{V_{in}}{(1-\alpha)}$$

$$\Rightarrow \quad (1-\alpha) = \frac{V_{in}}{V_o} = \frac{60}{120} = \frac{1}{2}$$

$$\boxed{\alpha = \frac{1}{2}}$$

From equation (i)

$$L_C = \frac{\frac{1}{2}\left(1-\frac{1}{2}\right)\times 60}{2\times 10\times 30\times 10^3} = 24\mu\text{F}$$

$$\boxed{L_C = 25\,\mu\text{H}}$$

Q–2(b) ***A signal m(t) = 2 cos(20πt) – cos(40πt), where the unit of time is millisecond, is amplitude modulated using the carrier frequency (f_c) of 600 kHz. The AM signal is given by***

s(t) = $5\cos 2\pi f_c t + m(t)\cos 2\pi f_c t$

(i) Sketch the magnitude spectrum of s(t). What is its bandwidth?

(ii) What is the modulation index?

(iii) The AM signal is passed through a high pass filter with cut-off frequency 595 kHz (i.e the filter passes all frequencies above 595 kHz, and cuts off all frequencies below 595 kHz). Find an explicit time domain expression for the quadrature component of the filter output with respect to a 600 kHz frequency refrerence.

Sol: **(i)** For an amplitude modulated signal the amplitude of modulated signal varies with the message signal.

Here is message signal we have more than are frequency signal multione AM

$$S_{AM(t)} = 5\left[1+\frac{m(t)}{5}\right]\cos 2\pi f_c t$$

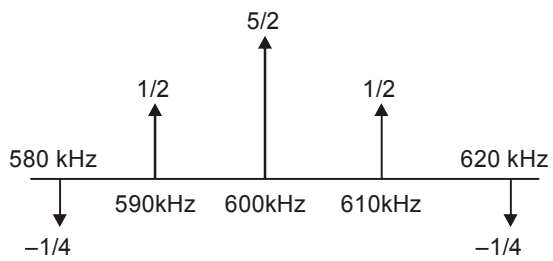
$$S_{AM(t)} = 5\left[1+\frac{2}{5}\cos(20\pi t)-\frac{1}{5}\cos 40\pi t\right]\cos 2\pi f_c t$$

$$S_{AM(t)} = A_c\left[1+\mu_1\cos 2\pi + m_1 t + \mu_2\cos 2\pi + m_2 t\right]\cos 2\pi f_c t$$

$$f_{m1} = [10\text{ Hz}]\,[10^3]$$

$$f_{m2} = [20\text{ Hz}]\,[10^3]\ f_i = 600\text{ kHz}$$

Magnitude spectrum



$$Bw = 2f_{m1} = 2\,f_{m2} = 2(20) = 40\text{ kHz}$$

(ii)
$$\mu_t = \sqrt{\mu_1^2+\mu_2^2} = \sqrt{\left(\frac{2}{5}\right)^2+\left(\frac{1}{5}\right)^2}$$

$$= \frac{1}{\sqrt{5}} = 0.4472$$

(iii) Before filter the signal was $A_c\cos(2\pi f_c t)+\frac{A_c\mu_1}{2}\cos(2\pi)(f_c+f_{m1})t+\frac{A_c\mu_1}{2}\cos\pi(f_c m_t)t$

$$+\frac{A_c\mu_2}{2}\cos 2\pi(f_c+f_{m2})t+\frac{A_c\mu_2}{2}\cos 2\pi(f_c-f_{m2})t$$

After filter it will be

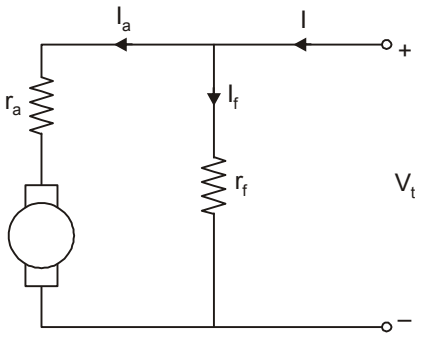
$$A_c\cos(2\pi f_c t)+\frac{A_c\mu_2}{2}\cos 2\pi(f_c+f_{m1})t+\frac{A_c\mu_2}{2}\cos 2\pi(f_c+f_{m2})t$$

$$S_{AM(t)} = 5\cos(2\pi f_c t) + \cos(2\pi)(f_c + f_{m1})t - \frac{1}{2}\cos(2\pi)(f_c + f_{m2})t$$

$$S_{AM(t)} = 5\cos(1200\pi t) + \cos(1220\pi t) - 0.5\cos(1240\pi t)$$

Q–2(c) ***A 400 V DC shunt motor has armature and fild resistances of 0.2 Ω and 200 Ω respectively. It draws current of 6 on no load and 70 A on full load. If its no load and full load speeds are the same, determine the field weakening due to load current as percentage of no load flux.***

Sol: Given, $V_t = 400$ V, $r_a = 0.2\Omega$ and $r_f = 200\Omega$



DC shunt motor

We know that speed a DC motor

$$\omega_m = \frac{E_a}{k_a\phi} = \frac{V_L - I_a r_a}{k_a\phi}$$

no load speed $\omega_{m0} = \dfrac{V_t - I_a r_a}{k_a\phi}$

and pull load speed $\omega_{mf1} = \dfrac{V_L - I_{a_2} r_a}{k_a\phi_2}$

In DC shunt motor $I = I_a + I_f$

and $I_f = \dfrac{V_t}{r_f} = \dfrac{400}{200} = 2A$

at no-load, $I = 6A$

$\therefore$ $I_{a1} = I - I_f = 6 - 2 = 4A$

and at full load, $I = 70A$

So, $I_{a2} = I - I_f = 70 - 2 = 68A$

Given that no load speed is same as full load speed

So, $$\frac{V_t - I_{a_1} r_a}{k_a\phi} = \frac{V_t - I_{a_2} r_a}{k_a\phi_2}$$

$\therefore$ $$\frac{\phi_2}{\phi_1} = \frac{V_t - I_{a_2} r_a}{V_t I_{a_1} r_a} = \frac{400 - 68 \times 0.2}{400 - 4 \times 0.2}$$

or, $$\frac{\phi_2}{\phi_1} = \frac{400-13.6}{400-0.8} = \frac{386.4}{399.2}$$

or, $$\frac{\phi_2}{\phi_1} = 0.968$$

or, $$\frac{\phi_2}{\phi_1} - 1 = 0.968 - 1$$

or, $$\frac{\phi_2 - \phi_1}{\phi_1} = -0.032$$

or, $$\frac{\phi_2 - \phi_1}{\phi_1} \times 100 = -3.2$$

$\therefore$ Field weaken by 3.2% of no-load flux.

Q–3(a) ***A salient pole star connected altenator is connected to infinite bus operating at 1.0 p.u*** voltage. The alternator has X_d ***= 0.75 p.u and Xq = 0.5 p.u on per phase basis. It is delivering 1.0 p.u power to the infinite bus at 0.8 p.f lag. Calculate (i) the load angle and excitation voltage under this condition (ii) the maximum power that can be delivered by the alternator with same excitation and the corresponding load angle (iii) the armature current and p.f under maximum power condition, and (iv) the theoretical value of maximum power that the alternator can deliver when its field circuit is suddenly disconnected due to fault.***

Sol. Given

$$x_d = 0.75 \text{ P.u.}$$
$$x_q = 0.5 \text{ P.u.}$$
$$V = 1 \text{ P.u.}$$
$$\text{Power } P = 1 \text{ p.u.}$$
$$\text{P.F.} = 0.8 \text{ lag}$$

(i) As we know for lagging p.f. alternator

$$\left.\begin{aligned} \tan\psi &= \frac{V\sin\phi + I_a x_q}{V\cos\phi + I_a R_a} \\ \text{and } \psi &= \delta + \phi \\ I_d &= I_a \sin\psi \\ I_q &= I_a \cos\psi \end{aligned}\right\} \begin{aligned} &\because R_a = 0 \\ &\\ &\text{from phasor diagram} \end{aligned}$$

$$E = V\cos\delta + I_d x_d \Rightarrow \text{ for } R_a = 0$$
$$\text{Delivered power} = 1$$
$$\text{Power} = V_{pu} \cdot I_{pu} \cdot \cos\phi$$
$$I = 1 \times I_a \times 0.8$$
$$I_a = 1.25 \text{ p.u.}$$

So, $$\tan\psi = \frac{1\times\sin(36.86°) + 1.25\times 0.5}{1\times\cos(36.86°) + 0}$$
$$\psi = 56.85°$$

$\because$ $$\psi = \delta + \phi$$
$$\delta = \psi - \phi$$
$$= 56.85° - 36.86°$$

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$\boxed{\delta \approx 20°}$ Ans.

then $I_d = I_a \sin\psi = 1.25 \times \sin 56.85$

$I_d = 1.046$ P.u.

So, $E_f = V \cos\delta + I_d X_x$

$E_f = 1 \times \cos 20° + 1.046 \times 0.75$

$\boxed{E_f = 1.724 \text{ p.u.}}$ Ans.

(ii) Power equation of salient pole alternator is

$$P = \frac{VE_f}{X_d}\sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\sin 2\delta$$

Here $E_f = 1.724$ p.u.

$V = 1$ Pu

then

$$P = \frac{1 \times 1.724}{0.75}\sin\delta + \frac{1^2}{2}\left[\frac{1}{0.5} - \frac{1}{0.75}\right]\sin 2\delta$$

$$P = 2.298\sin\delta + 0.33\sin 2\delta \quad \ldots(i)$$

for maximum power $\frac{dp}{d\delta} = 0$

$$\frac{dp}{d\delta} = 2.298\cos\delta + 0.66\cos 2\delta = 0$$

Using numerical method

$\delta = 74.98°$

Put δ value in eqn. (i)

$P_{max} = 2.298 \times \sin 74.98 + 0.3 \sin(2 \times 74.98)$

$P_{max} = 2.3846$ P.u.

(iii) Armature current and p.f. under maximum power condition.

$E_f = 1.724$

$V = 1$ P.u.

$\delta_{max} = 74.98°$

So alternator equation

$E = V \cos\ V\cos\delta + I_d x_d$

$1.724 = 1 \times \cos 74.98° + I_d \times 0.75$

$I_d = 1.9531$ p.u.

Power $P_{max} = VI_a \cos\phi$ $\{\because \psi = \delta + \phi$

$2.3846 = 1 \times I_a \cos(\psi - \delta_{max})$

$2.3846 = I_a \cos\psi \cos 74.98° + I_a \sin\psi \sin 74.98°$

$2.3846 = I_q \times 0.259 + I_d \times 0.9658$

$$2.3846 = I_q \times 0.259 + 2.253 \times 0.9658$$

$$I_q = 1.9224 \text{ p.u.}$$

so, $$I_a = \sqrt{I_d^2 + I_q^2} = \sqrt{(1.9531)^2 + (1.9224)^2} = 2.7405 \text{ p.u.}$$

$\because$ $$I_q = I_a \cos\psi$$

$$\cos\psi = \frac{I_q}{I_a} = \frac{1.9224}{2.7405}$$

$$\psi = 45.45°$$

$\therefore$ $$\phi = \psi - \delta = 45.45° - 74.98°$$

$$\approx -30°$$

$$\text{p.f.} = \cos 30° = 0.866 \text{ leading}$$

$$\boxed{\begin{array}{l} I_a = 2.7405 \text{ p.u.} \\ \text{P.f.} = 0.866 \text{ leading} \end{array}}$$

(iv) Max power when $E_f = 0$

$$P_{max}\big|_{\text{at } \delta=45°} = \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\sin 2\delta$$

$$P_{max} = \frac{1}{2}\left[\frac{1}{0.5} - \frac{1}{0.75}\right]\sin 90°$$

$$\boxed{P_{max} = 0.333 \text{ p.u.}}$$ **Ans.**

Q–3(b) ***A closed loop system with unit feedback and having the forwad loop transfer function as***

$$G(s) = \frac{14.4}{s(1+0.1s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot.

