## UPSC ESE 2023

# Mains Exam Solution 

## MECHANICAL ENGINEERING

## Paper-II

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# ESE 2023 

## SECTION-A

Q-1(a): $\quad$ A circular bar $A B C, 4 \mathrm{~m}$ long, is rigidly fixed at its ends $A$ and $C$. The portion $A B$ is 2.8 m long and of 50 mm diameter whereas $B C$ is 1.2 m long and of $\mathbf{2 5} \mathbf{~ m m}$ diameter. If the twisting moment of 700 Nm is applied at $B$, determine the values of the resisting moments at $A$ and $C$ and the maximum stress in each section of the shaft. For the material of the shaft $G=80 \mathrm{GN} / \mathrm{m}^{2}$.
[12 MARKS]
Sol:


Given,

$$
\begin{aligned}
D_{A B} & =50 \mathrm{~mm} \\
D_{B C} & =25 \mathrm{~mm} \\
T_{B} & =T_{0}=700 \mathrm{~N} / \mathrm{m} \\
\frac{T_{1} L_{1}}{J_{1}} & =\frac{T_{2} L_{2}}{J_{2}} \\
T_{1} & =\frac{\frac{\pi}{32} \times 50^{4}}{\frac{\pi}{32} \times 25^{4}} \times \frac{1.2}{2.8} \times T_{2} \\
T_{1} & =6.857 \mathrm{~T} 2 \\
T_{1}+T_{2} & =700 \mathrm{Nm} \\
T_{2} & =89.091 \mathrm{Nm} \\
T_{1} & =610.3 \mathrm{Nm}
\end{aligned}
$$

For AB

$$
\tau_{\max }=\frac{\operatorname{Tr}}{\mathrm{J}}=\frac{610.909 \times 25 \times 10^{3}}{\frac{\pi}{32} \times 50^{4}}
$$

$=24.891 \mathrm{MPa}$

For BC

$$
\tau_{\max }=\frac{89.091 \times 12.5 \times 10^{3}}{\frac{\pi}{32} \times 25^{4}}
$$

$=29.039 \mathrm{MPa}$

Q-1(b): What are supporting forces for the frame? Ngelect all weights except the 10 kN weight.

[12 MARKS]
Sol:


At D:

$D_{x}=-T \cos \theta=-9.95 \mathrm{kN}$
$\theta=\tan ^{-1}\left(\frac{0.3}{3}\right)=5.71^{\circ}$

Now, in member BC
Taking moment about ' E '.
$\sum M_{E}=0$

$$
(T \cos \theta) \sqrt{3}-B_{x} \times 2.1547=0
$$

$\mathrm{B}_{\mathrm{x}}=\frac{(10 \times \cos (5.71)) \sqrt{3}}{2.1547}$
$\mathrm{B}_{\mathrm{x}}=7.9985 \mathrm{kN}$
$\sum E_{x}=0$
$\Rightarrow \quad B_{x}-E_{x}+T \cos \theta=0$
$\mathrm{E}_{\mathrm{x}}=17.948 \mathrm{kN}$



Now, in AED:
$\sum \mathrm{E}_{\mathrm{x}}=0$
$\Rightarrow \quad A_{x}-E_{x}-D_{x}=0$
$\Rightarrow \quad \mathrm{A}_{\mathrm{x}}=17.948-9.95=7.998 \mathrm{kN}$
$\sum F_{y}=0$

$$
\begin{aligned}
\Rightarrow \quad & -D_{y}-E_{y}+A_{y}=0 \\
& A_{y}=-10.362+11=0.638 \mathrm{kN}
\end{aligned}
$$



$$
\mathrm{R}_{\mathrm{A}}=\sqrt{\mathrm{A}_{\mathrm{x}}^{2}+\mathrm{A}_{\mathrm{y}}^{2}=8.0234 \mathrm{kN}}
$$

Q-1(c): An electronic instrument is to be isolated from a panel that vibrates at frequencies range from 25 Hz to 35 Hz . it is estimated that at least $80 \%$ vibration isolation must be achieved to prevent damage to the instrument. If the instrument weighs 85 N , find the necessary static deflection of the isolator.
[12 MARKS]
Sol: The initial natural frequency ( $\omega_{1}$ ) of system as follows:

$$
\omega_{1}=2 \pi f_{1}
$$

Here the initial vibrating frequency is $f_{1}$

Substitute

$$
\mathrm{f}_{1}=25 \mathrm{~Hz}
$$

then,

The final natural frequency ( $\omega_{2}$ ) of system as follows

$$
\begin{aligned}
\omega_{2} & =2 \pi \mathrm{f}_{2} \\
\mathrm{f}_{2} & =\text { final vibrating frequency } \\
\mathrm{f}_{2} & =35 \mathrm{~Hz} \text { (given) } \\
\omega_{2} & =2 \pi(35)=219.91 \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

The permissibility force $T_{f}$ as follows:

$$
\begin{aligned}
& T_{f}=1-R \\
& R=\text { Vibration isolation }
\end{aligned}
$$

Substitute $R=0.8$ in above equation.

$$
r=\sqrt{\frac{1+\mathrm{T}_{\mathrm{f}}}{\mathrm{~T}_{\mathrm{f}}}}
$$

Substitute $\mathrm{T}_{\mathrm{f}}=0.2$ in above equation.

$$
r=\sqrt{\frac{1+0.2}{0.2}}=\sqrt{6}=2.449
$$

The static deflection ( $\delta_{\text {st }_{1}}$ ) at initial frequency as follows.

$$
\delta_{\mathrm{st} 1}=\frac{\mathrm{gr}^{2}}{\omega_{1}^{2}}
$$

Substitute $r=2.449$ and $\omega_{1}=157.08 \mathrm{rad} / \mathrm{sec}$

$$
\delta_{\mathrm{st} 1}=\frac{9.81(6)}{(157.08)^{2}}=0.002385 \mathrm{~m}=2.385 \mathrm{~mm}
$$

The static deflection ( $\delta_{\mathrm{st1}}$ ) at final frequency as follows.

$$
\delta_{\mathrm{st} 2}=\frac{\mathrm{gr}^{2}}{\omega_{2}^{2}}
$$

Substitute $r=2.449$ and $\omega_{2}=219.912 \mathrm{rad} / \mathrm{sec}$ then

$$
\delta_{\mathrm{st} 2}=\frac{9.81(6)}{(219.912)^{2}}=0.001217 \mathrm{~m}=1.127 \mathrm{~mm}
$$

Since $\delta_{\mathrm{st} 1}>\delta_{\mathrm{st} 2}$ then the greater is the required static deflection of the system. The necessary static deflection of the isolator is 2.385 m .

## Q-1(d): Describe all the inversion of a slider-crank mechanism.

[12 MARKS]
Sol: These inversion are found in the slider-crank mechanisms.

1. Pendulum pump or bull engine: In this mechanism, the inversion is obtained by fixing the cylinder or link 4(i.e., sliding pair), as shown in figure below. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link, as shown in figure.

2. Oscillating cylinder engine: The arrangement of oscillating cylinder engine mechanism, as shown in figure below is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

3. Rotary internal combustion engine or Gnome engine: Sometimes back, rotary internal combustion engines were used in aviation. But now-a-day gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre $D$, as shown in figure below, while the crank (link 2) is fixed. In this mechanism, when he connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

4. Crank and slotted lever quick return motion mechanism: This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (i.e., link 3) forming the turning pair is fixed, as shown in figure below. The link 3 corresponds to be connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at $B$ slides along the slotted bar AP and thus cause AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke $R_{1} R_{2}$. The line of stroke of the ram (i.e., $R_{1} R_{2}$ ) is perpendicular to AC produced.


In the extreme positions, $\mathrm{AP}_{1}$ and $\mathrm{AP}_{2}$ are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position $\mathrm{CB}_{1}$ to $\mathrm{CB}_{2}$ (or through and angle $\beta$ ) in the clockwise direction. The return stroke occurs when the crank rotates from the position $\mathrm{CB}_{2}$ to $\mathrm{CB}_{1}$ (or through angle $\alpha$ ) in the clockwise direction. Since the crank has uniform angular speed, therefore.

$$
\frac{\text { Time of cutting stroke }}{\text { Time of return stroke }}=\frac{\beta}{\alpha}=\frac{\beta}{360^{\circ}-\beta} \text { or } \frac{360^{\circ}-\alpha}{\alpha}
$$

Q-1(e): A structure is composed of circular members of diameter d. At a certain position along one member the loading is found to consist of a shear force of 10 kN along with an axial tensile load of 20 kN . If the elastic limit in tension of the material of the members is $300 \mathrm{MN} / \mathrm{m}^{2}$ and there is to be a factor of safety of 3 , estimate the magnitude of $d$ required according to the maximum shear strain energy per unit volume theory. Poisson's ratio $v=0.3$.
[12 MARKS]
Sol: Given:
Shear force $=10 \mathrm{kN}$
Axial tensile load $=20 \mathrm{kN}$

$$
f_{y}=300 \mathrm{MN} / \mathrm{m}^{2}
$$

Factor of safety $=3$
Poisson's ratio $=0.3$
Now,

$$
\text { Shear stress }=\frac{10}{\frac{\pi}{4} d^{2}}=\frac{25.46}{d^{2}}
$$

Now, using maximum shear strain energy theory,

$$
\text { Principal stress, } \begin{aligned}
\sigma_{1} / \sigma_{2} & =\left(\frac{\sigma_{x}+0}{2}\right) \pm \sqrt{\left(\frac{\left(\sigma_{x}-0\right)}{2}\right)^{2}+\tau_{x y}^{2}} \\
& =\left(\frac{25.4}{2 d^{2}}\right) \pm \sqrt{\left(\frac{25.4}{2 d^{2}}\right)^{2}+\left(\frac{12.73}{d^{2}}\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{25.4}{2 d^{2}} \pm \frac{1}{d^{2}} \sqrt{323.3429} \\
& =\frac{25.4}{2 d^{2}} \pm \frac{17.98}{d^{2}} \\
\sigma_{1} & =\frac{25.4}{2 d^{2}}+\frac{17.98}{d^{2}}=\frac{30.68}{d^{2}} \\
\sigma_{2} & =\frac{25.4}{2 d^{2}}-\frac{17.98}{d^{2}}=-\frac{5.28}{d^{2}} \\
\frac{\left(\sigma_{1}^{2}+\sigma_{2}^{2}+\left(\sigma_{1}-\sigma_{2}\right)^{2}\right)}{2} & \leq\left(\frac{f_{y}}{F O S}\right)^{2} \\
\left(\frac{\sigma_{1}^{2}+\sigma_{2}^{2}+\left(\sigma_{1}-\sigma_{2}\right)^{2}}{}\right. & \leq\left(\frac{f_{y}}{3}\right)^{2} \times 2 \\
\sigma_{1}^{2}+\sigma_{2}^{2}+\left(\sigma_{1}-\sigma_{2}\right)^{2} & \leq\left(\frac{300 \times 1000}{30.68}\right)^{2} \times 2 \\
+\left(\frac{-5.28}{d^{2}}\right)^{2}+\left(\frac{30.68+5.28}{d^{2}}\right)^{2} & \leq 0.02 \\
\frac{1}{d^{2}}[2262.26] & \leq 0.02 \\
d^{4} & \geq \frac{2262.26}{0.02} \\
d & \geq 18.339 m m
\end{aligned}
$$

Q-2(a): The rod AD is pulled at A and it moves to the left. If the coefficient of dynamic friction for the rod at $A$ and $B$ is 0.4 , what must the minimum of $W_{2}$ be to prevent the block from tipping when $\alpha=20^{\circ}$ ? With this value of $W_{2}$, determine the minimum coefficient of static friction between the block and the supporting plane needed to just prevent the block from sliding. Take $W_{1}=100 \mathrm{~N}$.