Sol: We know that

The transfer function of proportional can derivative controller = $k_p + k_{DS}$

Given:

Forward loop transfer function G(s) = $\dfrac{14.4}{S(1+0.15)}$

$\therefore$ Open loop transfer function of new system

$$= \frac{14.4}{S(1+0.15)}(k_p + k_{Ds})$$

$\therefore$ Characteristics equation is $S(1 + 0.1\,S) + 14.4\,k_p + 14.4\,k_{DS} = 0$

$\Rightarrow$ $$0.1\,S^2 + S + 14.4\,k_{DS} + 14.4\,k_p = 0$$

$\Rightarrow$ $$S^2 + (144\,k_D + 10)S + 144\,k_p = 0$$

Comparing with standard

Equation $S^2 + 2S\omega_n 5 + \omega_n^2 = 0$

We get,

$$\omega_n^2 = 144\, k_p$$

$$\omega_n = 12\sqrt{k_p} \text{ rad/sec}$$

$$\Rightarrow \quad k_p = 1$$

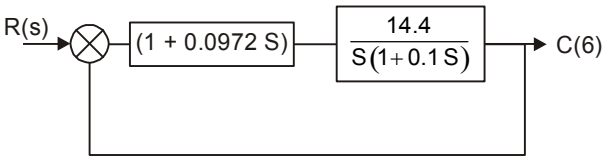
$$\Rightarrow \quad \omega_n = 12\sqrt{1}$$

$$\omega_n = 12 \text{ rad/sec}$$

$$\frac{2\omega_n - 10}{144} = k_D$$

$$\Rightarrow \quad \boxed{k_D = 0.0972}$$

$\therefore$ Overall system will be:



Q–3(c) ***Two 11 kV, 30 MVA, three phase synchronous generators operate in parallel supplying a sub station through a feeder having an impedance of (0.6 + j0.8) ohms to positive and negative sequence currents and (1.0 + j2.6) ohms to zero sequence currents. Each generator has x_1 = 0.8 ohms, x_2 = 0.5 ohms and x_0 = 0.2 ohms and has its neutral grounded through a reactance of 0.2 ohms. Evaluate the fault currents in each line the potential above earth occuraence of earth fault on the y and B phases at the sub station.***

Sol. Given, two 3-phase synchronous generator operating in parallel.

	z_1	z_2	z_0
Feeder	$(0.6 + j0.5)\Omega$	$(0.6 + j0.8)\Omega$	$(1 + j2.6)\Omega$
Each generator	$j0.8\Omega$	$j0.5\Omega$	$j0.2\Omega$

Z_n = nutral impedance = $j0.2\,\Omega$

$\because$ Both the generator are concentred in parallel.

$\therefore$ $(Z_g)_1$ = + ve sequence equivalent impedance of the parallel combination

(za_2) = –ve sequence equivalent impedance of the parallel combination

(z_{go}) = Zero sequence equivalent impedance of the parallel combination

$$(Z_g)_1 = \frac{j0.8}{2}\Omega = j0.4\Omega, (z_g)_2 = \frac{j0.5}{2}\Omega = j0.25\Omega$$

$$(z_g)_0 = \frac{j0.2 + 3 \times j0.2}{2} j0.4\Omega$$

$\therefore$ The net +ve sequence impedence from the generator to the substation

$$(Z_1)_{eq} = (z_g)_1 + (zf)_1$$
$$= j\,0.4 + 0.6 + j\,0.8$$

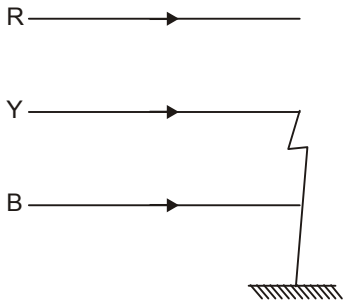
$= (0.6 + 1.2)\Omega$

The net –ve and zero sequence impedence from the generator to the substation

$(Z_2)_{eq} = (Z_9)_2 + (Zf)_2$, $(Z_o)_{eq} = (z_9)_1 + (Z_f)_0$

$= 0.6 + j\ 0.8 + j\ 0.25 = (0.6 + j\ 1.05)\Omega$

$= (j\ 0.4 + 1 + z\ 2.6 = 1 + j3)\Omega$



We have, the +ve sequence current
for a simultaneous earth fault at Y & B phase of the substitions,

Where $\xi_1 = +ve$ sequence generated voltage = $\frac{11}{\sqrt{3}}kV$

$$I_{R1} = \frac{E_1}{Z_1 + (y_0 \parallel z_2)}$$

$$IR_2 = \frac{-z_0}{z_0 + z_2} I_{R1}$$

$$IR_o = \frac{-z_2}{z_0 + z_2} I_{R1}$$

$$\therefore \quad \vec{I}R_1 = \frac{11/\sqrt{3}KV}{0.6 + j1.2 + (1 + j3) \parallel (0.6 + j1.05)} \quad I_{R2} = \frac{-70}{70 + 72} I_{R_1}$$

$$z_o \parallel z_2 = \frac{z_0 z_2}{z_0 + z_2} = \frac{(1 + j3)(0.6 + j1.05)}{(1.6 + j4.05)} = \frac{3.8243 < 131.82}{4.355 < 68.44}$$

$$\therefore \quad I_{R1} = \frac{11}{\sqrt{3}} \times \frac{1}{(0.6 + j1.2 + 0.3934 = j0.785)} \qquad = 0.878 < 63.38\Omega$$

$$\frac{6.351 < 0°}{(0.9935 + 51.985)} = \frac{6351}{2.22 < 63.414°}$$

$$\therefore \qquad \vec{I}_{R_1} = 2.8608 < -63.414 \text{ KA}$$

$$\vec{I}_{R_2} = \frac{-z_0}{(z_0 + z_2)} \times I_{R1}$$

$z_o + z_2 = 4.355 < 68.44\ \Omega$, $z_o = (1 + f_3)\Omega$

$= 3.162 < 71.565$

$$\therefore \qquad \vec{I}_{R_2} = \frac{-3.162 < 1.565}{4.355 < 68.44} \times 2.8608 < -63.414 \text{ kA}$$

$= -2.077 < -60.289$ kA

$$\vec{I}_{R0} = \frac{-Z_2}{Z_0+Z_2} I_{R1} = -\frac{(0.6+j1.05)}{4.355<68.44} \times 2.8608<-63.414 \text{ kA}$$

$$= \frac{1.21<60.255 \times 2.8608<-63.414}{4.355<68.44} \text{ kA}$$

$$= -0.7948 < -71.6 \text{ °KA}$$

$$\therefore \quad \vec{I}_R = \vec{I}_{R1} + \vec{I}_{R2} + \vec{I}_{R0} \qquad = -7948 < -71.6° \text{ KA}$$

$$= 0$$

$$I_y = I_{y1} + I_{Y2} + I_{Yo} = \alpha^2 I_{R1} + \alpha I_{R2} + I_{R0}$$

where, $\alpha = < 120°$, $\alpha^2 = < -120°$

$$\therefore \quad I_y = 2.8608 < (-63.414 - 120°) - 2.077 < (120° - 60.289) - 0.7948 < -71.6°$$

$$\therefore \quad I_y = (-4.154 - j\ 0.8689) \text{ KA}$$

$$= 4.244 < -168.186 \text{ kA}$$

$$I_B = I_{B1} + I_{B2} + I_{Bo}$$

$$= \alpha I_{R1} + \alpha^2 IR_2 + I_{RO}$$

$$= (2.8608 < 120° - 63.414 - 2.077 < -120° - 60.289° - 0.7948 < -71.6°) \text{ kA}$$

$$= (3.4015 + j\ 3.32) \text{ kA} = 4.624 < 42.638 \text{ °kA}$$

current through the neutral $= 3\ I_{ao}$

$$= 3 \times (0.7948 < -71.6°) \text{ kA}$$

$$= 2.3844 < 108.4 \text{ ° kA}$$

The potential above earth,

$$= 3\ I_{ao}\ Z_n$$

$$= 2.3844 < 108.4° \times 0.1 < 90°)$$

$$= 0.23844 < 198.4° \text{ kV}$$

$\therefore$ The different branch currents are,

$$\left\{ \begin{array}{l} \vec{I}_R = 0;\ \vec{I}_y = 4.244 < -168.186 \text{ kA};\ \vec{I}_B = 4.624 < 42.638 \text{ kA} \\ \vec{I}_{nutral} = 2.3844 < 108.4° \text{kA} \end{array} \right\}$$

The potential above earth = 0.23844 < 198.4°kV **Ans.**

Q–4(a) A signal g(t) band limited to B Hz is sampled by periodic pulse train p_T***(t) made up of a rectangular pulse of width*** $\frac{1}{8B}$ ***sec (centered at origin) repeating at the nyquist rate (2 B pulse per sec). Show that the sampled signal*** g_s***(t) is given by***

$$g_s(t) = \frac{1}{4} g(t) + \sum_{n=1}^{\infty} \left(\frac{2}{n\pi}\right) g(t) \cos(4n\pi Bt)$$

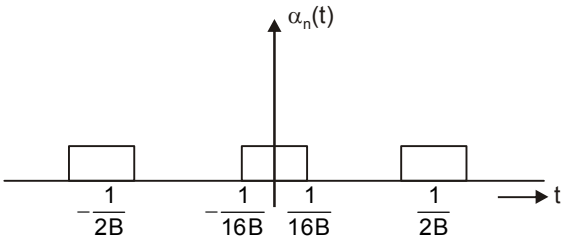
How will you recover g(t) from the signal g_s***(t)?***

Sol: Given,

$$\text{Rectangular pulse of width} = \frac{1}{8B}\text{sec}$$

$$\text{Nquist rate} = 2B \text{ pulsed per sec}$$

$$\therefore \quad \boxed{T_s = \frac{1}{2B}}$$



Fouriers series of $\alpha_T(t)$ can be written as

$$\alpha_n(t) = a_o + \sum_{x=1}^{\infty} a_n \cos n\omega_o t + b_n \sin \omega_o t$$

$$a_o = \frac{1}{T}\int_T \alpha_n(t)dt$$

$$D.a_x = \frac{2}{T}\int_T \alpha_n(t)\cos n w_o t dt$$

$$\text{and } bx = \frac{2}{T}\int_T \alpha_n(t)\sin n w_o t dt$$

$\because$ Given signal is even signal

$$\therefore \quad \boxed{bn = 0}$$

$$a_o = \frac{1}{T}\int_o^T \alpha_n(t)dt$$

$$= \frac{1}{T}\int_{-1/16B}^{1/16} 1\cdot dt$$

$$\boxed{a_o = \frac{1}{4}}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} \alpha_n(t)\cos n w_o t dt$$

$$a_n = \frac{2}{1/2B}\int_{1/-16B}^{1/16B} 1\cdot\cos(4B\pi nt)dt$$

$$= 4B\left[\frac{-\sin(-\pi n/4)+\sin(\pi n/4)}{4B\pi n}\right]$$

$$a_n = \frac{2}{\pi n}\sin\left(\frac{\pi n}{4}\right)$$

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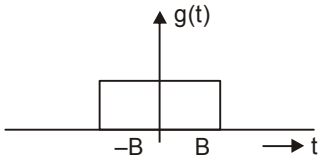
$$P_T(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin(\pi n/4) \cos(4B\pi nt)$$

$$g_s(t) = g(t) \cdot \alpha_n(t)$$

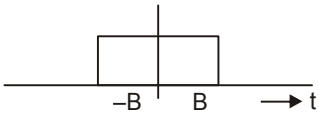
$$= \frac{g(t)}{4} + \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin\left(\frac{\pi n}{4}\right) g(t) \cos(4B\pi nt)$$

the g(t) is,

g(t) =

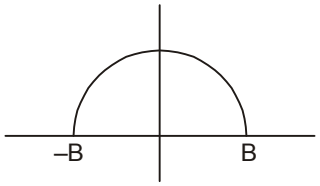


The fourier tranform of g(t)



$\longleftrightarrow \frac{2}{\pi} \sin B\omega$

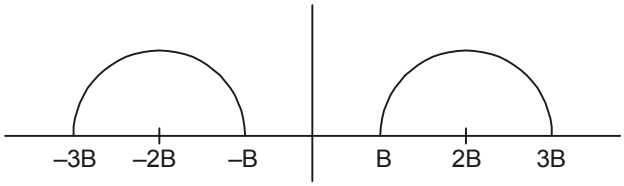
=



From multiplication property of fourier transform.

$$m_1(t) m_2(t) \longleftrightarrow M_1(f)(*)M_2(f)$$

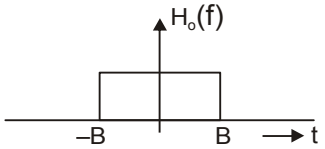
$$\therefore \quad \frac{2}{\pi} \sin\frac{\pi}{4} g(t)(\cos 4\pi Bt) \xleftrightarrow{FT}$$



When

the low pass filter output

=



i.e $\boxed{H(f) = 4\text{rect}\left(\frac{f}{2B}\right)}$

Q–4(b) ***A 3-phase half controlled rectifier with free wheeling diode is supplying a separately excited DC motor for speed control purpose. The AC input to the converter is 415 V, 3-phase, 50 Hz. The motor parameters are: V = 220 V DC, P = 10.5 kW***

Rated speed = 1100 rpm, armature resistance r_a = 0.4 Ω

The field current is kept constant at rated value. The motor is operated at rated speed

delivering half rated torque.

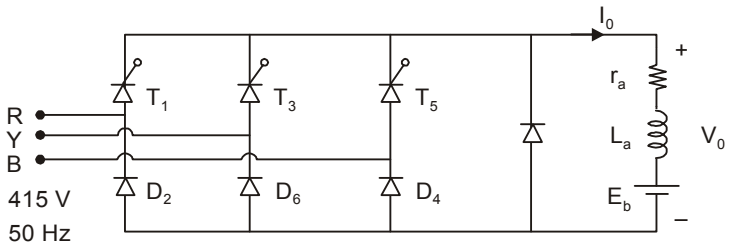
(i) Find motor terminal voltage and triggering angle of thyristor bridge.

(ii) Find the speed of the motor if one of the input phases to the converter is out due to fault and triggering angle is kept as before with same load torque.

(iii) Also find the new triggering angle if the motor speed is to be maintained at rated value with same load torque. (Neglect losses in the machine)

Sol: **(i)** Given:

$3-\phi$ half controlled rectifier:



$$V_o = 220 \text{ V}$$

$$r_a = 0.4\Omega$$

$$N = 1100 \text{ rpm}$$

$$P_0 = 1.5 \text{ kW}$$

$\because$ Field current is kept constant at rated value.

i.e $\phi \rightarrow$ constant

$$P_0 = V_0 I_0$$

$$10.5 \times 10^3 = 220 \times I_0$$

$$\text{Rated current } I_0 = \frac{10.5 \times 10^3}{220} = 47.73 \text{ A}$$

$\because$

$$V_0 = E_{b1} + I_0 r_a$$

$$E_{b1} = 220 - I_0 r_a$$

$$= 220 - 47.473 \times 0.4$$

$$E_{b1} = 220.91 \text{ V}$$

For seperately excited DC motor

i.e $\phi \rightarrow$ constant

$\therefore$ $\tau \propto I$

at rated torque $\tau_{rated} \propto I_0$ ...(i)

at half rated torque $\frac{\tau_{rated}}{2} \propto I_1$ ...(ii)

From equation (i) & (ii)

$$\frac{I_1}{I_0} = \frac{1}{2}$$

$$I_1 = \frac{I_0}{2}$$

$$I_1 = \frac{47.73}{2}$$

$$I_1 = 23.865 \text{ Amp.}$$

So,
$$V_0 = E_{b1} + I_1 r_a$$

$$V_0 = 200.91 + 23.865 \times 0.4$$

$$\boxed{V_0 = 210.456\,V}$$

As we know for half controlled rectifier

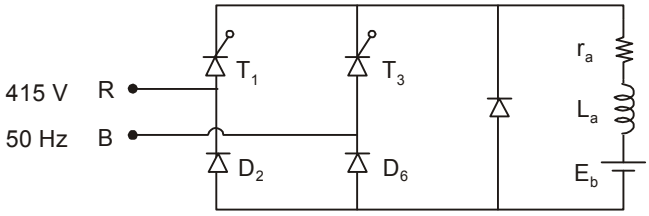
$$V_0 = \frac{3V_{mL}}{2\pi}(1+\cos\alpha)$$

$$210.456 = \frac{3 \times f_2 \times 415}{2\pi}(1+\cos\alpha)$$

$$\cos\alpha = -0.2493$$

$$\boxed{\alpha = 104.44^\circ}$$

(ii) If one of the input phase to the converter is out due to fault then its works like $1-\phi$ half controlled rectifier.



For this b/w
$$V_0 = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$V_0 = \frac{\sqrt{2} \times 415}{\pi}(1+\cos 104.44)$$

$$V_0 = 140.3 \text{ V}$$

$\because$
$$E_{b2} = V_0 - I_1 r_a$$

$$E_{b2} = 140.3 - 23.865 \times 0.4$$

$$= 130.755 \text{ V}$$

As we know,
$$E_b \propto N \qquad N \rightarrow \text{speed(rpm)}$$

$\therefore$
$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2}$$

$$\frac{200.91}{130.755} = \frac{1100}{N_2}$$

$$N_2 = 715.895 \text{ rpm}$$

(iii) If the motor speed is to be maintained at rated value with same load torque

i.e
$$E_{b1} = 200.91 \text{ V}$$

$$I_1 = 23.865 \text{ (Half rated torque)}$$

$\therefore$ $V_0 = E_{b1} + I_1 r_a$

$V_0 = 200.91 + 23.865 \times 0.4$

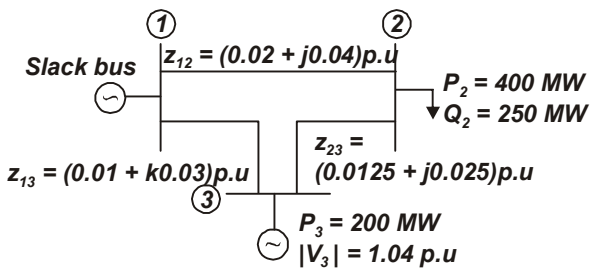
$V_0 = 210.456$ V

So, $V_0 = \frac{V_m}{\pi}(1+\cos\alpha)$

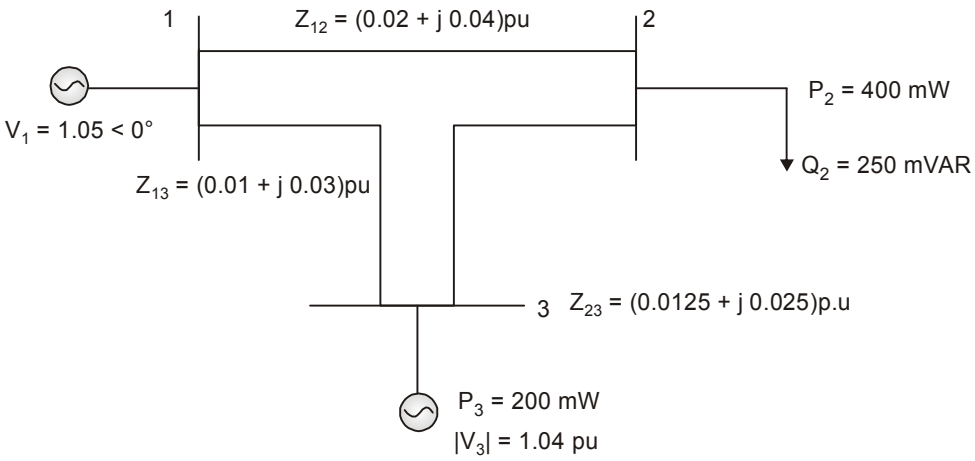
$$210.456 = \frac{\sqrt{2}\times 415}{\pi}(1+\cos\alpha)$$

$$\boxed{\alpha = 82.76^\circ}$$

Q–4(c) ***The figure below shows single line diagram of a power system with generators at bus-I and bus-3. The voltage at bus-1 is 1.05$\angle 0^\circ$ p.u and at bus-3, $|V|$ = 1.04 p.u. Line impedances are in p.u and line charging susceptances are neglected. Obtain state vector using fast decoupled load flow (FDLF) for one iteration.***



Sol. Given: The current diagram as:



$$y_{12} = \frac{1}{z_{12}} = \frac{1}{(0.02+j0.04)} = \frac{1}{0.04472 < 63.435^\circ} = 22.36 < -63.435 = (10 - j20)\ \mho$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{(0.0125+j0.025)} = \frac{1}{0.02795 < 63.435^\circ} = 35.778 < -63.435 = (16.45 - j32)$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{(0.01+j0.03)} = \frac{1}{0.0316 < 71.565} = 31.6456 < -71.565 = 10 - j30$$

$Y_{11} = y_{12} + y_{13} = 20 - j50$ $\quad$ $Y_{22} = y_{12} + y_{23} = 26 - j52$

$Y_{12} = -y_{12} = -10 + j20$ $\quad$ $Y_{23} = -y_{23} = -16 + j\ 32$

$Y_{13} = -y_{13} = -10 + j30$ $\quad$ $Y_{21} = -y_{12} = -10 + j20$

$Y_{33} = y_{13} + y_{23} = 26 - j\,62$

$Y_{31} = y_{13} = -10 + j30$

$Y_{32} = -y_{23} = -16 + j32$ p.u

$$[y]_{Bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

The expression of power P at Bus i is

$$P_i = \sum_{j=i}^{n} |v_i||v_j||y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \quad \ldots(i)$$

$$Q_i = -\sum_{j=i}^{n} |v_i||v_j||y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \quad \ldots(ii)$$

In fact decompled load flow soln.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & 0 & 0 \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & 0 & 0 \\ 0 & 0 & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ 0 & 0 & \frac{\partial \Delta_3}{\partial |V_2|} & \frac{\partial \Delta_2}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = \sum_{j=1}^{3} |V_2||V_j||V_{2j}|\sin(\theta_{2j} - \delta_2 + \delta_j) - |V_2|^2 |y_{22}|\sin\theta_{22}$$

or, $\frac{\partial P_2}{\partial \delta_2} = -Q_2 - |V_2|^2 B_{22}$

$B_{22} = |y_{22}| \sin \theta_{22}$

$\boxed{\text{assuming, } B_{ii} > Q_i \text{ and } |v_i|^2 \approx v_i}$

We obtain,

$$\frac{\partial P_2}{\partial \delta_2} = -|V_2| B_{22}$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