[20 MARKS]
Sol:


```
AB sin 20 = 120
```

$$
\begin{equation*}
A B=350.856 \tag{i}
\end{equation*}
$$

Friction force $=\mu N_{B}$

$$
\begin{equation*}
=0.4 \times 53.56=21.42 \tag{iii}
\end{equation*}
$$



In order to avoid tipping
$\sum M_{0}=0 \quad \mu N_{B} \cos 20 \times 120=N_{B} \cos 70 \times 120+W_{2} \times 30 \times 120$

$$
\begin{aligned}
\mathrm{N}_{\mathrm{B}}[120 \times \mu \times \cos 20-120 \cos 70] & =\mathrm{W}_{2} \times 30 \\
\mathrm{~W}_{2} & =7.253 \text { Newton }
\end{aligned}
$$

In order to avoid slipping $F_{\text {ext }}=\mu N$

$$
\begin{align*}
\mathrm{F}_{\mathrm{ext}} & =\text { force that produce tendency to slip } \\
\mathrm{F}_{\mathrm{ext}} & =21.42 \cos 20-53.56 \cos 70=1.8096 \mathrm{~N}  \tag{iv}\\
\mathrm{~N}_{2} & =\mathrm{N}_{\mathrm{B}} \sin 20+\mu \mathrm{N}_{\mathrm{B}} \sin 20+\mathrm{W}_{2} \\
& =53.56 \sin 70+0.4 \times 53.56 \sin 20+7.253 \\
& =64.9133 \mathrm{~N} \\
1.8096 & =\mu \mathrm{N}_{2} \\
\mu & =0.027
\end{align*}
$$

Minimum behaviour block and supporting plane to avoid sliding.

Q-2(b): (i) Define pitch point, addendum, module and pressure angle as applied to toothed gears. [8 MARKS]

Sol: - Pitch point: It is a common point of contact between two pitch circles.

- Addendum: It is the radial distance of a tooth from the pitch circle to the top of the tooth.


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- Module: It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m . Mathematically,

$$
\text { Module, } \mathrm{m}=\mathrm{D} / \mathrm{T}
$$

Pressure angle: It is also termed as the angle of obliquity. It is the angle between the tooth face and the gear wheel tangent. It is more precisely the angle at a pitch point between the line of pressure (which is normal is the tooth surface) and the plane tangent to the pitch surface.

Q-2(b) (ii) Compare involute curve with cycloidal curve for the profiles of gear teeth.
[12 MARKS]

## Sol: Cycloidal Tooth

- Pressure angle varies from a maximum at the beginning of the engagement, reduces to zero at the pitch point and again increases to a maximum at the end of the engagement resulting in the smooth running of gears.
- It involves double curves for the teeth epicycloid, and hypocycloid. This complicates the manufacturer.
- Exact center distance in required to transmit a constant velocity ratio.
- The phenomenon of interference does not occur at all.
- Interference can occur if the condition of minimum no of teeth on a gear is not followed.
- In this a convex flank always has contact with a concave face resulting in less wear.


## Involute Tooth

- The pressure angle is constant throughout the engagement of teeth. This results in the smooth running of the gears.
- It involves double curves for the teeth, epicycloid and hypocycloid. This complicates the manufacturer.
- These are simple to manufacture and thus are cheaper.
- A little variation in a center distance does not affect the velocity ratio.
- Interference can occur if the condition of minimum no of teeth on a gear is not followed.
- The teeth have radial flanks and thus are weaker as compared to the Cycloidal form for the same pitch.
- Two convex surfaces are in contact and thus there is more wear.

A single plate clutch (both sides effective) is required to transmit 27 kW at 1600 rpm . The outer diameter of the plate is limited to 30 cm , and intensity of pressure between the plates is not to exceed $0.1 \mathrm{~N} / \mathrm{mm}^{2}$. Assuming uniform wear and a coefficient of friction 0.3 , find the required inner diameter of the plates, and axial force necessary to engage the clutch.
[20 MARKS]
Sol:
Given, $n_{a}=2 ; P=27 \mathrm{~kW} ; N=1600$ r.p.m. $d_{1}=30 \mathrm{~cm} ; p_{\max }=6.87 \mathrm{~N} / \mathrm{cm}^{2} ; \mu=0.3$
Friction torque required to transmit 27 kW is

$$
T=\frac{P \times 60 \times 10000}{2 \pi N}
$$

or

$$
\mathrm{T}=\frac{27 \times 60 \times 1000}{2 \pi \times 1600}=161.14 \mathrm{~N}-\mathrm{m}
$$

With uniform rate of wear,
and as $r_{2}<r_{1}$, the pressure $p_{2}>p_{1}$

Hence,
and hence,
Thus, axial force,
Also, the friction torque

Substituting for W
or,
or,
By trial and error value of $r_{2}$ that satisfies the above equation is

Thus,
Therefore
The axial force required

$$
\begin{aligned}
r_{2} & =10.077 \mathrm{~cm} \\
r_{2} & =10.1 \mathrm{~cm}, \text { say } \\
d_{2} & =\text { inner diameter }=20.2 \mathrm{~cm} \\
\mathrm{~W} & =2 \pi C\left(r_{1}-r_{2}\right) \\
& =2 \pi(10.1 \times 6.87)(15-10.1)=2136.26 \mathrm{~N}
\end{aligned}
$$