* $\boxed{\text{assuming, } \delta_2 - \delta_3 \text{ is too small \& } |V_3| = 1.04 \approx 1\text{P.u}}$

$$\therefore \quad \frac{\partial P_2}{\partial \delta_3} = -|V_2||Y_{23}|\sin\theta_{23}$$

$$= -|V_2||B_{23}|$$

similarly, $\frac{\partial P_3}{\partial \delta_3} = -|V_3||B_{33}|$ and $\frac{\partial P_3}{\partial P_2} = -|V_3|B_{32}$

$$\therefore \quad \begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix} = -\begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} \quad \ldots(I)$$

$$B_{22} = |y_{12}|\sin\theta_{22} = -52$$

$$B_{23} = |y_{23}|\sin\theta_{23} = 32$$

$$B_{32} = |y_{32}|\sin\theta_{32} = 32$$

$$B_{33} = |y_{33}|\sin\theta_{33} = -62$$

$$\therefore \quad [B] = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix} \qquad \therefore [B]^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

$$\frac{\partial Q_2}{\partial |V_2|} = |v_2||y_{22}|\sin\theta_{22} + Q_2$$

where, $$Q_2 = -\sum_{j=1}^{3} |V_2||V_i||Y_{2j}|\sin(\theta_{2j} - \delta_2 + \delta_j)$$

$\boxed{\text{assuming, } \beta_{22} = |y_{22}|\sin\theta_{22} >> \theta_2}$

then, $$\frac{\partial Q_2}{\partial |V_2|} = -|y_i|\,|y_{22}|\sin\theta_{22}$$

and $$\frac{\partial Q_2}{\partial |V_3|} = 0$$

$$\Delta Q_2 = \frac{\partial Q_2}{\partial |V_2|} \times \Delta|V_2| \quad \begin{bmatrix} \Delta|V_3| = 0 \\ \text{as Bus 3 is PV Bus} \end{bmatrix}$$

or, $$\frac{\Delta Q_2}{|V_2|} = -[B_{22}][\Delta|V_2|] \quad \ldots(ii)$$

assuming a base of 100 MVA

$$(400 + j250) = \frac{400 + j250}{100} \text{P.u}$$

$$= (4 + j2.5) \text{ p.u}$$

$$\therefore \quad \Delta P_2 = P_{2(\text{Scheduled})} - P_2(\text{cal}) \quad \begin{vmatrix} P_2(\text{sch}) = 0 - 4 = -4 \text{ p.u} \\ Q_2(\text{sch}) = j(0 - 2.5) = -j2.5 \end{vmatrix}$$

$$P_2 \text{ (Cal)} = \sum_{j=1}^{n} |V_2||V_j|\cos(\theta_{2j} - \delta_2 + \delta_j) \times |y_{ij}|$$

assuming, similarly $\delta_2 = 0°$ & $\delta_3 = 0°$ & $|V_2| = 1.0$ p.u

$$P_2 \text{ (cal)} = 1.05\cos(\theta_{21} - \delta_2 + \delta_1)y_{12} + 1.04\cos(\theta_{23} - \delta_2 + \delta_3)y_{23} + \cos_{22} \times y_{22}$$

$$\theta_{21} = 116.656, \ \theta_{23} = 116.565, \ \theta_{22} = -63.435$$

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$= -1.14$ p.u

$\therefore \quad \Delta P_2 = -4 - (1.14) = -2.86$ p.u.

similarly,

$$Q_{2(cal)} = \sum_{j=i}^{n} |V_2||V_j| \sin(\theta_{2j} - \delta_2 + \Delta_j) \times |Y_2|$$

$Q_{2\ (cal)} = -2.28$ p.u.

$\therefore \quad \Delta Q_2 = \Delta Q_{2(sch.)} - Q_{2(cal.)}$

$= -2.5 - (-2.8) = -0.22$

$$\begin{bmatrix} \Delta\delta_2^{(1)} \\ \delta_3^{(0)} \end{bmatrix} = [B]^{-1} \begin{bmatrix} \Delta P_2 / |V_2| \\ \Delta P_3 / |V_3| \end{bmatrix}$$

as obtained earlier, from eqn. (1)

$$[B]^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

$\therefore \quad \delta_2^{(1)} = -0.060483$

$\delta_3^{(1)} = -0.008909$

Similarly, from equation (ii)

$$\Delta|V_2| = -\frac{1}{\Delta|V_2|[B_{22}]}\left[\frac{\delta Q_2}{|V_2|}\right]$$

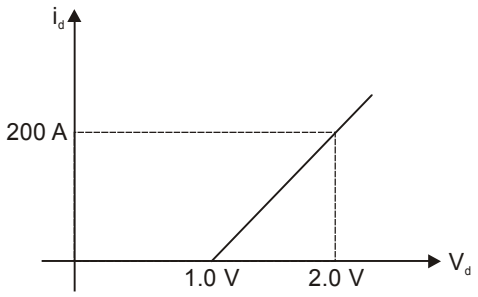
$$= -\frac{1}{52}\left(\frac{-2.2}{1.0}\right) = -0.0042308 \text{ p.u}$$

$$\left\{\begin{aligned} \delta_2^{(1)} &= 0° + (-0.060483° = -0.060483 \\ \delta_3^{(1)} &= 0 - 0.008909 = -0.008909 \\ |V_2^{(1)}| &= 1 + (-0.0042308) = 0.995769 \end{aligned}\right\}$$

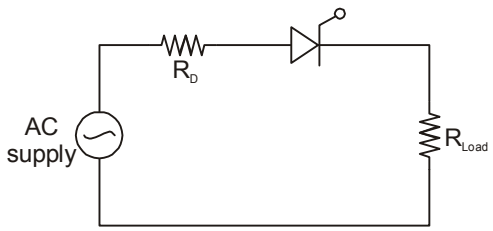
There are the various after 1st interal

SECTION-B

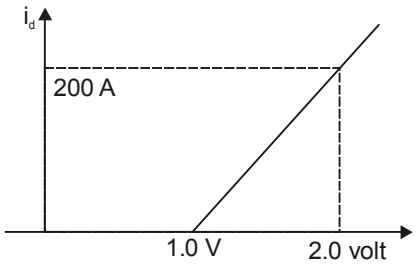
Q–5(a) ***A thryristor is having the I-V characteristic as given in the figure below. It is used in a half wave rectifier circuit with resistive load operating at $\alpha = 30°$ and carrying a peak load current of 100 A. Determine the average conduction loss in the thyristor.***



Sol.



(a)



From figure (b)

Voltage drop across thyristor = 1 V

Resistance of thyristor = 1/200

$$\boxed{R_D = 5\times10^{-3}\,\Omega}$$

Given I_p = 100 A

α = 30°

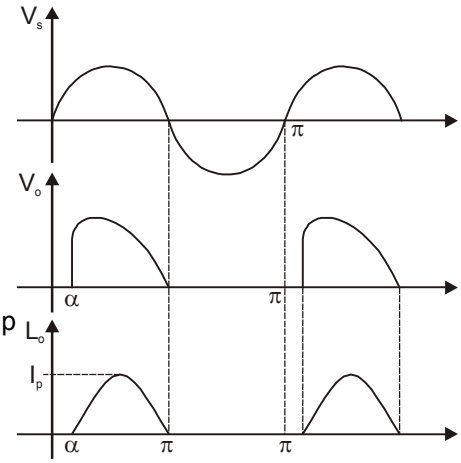
output average current

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t\, d\omega t$$

$$= \frac{I_m}{2\pi}(-\cos\omega t)_{\alpha}^{\pi}$$

$$\boxed{I_o = \frac{I_m}{2\pi}(1+\cos\alpha)}$$

$$I_{0ay} = \frac{100}{2\pi}(1+\cos 30) = 39.69\,\text{Amp}$$



Output rms current

$$I_{0ms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}(I_m \sin\omega t)^2\, d\omega t\right]^{1/2}$$

$$= \left\{\frac{I_m^2}{2\pi}\int_{\sigma}^{\pi}(\sin^2\omega t)\, d\omega t\right\}^{1/2}$$

$$I_{0rms} = \frac{I_m}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2}$$

$$= \frac{100}{2\sqrt{\pi}}\left[(\pi-\pi/6)+\frac{1}{2}\sin 60\right]^{1/2}$$

$$\boxed{I_{0m} = 49.27\ \text{Amp}}$$

Conduction loss of thyristor

$$P_{loss} = I_{0rm}^2 \times R_D + I_{0ay} \times V_T$$

$$P_{loss} = (49.27)^2 \times 5 \times 10^{-3} + 29.69 \times 1$$

$$\boxed{P_{loss} = 41.82} \text{ watt}$$

Q–5(b) ***A three-phase equilateral transmission line has a total corona loss of 55 kW at 110 kV and 100 kW at 114 kV. What is the disruptive critical voltage between lines ? What is the corona loss at 120 kV ?***

Sol. Given,

Sl. No.	Voltage	Corresponding Crona loss
1.	110 kV	55 kW
2.	114 kV	100 kW

Corona loss $$P_c = 243.5\frac{(f+25)}{\delta}\sqrt{\frac{r}{D}}(V - V_d)^2 \times 10^{-5} \text{ kN/km/hour}$$

where,
V = Phase voltage in kv (rms value)
V_d = Disruptive critical voltage in kv (rms value)
r = Radius of conductor in metres
D = Spacing between conductors in metres
f = System frequency

So, $$P_c \propto (V - V_d)^2$$

$$\therefore \quad \frac{P_{c_1}}{P_{c_2}} = \frac{(V_1 - V_d)^2}{(V_2 - V_d)^2}$$

or $$\frac{55}{100} = \frac{(110 - V_d)^2}{(114 - V_d)^2}$$

or $$(110 - V_d) = \sqrt{0.55}(114 - V_d)$$

or $$V_d - 0.7416\, V_d = 110 - 84.5447$$

or $$0.2584\, V_2 = 25.4553$$

or $$V_{d(L)-L} = 98.51 \text{ V}, \; V_D = (Ph) = \frac{98.51}{\sqrt{3}} = 56.876$$

Corona loss at 120 kV

$$\frac{P_c}{100} = \frac{(120 - 98.51)^2}{(114 - 98.51)^2} = \frac{(21.49)^2}{(15.49)^2}$$

or $$P_c = 100 \times \frac{461.8201}{239.9401} = 192.473 \text{ kW}$$

Let power loss at 120 kV be 'P'

$$\therefore \quad \frac{P}{55} = \frac{(120-98.52)^2}{(110-98.52)^2} = 35$$

$$P_L = 3.5 \times 55 \text{ kW} = 192.55 \text{ kW}$$

$$P_{corona} \text{ at } 120 \text{ kV} = 192.55 \text{ kW Ans.}$$

Q–5(c) ***A Gaussian pulse is specified by***

$g(t) = Ae^{-\alpha^2 t^2}$,

where α is an arbitrary attenuation coefficient and A is constant. Show that the Fourier transform of g(t) is also Gaussian.

Sol. Gaussian pulse

$$g(t) = Ae^{-\alpha^2 t^2}$$

As we know, fourier transform of g(t):

$$G(j\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t} dt$$

$$G(j\omega) = A\int_{-\infty}^{\infty} e^{-\alpha^2 t^2} e^{-j\omega t} dt$$

$$= A\int_{-\infty}^{\infty} e^{-(\alpha^2 t^2 + j\omega t)} dt \quad \text{... (i)}$$

Let

$$\alpha^2 t^2 + j\omega t = P^2 2pq$$

$$\alpha^2 t^2 + j\omega t = (p+q)^2 - q^2$$

$$p^2 = \alpha^2 t^2 \quad 2pq = j\omega t$$

$$p = \alpha t \quad q = \frac{j\omega t}{2P} = \frac{j\omega t}{2\alpha t}$$

$$q = \frac{j\omega}{2\alpha}$$

So,

$$(p+q)^2 - q^2 = \left(\alpha t + \frac{j\omega}{2\alpha}\right)^2 - \left(\frac{j\omega}{2\alpha}\right)^2$$

Let

$$\alpha dt = du$$

$$dt = \frac{du}{\alpha}$$

So, from equation (1)

$$G(j\omega) = A\int_{-\infty}^{\infty} e^{-\left(u^2 + \frac{\omega^2}{4\alpha^2}\right)} \frac{du}{\alpha}$$

$$= A\int_{-\infty}^{\infty} e^{-u^2} . e^{-\frac{\omega^2}{4\alpha^2}} \frac{du}{\alpha}$$

$$= \frac{A}{\alpha} e^{-\frac{\omega^2}{4\alpha^2}} \int_{-\infty}^{\infty} e^{-u^2} du$$

$$= \frac{A}{\alpha} e^{-\frac{\omega^2}{4\alpha^2}} \sqrt{\pi} . \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-u^2 du}$$

$$\boxed{G(j\omega) = \frac{A}{\sqrt{\pi\alpha}} e^{-\frac{\omega^2}{4\alpha^2}}} \qquad \because \frac{1}{\pi}\int_{-\infty}^{\infty} e^{-u^2} du = 1$$

$$G(\omega) = \frac{1}{\sqrt{\pi}} \frac{A}{\alpha} e^{-\frac{\pi^2 f^2}{\alpha^2}}$$

This is also a gaussian function.

So, we can say fourier transform of g(t) is also gussian.

Q–5(d) ***What are the advantages and limitations of lead and lag networks in a practical control system ?***

Sol: **Lag Network**

Advantages:

- A phase lag Network offers high gain at low frequency. This it performs the function of a low pass filter.
- Increases the steady state performance of the system.
- The lag network offers a reduction in bandwidth & this provides longer rise time & setting time.

Disadvantages

- Its shows more sensitivity towards variations in the parameter.
- Due to the addition of external network, some attenuation is introduced. So the overall gain must be increased to handle the attenuation. But this will increase the requirement of more elements & so the cost & space requirement.
- Adversely affect the stability of the system.
- Decrease bandwidth, because response is quite slow.

$$\boxed{BW \times t_r = 0.35}$$

Lead Network:

Advantages

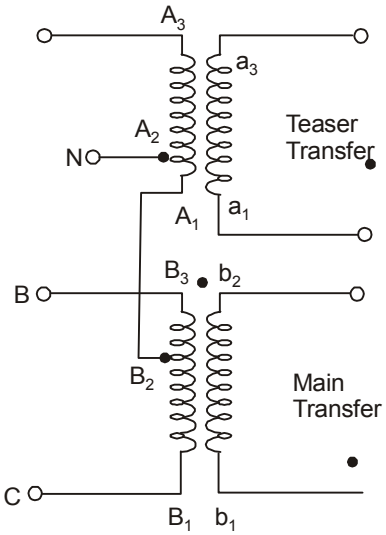
- Improves damping of the overall system.
- Transient respones gets improved.
- Improve the phase margin.
- Quick response therefore increase bandwidth.
- It maximizes the velocity constant of the system.

Disadvantages:

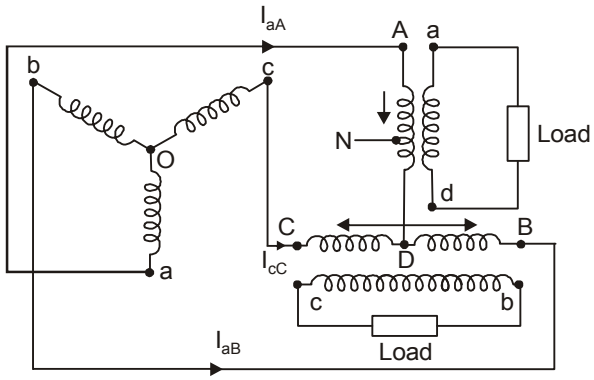
- Its also adds some attenuation to it.
- Thus to compensate the attenuation there must be an additional gain enhancement. But with an increase in gain the requirement of more element increases. This leads to cause cost enhancement as well as more weight & greater space.
- The lead network increases B.W but with increased B.W the system becomes more susceptible to noise.
- This sometimes makes the system conditionally stable.

Q–5(e) ***For a Scott connected transformer, prove that the number of turns on primary of the teaser transformer is $\frac{\sqrt{3}}{2}$ times the number of turns in primary of main transformer.***

Sol. Scott-connection and its schematic connection diagram are shown in



Scott-connection



Its schematic connection diagram

Let the positive direction of primary voltage and current be taken arbitrarily along the arrows from A to D and from B to C as shown in figure (i).

Neglecting the transformer internal impedance drops the voltage V_{ab} or V_{AB} must be equal to the voltags induced in AD and DB.

$$\vec{V}_{ab} = \vec{V}_{AB} = \vec{V}_{AD} + \vec{V}_{DB}$$

$$\therefore \quad \vec{V}_{AD} = \vec{V}_{AB} - \vec{V}_{DB} = \vec{V}_{AB} + \vec{V}_{BD}$$

If $V_{AB} = V_{BC} = V_{CA} = V_L$ (line voltage) then with V_{BC} as reference phasor as shown in figure (ii)

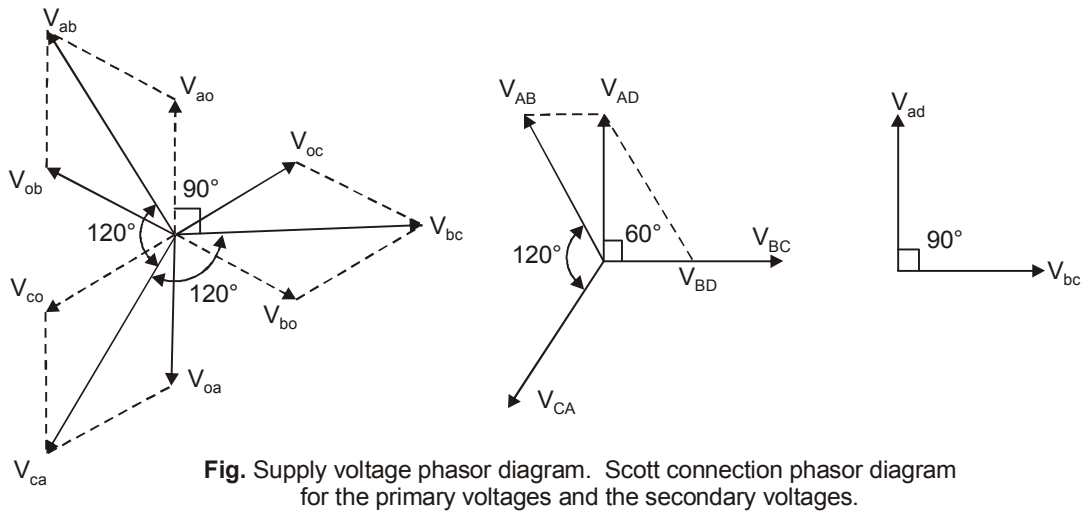


Fig. Supply voltage phasor diagram. Scott connection phasor diagram for the primary voltages and the secondary voltages.

$$\vec{V}_{BC} = V_L\angle 0°, \vec{V}_{BD} = \frac{1}{2}\vec{V}_{BC} = \frac{1}{2}V_L\angle 0°, \vec{V}_{AB} = V_L\angle 120° \text{ and } \vec{V}_{CA} = V_L\angle -120°$$

Now, $$\vec{V}_{AD} = V_L\angle 120° + \frac{1}{2}V_L\angle 0°$$

or $$\vec{V}_{AD} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}V_L + \frac{1}{2}V_L = j\frac{\sqrt{3}}{2}V_L$$

or $$\vec{V}_{AD} = \frac{\sqrt{3}}{2}V_L\angle 90°$$

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Hence, V_{AD} the voltage across the teasear, is $\frac{\sqrt{3}}{2}$ of that across the main and leads it by 90°

Since, $V_{AD} = \frac{\sqrt{3}}{2} V_L$ and $V_{BC} = V_L$ the primaries of both the transformers would have equal voltage per turn only if the teaser primary turns are $\frac{\sqrt{3}}{2}$ times the primary turns of the main transformer.

Q–6(a) ***A 15 kW, 400 V, 3-phase, star connected synchronous motor has synchronous impedance of $0.4 + j4\Omega$. Find the motor excitation voltage for full load output at 0.866 leading power factor. Take the armature efficiency of 95%.***

Sol. Given:

$$Z_s = 0.4 + j4 = 4.02\angle 84.28°$$

$$\text{Efficiency } (\eta) = 95\%$$

$$\text{output power} = 15 \text{ kW}$$

$$V_L = 400 \text{ V}$$

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\therefore \quad \text{Input power} = \frac{\text{o/p power}}{\eta}$$

$$= \frac{15000}{0.95} = 15789.47 \text{ watt}$$

So Armature current

at full load

$$I_a = \frac{15789.47}{\sqrt{3} \times 400 \times 0.866}$$

$$= 26.316 \angle 30°$$

As we know for motor

$$E_{ph} = V_{ph} - I_a z_s$$

$$E_{ph} = \frac{400}{\sqrt{3}} - 26.316\angle 30° \times 4.02\angle 84.28$$

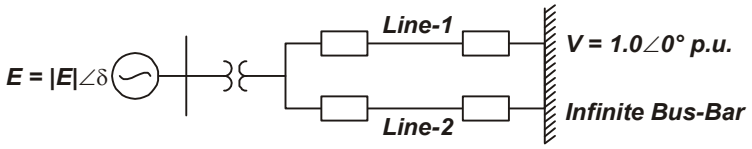
$$= 290.88 \angle -19.36°$$

So

$$E_{line} = \sqrt{3} \times E_{ph} = \sqrt{3} \times 290.88$$

$$= 503.818\angle -19.36° \text{ Ans.}$$

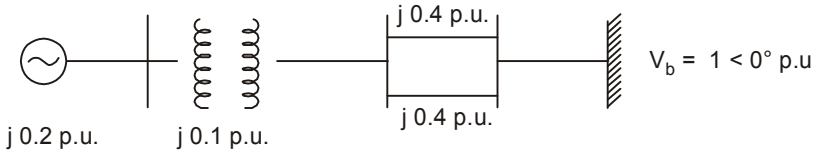
Q–6(b) ***A synchronous machine is connected to an infinite bus through a transformer and a double circuit line shown in the figure. The infinite bus voltage is $V = 1.0\angle 0°$ p.u. The direct axis transient reactance of the machine is 0.20 p.u., the transformer reactance is 0.10 p.u. and the reactance of each of the transmission lines is 0.4 p.u., all to a base of the rating of the synchronous machine. Initially the machine is delivering 0.8 p.u. power with a terminal voltage of 1.05 p.u. The inertia constant H = 5 MJ/MVA. All resistances are neglected. Determine the equation of motion of the machine rotor.***



Sol. Given,

$$xd = j.2 \text{ p.u}, \; x_{ime} \; j0.4 \text{ p.u}$$

$$x_{tr} = j\ 0.1 \text{ p.u.}, \; V_{bar}\ 1 < 0° \text{p.u}$$



$$Pe = \frac{V_t \times V_b}{(0.1-0.2)} \sin\delta_t \quad \text{or,} \quad \delta_t = \delta_m^{-1} \frac{0.8 \times 0.3}{1.05}$$

$$\delta_L = 13.2 \text{ B°}$$

$$i = \frac{1.05\angle 13.213° - \angle 0°}{0.3\angle 90°}$$

$$= \frac{0.0222 + j0.24}{0.3 \angle 90°} = \frac{0.241 \angle 84.715°}{0.3 \angle 90°}$$

$$= 0.8033 \angle -5.258$$

$$E_f = v_t < \theta + i \times jxd$$

$$= 1.05 < 13.213 + 0.8033 \times 0.2 < 90° - 5.285°$$

$$= 1.05 < 13.213 + 0.161 < 84.715°$$

$$= 1.04 + j0.4 = 1.1143 < 21.04\ °$$

$$X_{eq} = j0.20 + j0.10 + j\ 0.20$$

$$j\ 0.50 \text{ p.u.}$$

$$P_e = \frac{E_f v_t}{x_{eq}} \sin\delta = \frac{1.114 \times 1}{0.5} \sin\delta = 2.228 \sin\delta$$