Q-3(a): Find the slope and deflection at the tip of the cantilever shown in the figure. What load P must be applied upwards at mid-span to reduce the deflection by half? EI $=20 \mathrm{MN} / \mathrm{m}^{2}$

[20 MARKS]
Sol: Given:

$$
\mathrm{El}=20 \mathrm{MN} / \mathrm{m}^{2}
$$



Total deflection due to UDL and 30 kN load at the end,

$$
=\frac{20 \times 4^{4}}{8 E I}+\frac{30 \times 4^{3}}{3 E I}
$$

After applying ' $P$ ' load at centre, the deflection due to ' $P$ ' load at the end

$$
=\frac{\mathrm{P} \times 2^{3}}{3 E \mathrm{E}}+\frac{\mathrm{P} \times 2^{2}}{2 \mathrm{EI}} \times 2
$$

Now, it is given after applying ' $P$ ' load the deflection become half,

$$
\begin{aligned}
\frac{\mathrm{P}}{\mathrm{EI}} \times\left(\frac{8}{3}+\frac{8}{2}\right) & =\frac{1}{2 \mathrm{EI}}\left(\frac{20 \times 64 \times 4}{8}+\frac{30 \times 64}{3}\right) \\
\mathrm{P} & =\frac{240 \times 64+240 \times 64}{8 \times 40}=96 \mathrm{kN}
\end{aligned}
$$

Now the slope at end will be

$$
\begin{aligned}
& =\frac{20 \times 4^{3}}{6 E I}+\frac{30 \times 4^{2}}{2 E I}-\left(\frac{96 \times 2^{2}}{2 E I}\right) \\
& =\frac{261.33}{E I}=\frac{261.33}{20 \times 10^{3}}=0.013 \text { radian or } 0.013 \times \frac{180}{\pi} \text { degree } \\
& =0.749^{\circ}
\end{aligned}
$$

Total deflection at the end,

$$
\begin{aligned}
& =\frac{20 \times 4^{4}}{8 \mathrm{EI}}+\frac{30 \times 4^{3}}{3 \mathrm{EI}}-\left(\frac{96 \times 2^{3}}{3 \mathrm{EI}}+\frac{96 \times 2^{3}}{2 \mathrm{EI}}\right) \\
& =\frac{3 \times 20 \times 4^{4}+8 \times 30 \times 4^{3}-8 \times 96 \times 2^{3}-12 \times 96 \times 2^{3}}{24 \mathrm{EI}} \\
& =\frac{15360}{24 \times 20 \times 1000} \\
& =32 \times 10^{-3} \mathrm{~m} \text { or } 32 \mathrm{~mm}
\end{aligned}
$$

The axes of a three-cylinder air compressor are $120^{\circ}$ apart and their connecting rods are connected to a common crank. The length of each connecting rod is 200 mm and the strokes is 160 mm . The mass of the reciprocating parts per cylinder is 2 kg . Find the maximum primary and secondary forces acting on the frame of the compressor when running at 2500 rpm.
[20 MARKS]

## Sol:



Given:

$$
\begin{aligned}
\mathrm{L} & =160 \mathrm{~mm} \\
\mathrm{r} & =\frac{\mathrm{L}}{2}=\frac{160}{2}=80 \mathrm{~mm}=0.08 \mathrm{~m} \\
\ell & =200 \mathrm{~mm} \\
\mathrm{~N} & =2500 \mathrm{rpm} \\
\mathrm{~m} & =2 \mathrm{~kg} \\
\omega & =\frac{2 \pi \mathrm{~N}}{60}=\frac{2 \times \pi \times 2500}{60}=261.79 \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

The position of three cylinder is show in figure

$$
\begin{aligned}
& \theta=0^{\circ} \text { for cylinder } 1 \\
& \theta= \pm 120^{\circ} \text { for cylinder } 2 \\
& \theta= \pm 240^{\circ} \text { for cylinder } 3
\end{aligned}
$$

So primary direct crank angle is $240^{\circ}$ clockwise and primary reverse crank angle is $240^{\circ}$ anticlockwise from the line of stroke of cylinder 3.


From above figure we see that primary reserve crank form a balanced system.
We can see from figure, resultant primary force is equivalent to centrifugal force of mass

$$
\left(\frac{m}{2}+\frac{m}{2}+\frac{m}{2}\right)=\frac{3 m}{2}
$$

Note: There is no unbalance primary force due to reverse crank

$$
\begin{aligned}
\text { Maximum primary force } & =\frac{3 \mathrm{~m}}{2} \times \omega^{2} \times r \\
& =\frac{3}{2} \times 2 \times(261.79)^{2} \times 0.08 \\
& =16.448 \mathrm{kN}
\end{aligned}
$$

Secondary direct and reverse crank position are shown in figure.

(a) Direct secondary cranks.

(b) Reverse secondary cranks.

| Secondary crank angle | Cylinder |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |  |
| $\theta$ (direct) | 0 | 240 | 480 |  |
| $\theta$ (indirect) | 0 | -240 | -480 |  |

$\begin{aligned} \text { Maximum secondary force } & =\frac{3 m}{2} \times(2 \omega)^{2} \times \frac{r}{4 n} \\ & =\frac{3 \times 2}{2} \times(2 \times 261.79)^{2} \times\left(\frac{r}{4 \times \frac{\ell}{r}}\right)\end{aligned}$

$$
=1.5 \times 2 \times(2 \times 261.79)^{2} \times\left(\frac{0.08}{4 \times \frac{200}{1000}}\right)
$$

$=6.58 \mathrm{kN}$

Q-3(c): A simply supported beam AB is shown in the figure. A bar CD is welded to the beam. After determining the supporting forces, sketch the shear force and bending moment diagrams and determine the maximum bending moment.

[20 MARKS]
Sol:


Calculation of reactions:

$$
R_{A}+R_{B}=10+2 \times 8=26 \mathrm{kN}
$$

Taking moment at $B=0$

$$
\begin{aligned}
R_{A} \times 24-10 \times 16+5 & \times 0.5-2 \times 8 \times 12=0 \\
R_{A} \times 24 & =349.5 \\
R_{A} & =14.56 \mathrm{kN} \\
R_{B} & =26-14.56=11.437 \mathrm{kN}
\end{aligned}
$$



Sign convection: SFD $\uparrow \oplus$


## For AE

$x=(0-8 \mathrm{~m})$, Taking origin at $A$

$R_{A}-V_{x}=0$
$\mathrm{V}_{\mathrm{x}}=\mathrm{R}_{\mathrm{A}}=14.56 \mathrm{kN} \downarrow$
i.e., $S F$ for $x=0-8 \mathrm{~m}$ is +14.56 kN
$\sum \mathrm{M}=0, \oplus$
$M-R_{A} \times x=0$
$M=14.56 x \mathrm{kNm} \oplus$
At $E, \quad \sum F_{y}=0$

$$
\begin{aligned}
& R_{A}-10-V=0 \\
& \therefore \quad V=R_{A}-10=14.56-10 \\
& \quad=4.56 \mathrm{kN} \oplus
\end{aligned}
$$

For ED, $x=(0-8 \mathrm{~m})$

$\sum \mathrm{F}_{\mathrm{y}}=0$
$R_{A}-10-2 x-V=0$
or $\quad V=14.56-10-2 x=4.56-2 x$

$$
\begin{aligned}
\sum M & =0 \oplus) \\
& +R_{A} \times(8+x)-10 x-2 x^{2}-M=0
\end{aligned}
$$

$\Rightarrow \quad M=-\left[2 x^{2}-10 x+14.56 \times(8+x)\right]$
For BD, $x=0-8 \mathrm{~m}$

$\sum F_{y}=0$

$$
\begin{aligned}
& V+R_{B}=0 \\
\therefore \quad & V=-R_{B}=-11.437 \mathrm{kN}
\end{aligned}
$$

i.e. $\quad V=11.437 \mathrm{kN} \downarrow$
$\sum M=0 \oplus$
$\mathrm{M}-\mathrm{R}_{\mathrm{B}} \times \mathrm{x}=0$
$\therefore \quad \mathrm{M}=11.437 \times \mathrm{xkNm}$
At D

$$
2.5+M-11.437 \times 8=0
$$

$\therefore \quad \mathrm{M}=88.996 \mathrm{kNm}$
Shear force diagram:


Maximum bending moment $=121.7 \mathrm{kNm}$

Q-4(a): A uniform $T$-section beam is 100 mm wide and 150 mm deep with flange thickness of $\mathbf{2 5} \mathbf{~ m m}$ and a web thickness of 12 mm . If the limiting bending stresses for he material of the beam are $80 \mathrm{MN} /$ $m^{2}$ in compression and $160 \mathrm{MN} / \mathrm{m}^{2}$ in tension, find the maximum u.d.I. that the beam can carry over a simply supported span of 5 m .
[20 MARKS]

Sol:

$$
\begin{aligned}
\overline{\mathrm{y}} & =\frac{(12 \times 125) \frac{125}{2}+(100 \times 25) \times 137.5}{2500+12 \times 125} \\
\overline{\mathrm{y}} & =109.375 \mathrm{~mm} \\
\sigma_{\text {compression }} & =80 \mathrm{MN} / \mathrm{m}^{2} \\
\sigma_{\text {tension }} & =160 \mathrm{MN} / \mathrm{m}^{2} \\
\sigma_{\mathrm{x}} & =\frac{\mathrm{My}}{1} \\
\mathrm{I}_{\mathrm{NA}} & =\frac{12 \times 125^{3}}{12}+(12 \times 125) \times(109.35-62.5)^{2}+\frac{100 \times 25^{3}}{12}+(100 \times 25) \times(137.5-109.375)^{2} \\
& =7353256.146 \mathrm{~mm}^{2} \\
-80 \mathrm{MN} / \mathrm{m} & =\frac{\mathrm{M}_{1} \times}{7353256.146 \times 10^{-12} \mathrm{~m}^{4}} \text { for compression } \\
\mathrm{M}_{1} & =\frac{80 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2} \times 7353256.146 \times 10^{-12} \mathrm{~m}^{4}}{40.625 \times 10^{-3} \mathrm{~m}} \\
\mathrm{M}_{1} & =-14.48 \mathrm{kNm} \\
160 & =\frac{\mathrm{M}_{2} \times 109.375}{7353256.146 \times 10^{-12}} \text { for tension } \\
\mathrm{M}_{2} & =10.75 \mathrm{kNm}
\end{aligned}
$$

Maximum moment occurs in UDL $=\frac{\mathrm{w} \ell^{2}}{8}$

$$
\begin{aligned}
\frac{w \ell^{2}}{8} & =3.73 \mathrm{kNm} \\
w & =1.1936 \mathrm{kN} / \mathrm{m} \quad \text { or } \quad w=1193 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