Solving equation of the roler is given as

$$\frac{gH}{\pi f}\left(\frac{d^2\delta}{dt^2}\right) = p_m - p_e$$

$$= 0.8 - 2.228 \sin\delta$$

or,
$$\frac{1 \times 5}{\pi \times 50}\left(\frac{d^2\delta}{dt^2}\right) = 0.8 - 2.228 \sin\delta$$

or,
$$\frac{d^2\delta}{dt^2} = 31.4(0.8 - 0.228 \sin\delta)\frac{\text{rad}}{\text{sec}^2} \quad \text{Ans.}$$

Q–6(c) ***The open loop transfer function of unity feedback control system is given by***

$$G(s) = \frac{K}{s(s+a)(s+b)} \; 0 < a \le b$$

(i) ***Find the range of the gain constant K (> 0) for stability using Routh-Hurwitz criterion.***

(ii) ***What type of control do you use if the system is required to have zero steady-state error for ramp input ? Let 'A' be the parameter that can be varied in the introduced control. Find the range of 'K' for stability in terms of parameters a, b and A using Routh-Hurwitz criterion.***

Sol. **(i)** Characteristic equation figure given open loop function

$$(S^2 + Sa)(S + b)\,k = 0$$

S^3	1	ab
S^2	$a+b$	k
S^1	$ab-\dfrac{k}{a+b}$	0
S^0	k	

$$k > 0$$

$$ab > \frac{k}{a+b} \Rightarrow 0 < k < ab(a+b)$$

(ii) Transfer of PI controller $= \left(1+\dfrac{A}{s}\right)$

Characteristic equation is:

$$1+\frac{(A+s)k}{s^2(s+a)(s+b)} = 0$$

$$\Rightarrow \quad s^4+(a+b)s^3+abs^2+ks+ak = 0$$

$$\Rightarrow \quad \begin{array}{c|ccc} s^4 & 1 & ab & ak \\ s^3 & a+b & k & \\ s^2 & \dfrac{ab(a+b)k}{a+b} & Ak & \\ s^1 & \dfrac{\left(\dfrac{ab(a+b)-k}{a+b}\right)-(a+b)Ak}{\dfrac{(a+b)ab-k}{a+b}} & & \\ s^0 & Ak & & \end{array}$$

For stability: $ak > 0$

$k > 0$

and $\left[\dfrac{(a+b)ab-k}{a+b}-(a+b)A\right]k > 0$

$\Rightarrow (a + b)\,ab - k - (a + b)^2 A > 0$

$\Rightarrow (a + b)\,ab - (a + b)^2 A > 0$

$\Rightarrow k < (a + b) - (a + b)^2 A$

and $\dfrac{(a+b)ab-k}{a+b} > 0$

$\Rightarrow k\,(a + b)\,ab$

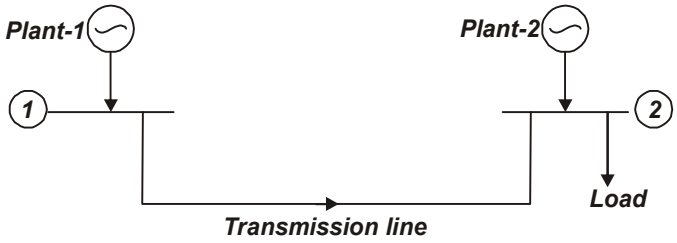
$\Rightarrow k < (a + b)\,ab - (a + b)^2 A$

Q–7(a) ***A system consists of two plants connected by a transmission line and a load is at power plant-2 as shown in the figure. Data for the loss equation consists of the information that 200 MW transmitted from plant-1 to the load results in transmission loss of 20 MW. Find***

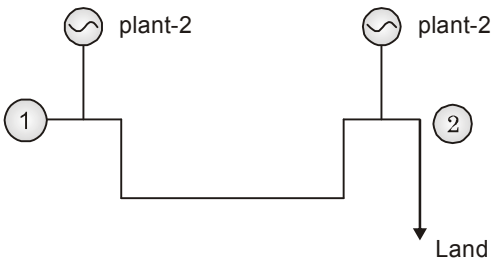
the optimum generation schedule considering transmission losses to supply a load of 204.41 MW. Also evaluate the amount of financial loss that may be incurred if at the time of scheduling transmission losses are not coordinated. The incremental fuel cost characteristics of plant-1 and plant-2 are given by

$$\frac{df_1}{dP_1} = 0.025\,P_1 + 14 \quad \text{Rs/MW-hr}$$

$$\frac{df_2}{dP_2} = 0.05\,P_2 + 16 \quad \text{Rs/MW-hr}$$



Sol.



The expression of the transmission line cross is gain as

$$P_1 = \beta_{11}P_1^2 + \beta_{22}P_2^2 + 2\beta_{12}P_1P_2$$

$\because$ The load is concluded to plant-2, $\therefore$ $\beta_{22} = \beta_{n2} = 0$

$\therefore$ $P_L = \beta_{11}P_1^2$

Given, when $P_1 = 200$ mW, $P_L = 20$ mW

$\therefore$ $P_L = 20 = \beta_{11} \times (200)^2$ or $\beta_{11} = \dfrac{20}{40000}$ (MW)

$\therefore$ $\boxed{P_L = 5 \times 10^{-4} \times P_1^2}$ $= 5 \times 10^{-4}$ (MW)

Given $\dfrac{df_1}{dp_1} = 0.025\,P_1 + 14$, $\dfrac{df_2}{dp_2} = 0.05P_2 + 16$,

When the losses are coordinated, then

$$\frac{\dfrac{df_1}{dp_1}}{1-\left(\dfrac{\partial P_L}{\partial P_1}\right)} = \frac{\dfrac{df_2}{dp_2}}{1-\dfrac{\partial P_1}{\partial P_2}} = \lambda$$

$$\frac{\partial P_2}{\partial P_1} = 10^{-3}\,P_1 \quad \frac{\partial P_2}{\partial P_2} = 0$$

$$\therefore \quad \frac{0.025P_1 + 14}{1 - 10^{-3}P_1} = \frac{0.05P_2 + 16}{1} = \lambda \quad \left[\because \frac{\partial P_L}{\partial P_2} = 0\right]$$

or, $0.025\ P_1 + 14 = (0.05\ P_2 + 16)\ (1 - 10^{-3}\ P_1)$ ...(1)

again, $P_1 + P_2 - P_L = P_{load} = 204.11$ ...(ii)

or, $P_2 = 204.41 - P_1 + \beta_{11}P_1^2$

$= 204.41 - P_1 + 5 \times 10^{-4}\ P_1^2$

from (1) & (ii), we obtain

$$0.025\ P_1 + 14 = \left\{0.05\left(204.41 - P_1 + 5\times10^{-4}P_1^2\right) + 16\right\}\{1 - 10^{-3}\ P_1\}$$

$$= \left(10.2205 + 25\times10^{-6}P_1^2 - 0.05P_1 + 16\right)(1 - 10\text{–}3\ P_1)$$

$$= \left(26.2205 + 0.25\times10^{-4}P_1^2 - 0.05P_1\right)\left(1 - 10^{-3}P_1\right)$$

$$0.025\ P_1 + 14 = 26.2205 - 0.0262205\ P_1 + 0.25 \times 10^{-4}\ P_1^2 - 0.25 \times 10^{-7}\ P_1^2 - 0.05\ P_1 + 0.05 \times 10^{-3}\ P_1^2$$

$$0.025\ P_1 + 14 = 26.2205 + 7.5 \times 10^{-5}\ P_1^2 - 0.25 \times 10^{-7}\ P_1^3 - 0.0762\ P_1$$

or, $0.25 \times 10^{-7}\ P_1^3 - 7.5 \times 16^{-5}\ P_1^2 + 0.1212\ P_1 - 12.2205 = 0$

or, $P_1 = 133.35$ mW

$P_L = B_u\ P_1^2 = 5 \times 10^{-4} \times (133.35)^2$ mW

$= 8.891$ mW

$\therefore$ $P_2 = P_{load} + P_L - P_1$

$= 204.41 + 8.891 - 133.35$

$= 79.951$ mW

$\therefore$ When the losses are coordianted then for optimum performance,

$$\left\{\begin{matrix} P_1 = 133.35\ \text{MW}; P_2 = 79.951\ \text{MW}\ \& \\ P_L = 8.891\ \text{MW} \end{matrix}\right\}$$

When the losses, are not coordianted but included,

$$\frac{df_1}{dp_1} = \frac{df_2}{dp_2} = \lambda$$

$\therefore$ $0.025\ P_1 + 14 = 0.05\ P_2 + 16$ ...(i)

$P_1 + P_2 - P_L = P_{load}$

$\therefore$ $P_2 = 204.41 + 5 \times 10^{-4}\ P_1^2 - P_1$ ...(ii)

$\therefore$ showing the equations (i) and (ii), we get

$0.025\ P_1 + 14 = 0.05\ (204.41 + 5 \times 10^{-4}\ P_1^2 - P_1) + 16$

$0.25 \times 10^{-4}\ P_1^2 - 0.075\ P_1 + 126.2205 = 0$

or $P_1 = 2827.1$ MW , $P_2 = 172.91$ MW

$\therefore$ $P_L = B_u\ P_1^2 = 5 \times 10^{-4} \times (172.91)^2$ MW

$P_L = 14.96$ MW

$\therefore$ $P_2 = P_{load} + P_L - P_1$

$= (204.41 + 14.95 - 172.91)$ MW $= 46.451$ MW

$$\therefore \quad \left\{\begin{matrix} P_1 = 172.91\,\text{MW}; \quad P_2 = 46.451\,\text{MW} \\ P_L = 14.95\,\text{MW} \end{matrix}\right\} \quad \text{When the losses are coordinates)}$$

$\therefore$ Total savings indluced when the losses are coordinated is given as:

$$= ₹\left\{\int_{172.91}^{133.35}(0.025P_1+14)\,dP_1 + \int_{46.451}^{79.953}(0.05P_2-116)\,dP_2\right\}$$

$$= \left\{\begin{matrix} \frac{0.025}{2}(133.35^2-172.91^2)+14(133.35-172.91)+\frac{0.05}{2}(79.953^2-46.451^2) \\ +16(79.953-46.451) \end{matrix}\right\}$$

$$= \text{Rs.}63.4 \approx \boxed{63\text{Rs}}$$

$\therefore$ Total savings incurred when the losses are coordianted.

= Rs 63. Ans.

Q–7(b) A continuous-time integrator has a system function $H_a(s) = \frac{1}{s}$.

(i) Design a discrete-time integrator using bilinear transformation and find the difference equation relating the input x[n] to the output y[n] of the discrete-time system.