Q-4(b): In a spring loaded governor of Hartnell type, the weight of each ball is 5 kg and the lift of the sleeve is 5 cm . The speed at which the governor begins to float is 250 rpm , and at this speed the radius of the ball path is 10 cm . The mean working speed of the governor is 20 times the range of speed when friction is neglected. If the lengths of ball and roller arm of the bell crank lever are 12 cm and 10 cm respectively and if the distance between the centre of pivot of bell crank lever and axis of the governor spindle is 14 cm , determine the initial compression of the spring, taking into account obliquity of arms.
[20 MARKS]
Sol: Data given:

$$
\begin{aligned}
\mathrm{m} & =5 \mathrm{~kg} \\
\mathrm{~h} & =5 \mathrm{~cm}=0.05 \mathrm{~m} \\
\mathrm{~N} & =250 \mathrm{rpm} \\
\omega_{1} & =\frac{2 \pi \mathrm{~N}}{60}=\frac{2 \pi \times 250}{60}=26.179 \mathrm{rad} / \mathrm{sec} \\
\mathrm{r}_{1} & =10 \mathrm{~cm}=0.1 \mathrm{~m} \\
v & =12 \mathrm{~cm}=0.12 \mathrm{~m}
\end{aligned}
$$

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$$
\begin{aligned}
\mathrm{y} & =10 \mathrm{~cm} \\
\mathrm{r} & =0.1 \mathrm{~m} \\
& =0.14 \mathrm{~cm}
\end{aligned}
$$



Initial compression of the spring taking into account of obliquity of arms:

$$
\begin{aligned}
\omega & =\frac{\omega_{1}+\omega_{2}}{2}=\text { mean working speed } \\
\omega & =20\left(\omega_{2}-\omega_{1}\right) \\
\frac{\omega_{1}+\omega_{2}}{2} & =20\left(\omega_{2}-\omega_{1}\right) \\
\omega_{1}+\omega_{2} & =40 \omega_{2}-40 \omega_{1} \\
\omega_{2} & =\frac{41}{39} \omega_{1}=\frac{41}{39} \times 26.779 \\
& =27.52 \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

or,

The minimum and maximum position of the governor ball is shown in figure Let,

$$
r_{2}=\text { maximum radius of rotation }
$$

We know that lift of the sleeve,

$$
\begin{aligned}
& h=\left(r_{2}-r_{1}\right) \times y / x \\
& r_{2}=r_{1}+h \times \frac{x}{y}=0.1+0.05 \times \frac{0.12}{0.1} \\
& r_{2}=0.16 \mathrm{~m}
\end{aligned}
$$

We know that centrifugal force at minimum speed:

$$
\begin{aligned}
\mathrm{F}_{\mathrm{C} 1} & =\mathrm{m}\left(\omega_{1}\right)^{2} \times \mathrm{r}_{1}=5 \times(26.179)^{2} \times 0.1 \\
\mathrm{~F}_{\mathrm{C} 1} & =342.67 \mathrm{~N} \\
\mathrm{~F}_{\mathrm{C} 2} & =\mathrm{m}\left(\omega_{2}\right)^{2} \times \mathrm{r}_{2}=5 \times(27.52)^{2} \times 0.16 \\
\mathrm{~F}_{\mathrm{C} 2} & =605.8 \mathrm{~N} \\
\mathrm{a}_{1} & =\mathrm{r}-\mathrm{r}_{1} \\
& =0.14-0.1=0.04 \mathrm{~m}
\end{aligned}
$$

So,

$$
\begin{aligned}
& x_{1}=\sqrt{x^{2}-\left(a_{1}\right)^{2}}=\sqrt{(0.12)^{2}-(0.04)^{2}}=0.113 \\
& y_{1}=\sqrt{y^{2}-\left(h_{1}\right)^{2}} \\
& h_{1}=\frac{h}{2}=\frac{0.05}{2}=0.025
\end{aligned}
$$

$$
\begin{aligned}
y_{1} & =\sqrt{(0.1)^{2}-(0.025)^{2}}=0.0968 \\
a_{2} & =r_{2}-r \\
& =0.16-0.14=0.02 \mathrm{~m} \\
x_{2}=x_{1} & =0.113 \mathrm{~m} \\
y_{2}=y_{1} & =0.0968 \mathrm{~m}
\end{aligned}
$$

Taking moment about point O

$$
\begin{aligned}
\frac{S_{1}}{2} \times y_{1} & =F_{C 1} \times x_{1}-m g \times a_{1} \\
\frac{S_{1}}{2} \times 0.0968 & =342.67 \times 0.113-5 \times 9.8 \times 0.04 \\
S_{1} & =759.53 \mathrm{~N}
\end{aligned}
$$

Taking moment about point O for the maximum position:

$$
\begin{aligned}
\frac{\mathrm{S}_{2}}{2} \times \mathrm{y}_{2} & =\mathrm{F}_{\mathrm{C} 2} \times \mathrm{x}_{2}+\mathrm{mg} \times \mathrm{a}_{2} \\
\frac{\mathrm{~S}_{2}}{2} \times 0.0968 & =605.8 \times 0.113+5 \times 9.8 \times 0.02 \\
\mathrm{~S}_{2} & =1434.61 \mathrm{~N}
\end{aligned}
$$

Stiffness of spring:

$$
\begin{aligned}
& S=\frac{S_{2}-S_{1}}{h}=\frac{1434.61-759.53}{(5 \times 10) \mathrm{mm}} \\
& S=13.5 \mathrm{~N} / \mathrm{mm}
\end{aligned}
$$

Initial compression of spring:

$$
=\frac{\mathrm{S}_{1}}{\mathrm{~S}}=\frac{759.53}{13.5}=56.254 \mathrm{~mm}
$$

Q-4(c): (i) What are the assumptions made in the Lewis equation for beam strength? [8 MARKS]
Sol: Lewis equation for helical gears: Tooth of a helical gear is considered as a cantilever beam of length equal to height of the tooth (addendum + dedendum). Beam strength of a tooth is the maximum force, which it can take without causing failure in bending. The strength is calculated for the formulative imaginary gear of diameter and number of teeth.