(ii) Find the frequency response of the discrete-time integrator found in part (i) and determine whether or not this system is a good approximation of the continuous time system.

$$\left(\textit{For } \theta << 1,\ \sin\theta = \theta \textit{ and } \cos\theta = 1-\frac{\theta^2}{2}\right)$$

Sol: Given,

$$H_a(s) = 1/s$$

To convert s-domain to z-domain by bilinear transformation

Put
$$S = \frac{2}{T_s}\cdot\frac{1-z^{-1}}{1+z^{-1}}$$

$$T_s = \text{Sampling interval}$$

$$H(z) = \frac{1}{\frac{2}{T_s}\cdot\frac{1-z^{-1}}{1+z^{-1}}}$$

$$\Rightarrow \quad \frac{C(z)}{R(z)} = \frac{T_s(1+z^{-1})}{1-z^{-1}}$$

$$\Rightarrow \quad \frac{2}{T_s}(z)-\frac{2}{T_s}z^{-1}C(z) = R(z)+z^{-1}R(z)$$

Applying inverse z-tranform on both sides.

$$\boxed{\frac{2}{T_s}C(n)-\frac{2}{T_s}C(n-1) = r(n)+r(n-1)}$$

(ii) Now, put
$$z = e^{j\omega}$$

$$H(e^{j\omega}) = \frac{1+e^{-j\omega}}{\frac{2}{T_s}(1-e^{-j\omega})}$$

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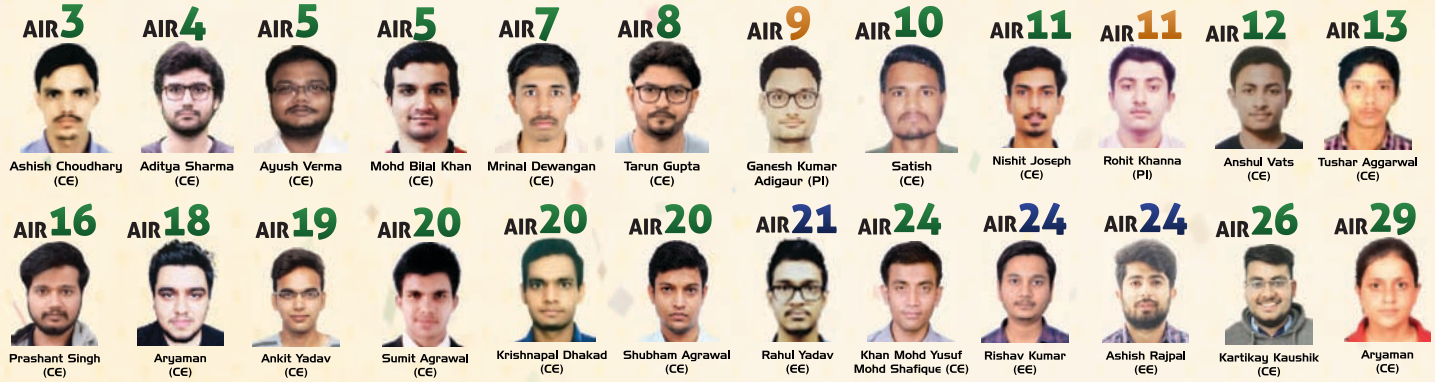

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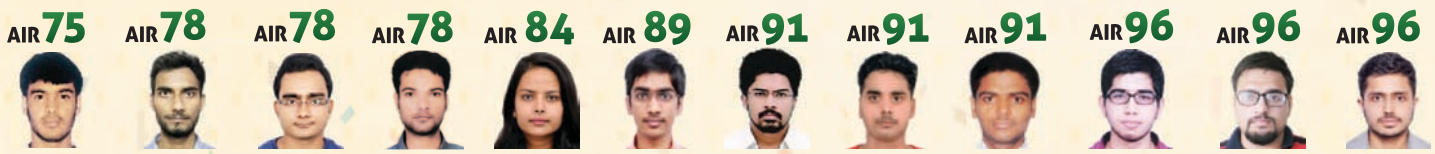

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$$\because \quad e^{-j\omega} = \cos\omega - j\sin\omega$$

$$H\left(e^{j\omega}\right) = \frac{(1+\cos\omega) - j\sin\omega}{\frac{2}{T_s}\left[(1-\cos\omega) + (j\sin\omega)\right]}$$

$$\Rightarrow \quad H\left(e^{j\omega}\right) = \frac{(1+\cos\omega) - j\sin\omega}{\frac{2}{T_s}(1-\cos\omega + j\sin\omega)} \times \frac{(1-\cos\omega - j\sin\omega)}{(1-\cos\omega - j\sin\omega)}$$

$$H\left(e^{j\omega}\right) = \frac{T_s}{2} \cdot \frac{1-\sin^2\omega - 2j\sin\omega - \cos^2\omega}{2-2\cos\omega}$$

$$H\left(e^{j\omega}\right) = \frac{T_s}{2} \cdot \frac{-2\sin\omega}{2(1-\cos\omega)} = \frac{-T}{2} j \frac{\sin\omega}{1-\cos\omega}$$

$$\Rightarrow \quad H\left(e^{j\omega}\right) = \frac{-T_s}{2} j \frac{\cos\omega/2}{\sin\omega/2}$$

Since by is given that if $\theta \ll 1$ then $\sin\theta = \theta$

$$\cos\theta = 1 - \frac{\theta^2}{2}$$

So, by using the above approximation,

$$\cos\frac{\omega}{2} = \frac{1-(\omega/2)^2}{2} = 1 - \frac{\omega^2}{8}$$

$$\sin\frac{\omega}{2} = \frac{\omega}{2}$$

$$\Rightarrow \quad H\left(e^{j\omega}\right) = -j\frac{T_s}{2}\frac{\left(1-\omega^2/8\right)}{\omega/2}$$

$$= -Tj\left(\frac{1-\omega^2/8}{\omega}\right)$$

$$\left|H\left(e^{j\omega}\right)\right| = \left|\frac{1-\omega^2/8}{\omega}\right|$$

$\omega = 0, \left|H\left(e^{j\omega}\right)\right| = \infty$

$\omega = 1, \left|H\left(e^{j\omega}\right)\right| = 7/8$

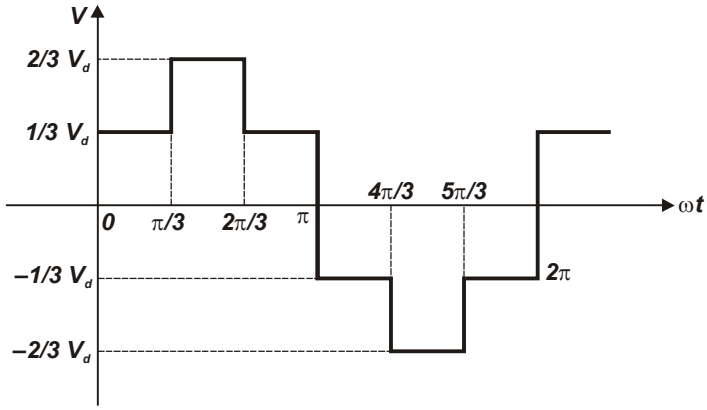
$\omega = 2, \left|H\left(e^{j\omega}\right)\right| = 1/4$

By seeing the magnitude response of discrete time integrator is seems to be the response of continuous time intervals.

Q–7(c) ***For a 3-phase, 50 Hz, 415 V, 4-pole induction motor, the standstill resistance and reactance are 3.0 Ω and 5.0 Ω at 50 Hz respectively. The machine has magnetising inductance of 350 mH and stator resistance of 1.2 Ω. The machine is supplied from a 3-phase voltage source inverter with quassi square wave output voltage waveform per phase as shown in the figure below. The DC bus voltage is 500 V.***

If the machine is operating at 4% slip, find (i) the fundamental input current, (ii) harmonic copper losses in the machine up to 13 harmonics, and (iii) input power factor.

Assume negligible core losses, equal distribution of stator and rotor leakage reactances and linear magnetic circuit.



Sol:

The input voltage can be written as

$$V_R = \sum_{n=6K\pm1}^{K=0-\infty} \frac{2V_s}{n\pi}\sin(n\omega t) \rightarrow \text{Per phase voltage}$$

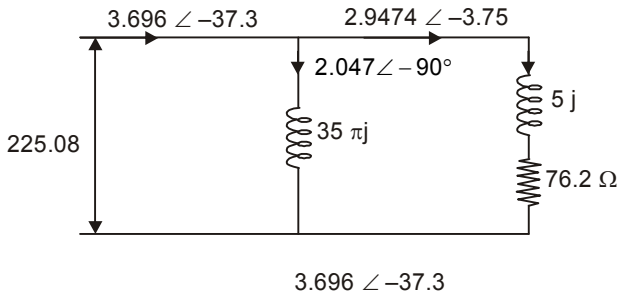
Solving in per phase

Harmonics present $\rightarrow$ 5, 7, 11, 13

$Z_{fundamental} \rightarrow$

$$V_{fundamental} = \frac{2(500)}{\sqrt{2}\pi}\text{Volt}$$

$$= 225.08 \text{ V}$$



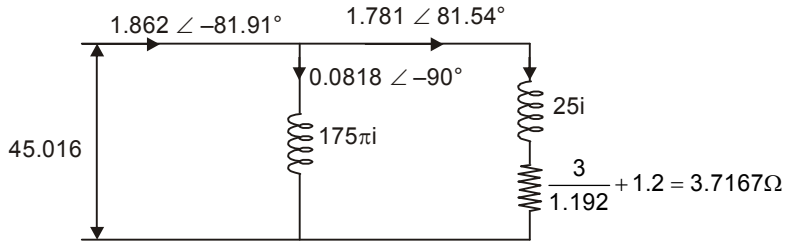
For 5th harmonic

$$\text{Slip} = \frac{5N_s + N_r}{5N_s}$$

[$\therefore$ as rotation of 5th harmonic is opposite of rotor rotation]

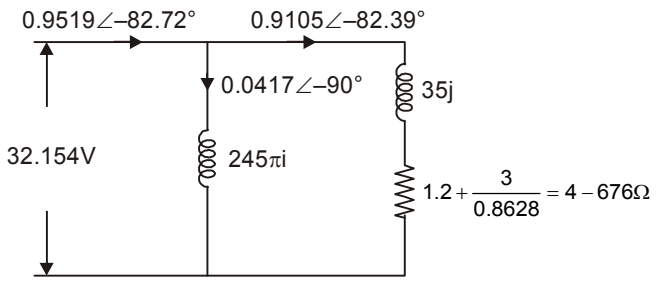
$$S_5 = 1.192$$

$N_r = 1440$ rpm, $N_s = 1500$ rpm



$$V_5 = \frac{(2)(500)}{5\sqrt{2}\pi} = 45.016 \text{ Volt}$$

For 7th harmonic



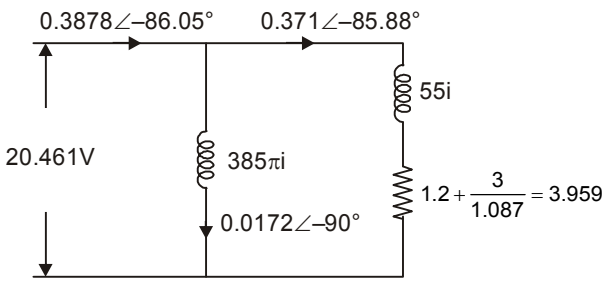
$$\text{Slip}_7 = \frac{7N_S - N_r}{7N_s} = 0.8628$$

$$V_7 = \frac{(\sqrt{2})(500)}{(7)(\pi)} = 32.154\,\text{V}$$

for 11th haromonic

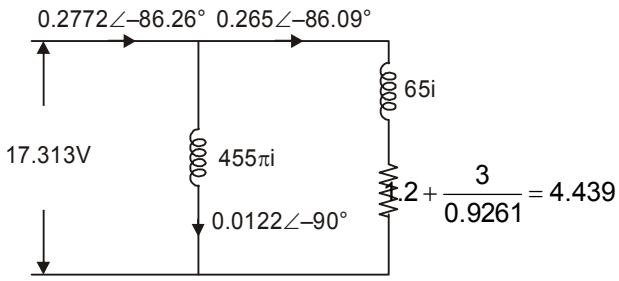
$$\text{Slip} = \frac{11N_s + N_r}{11N_s} = 1.087$$

$$V_{11} = \frac{500\sqrt{2}}{(11)(\pi)} = 20.461 \text{ Volt}$$



For 13th harmonic

$$\text{Slip} = \frac{13N_s - N_r}{13N_s} = 0.9261$$



$$V_{13} = \frac{500\sqrt{2}}{13\pi} = 17.313 \text{ Volt}$$

(i) $I_{fundamental} = 3.696$ Amps

(ii) Harmonic Copper Loss $= 3I^2R$

$$R = 1.2 + 3 = 4.2\Omega$$

$$\text{Harmonic Loss} = (4.2)(3)\left[(1.781)^2 + (0.9105)^2 + (0.371)^2 + (0.265)^2\right]$$

$$\text{Harmonic Loss} = 53.031 \text{ Watt}$$

(iii) $V_{srms}\, I_{srm}\, P_f = P_{out} = I^2R \qquad \left[R = R_1 + \frac{R_2}{5}\right]$

$$V_{srm} = \frac{500\sqrt{2}}{3} = 235.7 \text{ Volt}$$

$$I_{srm} = \sqrt{(3.696)^2 + (1.862)^2 + (0.9519)^2 + (0.3878)^2}$$

$$I_{srm} = 4.2642 \text{ Amp}$$

$$P_{out} = 16.501 + (2.9474)^2(76.2) = 678.463 \text{ watt}$$

$$P_f = \frac{678.463}{(4.2642)(235.7)} = 0.675$$

Q–8(a) ***A 50 Hz, 3-phase induction motor has a slip of 0.2 for maximum torque, when operated on rated frequency and rated voltage. If the motor is run on 60 Hz supply with application of rated voltage, find the ratio of***

(i) Starting currents

(ii) Starting torques

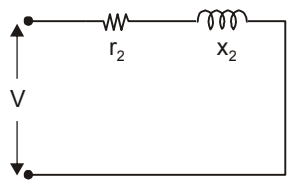
(iii) Maximum torques

With respective values at 50 Hz

Neglect the stator impedance.

Sol. Let rated volstage is v volt rotor leakage reactance is x_2 at 50 Hz and $x_1' = \frac{60}{50}x_2 = 1.2\,x_2$

It is stated that neglect stator impedance. So, the equivelent circuit is



Now, starting curvent

$$I_{st} = \frac{V}{\sqrt{r_2^2 + x_1^2}} \approx \frac{V}{x_2}\left(\text{as } r_2^2 \ll x_2^2\right)$$

Starting torque,

$$T_{est} = \frac{3}{w_s}\cdot\frac{V^2}{r_2^2 + x_2^2}r_2 \approx \frac{3}{w_s}\frac{v^2}{x_2^2}r_2$$

Maximum torque,

$$T_{em} = \frac{3v^2}{w_r}\cdot\frac{1}{2x_2}$$

(i) $$\frac{\text{Starting current at 60 Hz}}{\text{Starting current at 50 Hz}} = \frac{V}{x_2'}\times\frac{x_2}{V} = \frac{x_2}{x_2'}$$

$$= \frac{x_2}{1.2x_2} = \frac{1}{1.2} = 0.8333$$

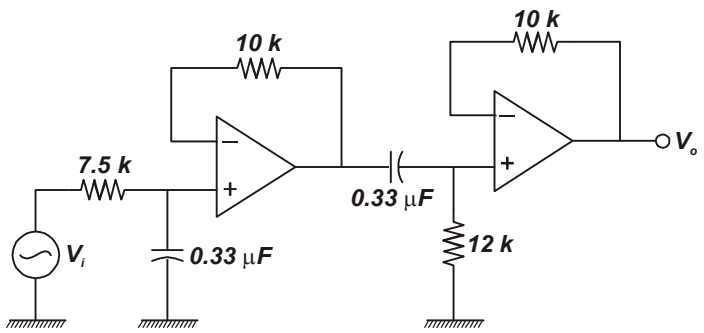
(ii) $$\frac{\text{Starting current at 60 Hz}}{\text{Starting current at 50 Hz}} = \frac{3V^2r_2}{60x_2'^2}\times\frac{50x_2^2}{3V^2r_2} = \frac{50}{60}\times\frac{x_2^2}{(1.2x_2)^2}$$

$$= \frac{1}{(1.2)^3} = \frac{1}{1.728} = 0.5787$$

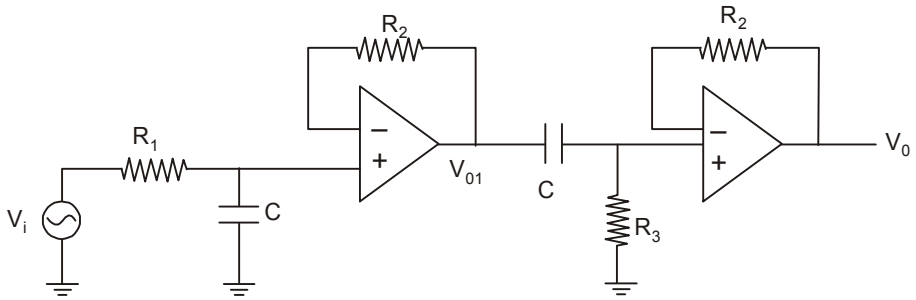
(iii) $$\frac{\text{Maximum torque at 60 Hz}}{\text{Maximum torque at 50 Hz}} = \frac{3V^2}{60\times 2x_2'}\times\frac{50\times 2x_2}{3V^2}$$

$$= \frac{50}{60}\times\frac{x_2}{1.2x_2} = \frac{1}{(1.2)^2} = \frac{1}{1.44} = 0.6944$$

Q–8(b) ***The current of an induction motor is sensed through a suitable arrangement and converted to equivalent voltage. The current contains fundamental and higher order 5th and 7th harmonics. In order is separate the fundamental, the equivalent voltage waveform is passed through the following circuit as given in figure. Find the (i) cut-off frequencies of each section, (ii) overall gain attenuation in dB for fundamental, 5th and 7th harmonics, and (iii) overall phase shift of the measured fundamental current.***



8 (b) (i)



$$V_{01} = \frac{V_i X_c}{\sqrt{R_1^2 + X_C^2}} = \frac{V_i}{\sqrt{1+(R_1\omega C)^2}}$$

Section–1 is LPF

$$(V_{0_1})_{w=0} = V_i$$

$$(V_{01}) = \frac{V_i}{\sqrt{1+(R_1 2\pi (f)^2}}$$

$$(V_{0_1})_{f=\text{cutoff}} = \frac{V_1}{\sqrt{2}} = \frac{V_i}{\sqrt{1+(R_1 (2\pi (f)^2)}}$$

$$2B, \pi (f = 1) \quad f_{cutoff} = \frac{1}{2\pi R_1 C}$$

For sation-2

$$V_0 = \frac{V_{0_1} R_3}{\sqrt{R_3^2 + X_C^2}} - \frac{V_{0_1} R_s \omega C}{\sqrt{1+(R_3\omega C)^2}}$$

It is a high pass filter

$$(V_0)_{w=\infty} = V_{0_1}$$

$$((V_0)_{\omega=\text{cuttoff}} = \frac{V_{01}}{\sqrt{2}} = \frac{V_{0_1} R_3}{\sqrt{R_3^2 + x_L^2}}$$

$$R_3^2 + x_L^2 = 2R_3^2$$

$$x_L^2 = R_b^2 \Rightarrow x_L = R_3$$

$$\frac{1}{\pi f_C} = R_3 \Rightarrow f_{cutoff} = \frac{1}{2CR_3}$$

$$f_{1\text{cuttoff}} = \frac{1}{(2\pi)(7.5)(10^3)(0.33)(10^{-6})} = 64.3\text{ Hz}$$

$$f_{2\text{ cuttoff}} = \frac{1}{(2\pi)(0.33)(10^{-6})(12)(10^3)} = 40.19\text{ Hz}$$

(ii) for fundamental with frequency = 50 Hz

$$V_{01} = (0.78944)\, V_{f_1}$$

$$V_0 = (0.77941)V_{0_1} = 0.6153\ V_{f_1}$$

$$\text{Gain in dB} = 20\log(0.6153) = -4.21\ \text{dB}$$

for 3rd harmonic f = 150 Hz

$$V_{0_1} = (0.39401)\ V_{f_3}$$

$$V_0 = (0.965928)\ V_{0_1} = 0.2459\ V_{f_5}$$

for 5th harmonic f = 250 Hz

$$V_{0_1} = (0.24911)V_{f_5}$$

$$V_0 = (0.9873)V_{0_1} = 0.2459\ v_{f_5}$$

$$\text{Gain in dB} = 20\log(0.2459) = -12.184\ \text{dB}$$

for 7th harmonic f = 350 Hz

$$V_{0_1} = 0.18\ V_{f_7}$$

$$V_0 = 0.993\ V_{0_1} = 0.178\ V_{f_7}$$

$$\boxed{-14.99\ \text{dB}}$$

(iii) $V_{0_1} = \dfrac{-(V_i)jX_c}{R_1 - jX_c}$ $\quad V_0 = \dfrac{V_{0_1}R_3}{-jX_c + R_3}$

$$V_0 = \left(\frac{R_3}{R_3 - jX_c}\right)\left(\frac{-jX_c}{R_1 - jX_c}\right)V_i \qquad X_C = 9645$$

$$V_0 = (0.7794\angle 38.79)(0.7894\angle -37.866)\ V_{f_1}$$

$$\text{Phase shift} = 0.924°$$

Q–8(c) ***Given the following facts about a real signal x(t) with Laplace transform X(s):***

A : X(s) has exactly two poles

B : X(s) has no zeros in the finite s-plane

C : X(s) has a pole at s = – 1 + j

D : e^{2t} x(t) is not absolutely integrable

E : X(0) = 8

Determine X(s) and specify its region of convergence.

Sol: **Point A:** X(s) has exactly two poles

$\because$ x(t) is real signal therefore poles & zeros will occurs in conjugate pairs (if they are complex)

Point C: X(s) has a pole at s = –1 + j

Poles are present in conjugate pairs

So,

$$P_1 = -1 + j$$

$$P_2 = -1 - j$$

Now,

Transfer function

$$X(s) = \frac{K}{(s - P_1)(s - P_2)} = \frac{K}{(s + 1 - j)(s + 1 + j)}$$

$$= \frac{K}{(s+1)^2+1}$$

$$X(s) = \frac{K}{s^2+2s+2} \quad \text{...(i)}$$

Point E: X(0) = 8

From equation (i)

$$X(0) = 8 = \frac{K}{0+0+2}$$

$$K = 16$$

So,

$$\boxed{X(s) = \frac{16}{s^2+2s+2}}$$

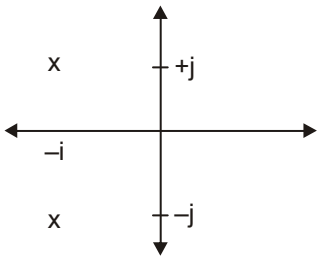
From point D:

$$g(t) = e^{2t}\, x(t)$$

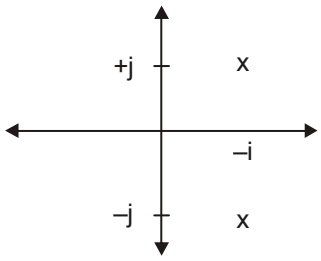
$$\Updownarrow$$

$$G(s) = X(s-2)$$

Pole zero plot of X(s)



Pole zero plot of G(s)



ROC of g(t) : $\sigma > 1$
Right sided

$$g(t) = e^{2t}\, x(t)$$