Following assumptions are made in the Lewis equation:

- Tangential component is uniformly distributed over the entire face of the tooth.
- Effect of radial component causing compressive stresses in neglected.
- Point of contact moves, which causes change in resultant force. This change is neglected.
- Although number of teeth that are in contact is more than one, but it is assumed that only one pair of teeth takes the entire load.
- Analysis is valid, when gears are stationary or rotating at very slow speed.
- Effect of the dynamic forces is neglected in strength calculations.
- Effect of stress concentration is neglected.

Beam strength for helical gear $S_{b}$ is given by Lewis equation as under

$$
S_{b}=\sigma_{b} \pi m_{n} b y^{\prime}
$$

where,

$$
\begin{aligned}
\sigma_{\mathrm{b}} & =\text { Bending strength }\left(\mathrm{N} / \mathrm{mm}^{2}\right) \\
\mathrm{m}_{\mathrm{n}} & =\text { Normal module }(\mathrm{mm}) \\
\mathrm{b} & =\text { Face width }(\mathrm{mm}) \\
\mathrm{y}^{\prime} & =\text { Lewis factor based on equivalent number of teeth }
\end{aligned}
$$

Q-4(c): (ii) A pair of spur grears with $20^{\circ}$ full depth involute teeth consists of a 20 teeth pinion meshing wth a 50 teeth gear. The pinion is mounted on a crank shaft of 5 kW engine running at 1200 rpm. The driven shaft is connected to a compressor. The pinion as well as the gear is made of steel having ultimate strength in tension equal to $500 \mathrm{~N} / \mathrm{mm}^{2}$. The module and face width of the gears are 4 mm and 44 mm . Assume service factor as 2 . Using the velocity factor to account for the dynamic load, determine the factor of safety. Take Lewis form factor for 20 teeth equal to 0.320 and for 50 teeth equal to 0.408 . Take velocity factor, $C_{v}=\frac{3}{3+v}$, where $v$ is the pitch line velocity in $\mathrm{m} / \mathrm{s}$.
[12 MARKS]
Sol: Data given:
$K W=5 \mathrm{~kW}, \mathrm{~N}=1200 \mathrm{rpm}, \mathrm{Z}_{\mathrm{p}}=20, \mathrm{Z}_{\mathrm{g}}=50, \mathrm{M}=4 \mathrm{~mm}, \mathrm{~b}=44 \mathrm{~mm}, \mathrm{C}_{\mathrm{s}}=2, \mathrm{~S}_{\mathrm{ut}}=500 \mathrm{~N} / \mathrm{mm}^{2}$,
$\sigma_{\mathrm{b}}=\frac{1}{3} \mathrm{~S}_{\mathrm{ut}}=\frac{500}{3} \mathrm{~N} / \mathrm{mm}^{2}, Y_{\mathrm{p}}=0.320, \mathrm{Y}_{\mathrm{g}}=0.408, \mathrm{C}_{\mathrm{v}}=\frac{3}{3+\mathrm{v}}$
Since pinion and gear made of same material so, pinion is weaker than the gear. So, w.r.to pinion,

$$
\begin{aligned}
S_{b} & =M \times b \times \sigma_{b} \times Y_{p} \\
& =4 \times 44 \times \frac{500}{3} \times 0.320=9386.67 \mathrm{~N}
\end{aligned}
$$

Tangential force due to rated torque:
or

$$
\begin{aligned}
& d_{p}^{\prime}=M Z_{p}=4 \times 20=80 \mathrm{~mm} \\
& M_{t}=\frac{K W}{W}=\frac{5 \times 60}{2 \pi \times 1200}=0.039788 \mathrm{~N}-\mathrm{m} \\
& M_{t}=39788.73 \mathrm{~N}-\mathrm{mm}
\end{aligned}
$$

$$
P_{t}=\frac{2 M_{t}}{d_{p}^{\prime}}=\frac{2 \times 39788.73}{80}=994.718 \mathrm{~N}
$$

## Effective load:

$$
\begin{aligned}
V & =\frac{\pi \mathrm{d}_{\mathrm{p}}^{\prime} \mathrm{N}}{6 \times 10^{3}}=\frac{\pi \times 80 \times 1200}{6 \times 10^{3}}=5.026 \mathrm{~m} / \mathrm{sec} \\
\mathrm{C}_{\mathrm{v}} & =\frac{3}{3+\mathrm{v}}=\frac{3}{3+5.026}=0.37378 \\
P_{e f f} & =\frac{C_{s}}{C_{v}} \times P_{t}=\frac{2}{0.37378} \times 994.718=5322.4 \mathrm{~N}
\end{aligned}
$$

## Factor of Safety

$$
\mathrm{f}_{\mathrm{s}}=\frac{\mathrm{S}_{\mathrm{b}}}{P_{\text {eff }}}=\frac{9386.67}{5322.4}=1.763
$$

## SECTION-B

Q-5(a): What is the distinction between hypoeutectoid and hypereutectoid steels? Explain the development of microstructure in a hypoeutectoid steel with the help of neatly labelled diagram.
[12 MARKS]
Sol: - A hypoeutectoid steel has a carbon concentration less than the eutectoid on the other hand hypereutectoid steel has a carbon content greater than eutectoid.

- In hypoeutectoid steel structure there is continuous network of cementite, which separates each pearlite colony.
- Hypoeutectoid steel consists of pearlite and ferrite whereas hypereutectoid steel consists of pearlite and cementite.


Development of microstructure in hypoeutectoid steel: It is a sample of a 0.4 percent $C$ plain-carbon steel (hypoeutectoid steel) is heated to about $900^{\circ} \mathrm{C}$ (point a in figure) for a sufficient time, its structure will become homogeneous austenite. Then, if this steel is slowly cooled to temperature $b$ in figure (about $775^{\circ} \mathrm{C}$ ), proeutectoid ferrite will nucleate and grow mostly at the austenitic grain boundaries. If this alloy is slowly cooled from temperature $b$ to $c$ in figure, the amount of proeutectoid ferrite formed will continue to increase until about 50 percent of the austenite is transformed. While the steel is cooling from $b$ to c , the carbon content of the remaining austenite will be increased from 0.4 to 0.8 percent. At $723^{\circ} \mathrm{C}$, if very slow cooling conditions prevail, the remaining austenite will transform isothermally into pearlite by the eutectoid reaction austenite $\rightarrow$ ferrite + cementite. The $\alpha$ ferrite in the pearlite is called eutectoid ferrite to distinguish it from the proeutectoid ferrite that forms first above $723^{\circ} \mathrm{C}$. Figure is an optical micrograph of the structure of a 0.35 percent C hypoeutectoid steel that was austenites and slowly cooled to room temperature.

Q-5(b): With the help of schematic diagram, discuss the following:
(i) Single manufcturing cell
(ii) Flexible manufacturing cell
(iii) Flexible manufacturing system
[12 MARKS]
Sol: (i) Single manufacturing cell: It is a production unit formed by NC machine, completed by the manipulation facility to change the object of production.
(a) Most common manufacturing system in industry.
(b) Operation is independent of other stations.
(c) Perform either processing or assembly operations.
(d) Can be designed for single model production or batch production or mixed model production.
(e) Shortest time taken to implement this model.

It is on automated production machine capable of operating unattended for longer than one work cycle.
(ii) Flexible manufacturing cell: It contains 2-3 machines. It is a manufacturing system, created by grouping several numerical control (NC) machines, determined for certain group of parts with similar sequence of the operations or the contain type of operations.
(iii) Flexible manufacturing system: it is a production method that is designed to easily adopt to changes in type and quantity of the product being manufactured. It contains 4 and more machines. In it grouping of machine is done without material dependence of their activity.

Q-5(c): (i) Express unilateral and bilateral tolerances with the help of diagram considering normal size 24.00 mm and tolerance 0.030 mm .
[6 MARKS]
Sol: Unilateral Tolerance System: In this system, tolerance zone is provided only on one side of reference.


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Bilateral Tolerance System: In this system, tolerance zone is placed on both the sides of reference.


Q-5(c): (ii) Three blocks A, B and C are to be assembed in a channel of dimension D as shown in figure. Determine the tolerance that must be assigned to $D$, if it is essential that the minimum gap $E$ is not less than 0.005 mm . The dimensions of blocks are:
$A=0.75 \pm 0.003 \mathrm{~mm}$
$B=1.0 \pm 0.005 \mathrm{~mm}$
$C=1.125 \pm 0.004 \mathrm{~mm}$
Consider basic dimension of channel $D=2.894 \mathrm{~mm}$.

[6 MARKS]

Sol:
$\mathrm{E}=0.005 \mathrm{~mm}$


$$
D_{\max }=E+A_{\max }+B_{\max }+C_{\max }
$$

$$
D_{\min }=E+A_{\min }+B_{\min }+C_{\min }
$$

$$
D_{\max }=0.005+0.753+1.005+1.129
$$

$$
D_{\max }=2.892 \mathrm{~mm}
$$

$$
D_{\min }=0.005+0.747+0.995+1.121
$$

$$
D_{\min }=2.868 \mathrm{~mm}
$$

Tolerance that must be assigned to $D$

$$
\begin{aligned}
& =D_{\max }-D_{\min } \\
& =2.892-2.868=0.024
\end{aligned}
$$

Q-5(d): (i) Why is it necessary to schedule debris sampling for wear debris?
[6 MARKS]

## Sol: Wear Debris Sampling

A critical aspect in the wear particle analysis approach is the wear debris sample. Care must be taken to ensure that the analyzed wear debris sample is representative of the total wear debris being generated in the monitored system.

In the case of oil lubrication components, the generated wear debris is picked up by the oil and circulated throughout the lubricant system. Sampling of oil borne wear debris can be accomplished by either an inline or off-line technique.

In-line sampling involves the monitoring of debris parameters directly in the lubrication system. This approach provides a real time indication of the monitored parameters. In order to implement this approach, the monitoring equipment must be tied directly into the monitored equipment lubricant system. In-line monitoring of wear debris quantity and size distribution parameters, is proposed for the detection health monitoring element of high cost and/or critical application machinery.

In-line sampling, although simple in theory, present several practical implementation problems. A major portion of these problems result from the operating parameters dictated by the direct lubrication system tie in. Such parameters as flow, pressure, and temperature as seen in a lubrication system, are not conductive to effective debris monitoring.

Off-line sampling involves the withdrawal of a lubricant sample containing wear debris, from a lubrication system. Samples can then be transported to a laboratory for analysis. Tis approach provides more analysis versatility than the in-line sampling approach. Off-line sampling is proposed for the diagnosis health monitoring element. This element requires parameter analysis that presently can only be practically applied in a laboratory environment.
Although more versatile than the in-line approach, off-line sampling is very dependent on sampling technique. Such factors as sampling interval, time after shutdown, and sampling location can drastically effect sample wear debris quantity and distribution.

The above two sampling approaches rely heavily on the fact that significant wear debris is picked by the lubricant and circulated throughout the lubrication system. A prime factor affecting this debris circulation is the lubricant filter. This filter is designed to remove debris from the lubricant. Traditionally, lubricant filtration has been relatively coarse thus ineffective in removing significant wear debris from the lubricant. Recently, however, equipment lubricant filtration has been improving. This improvement is being driven by the realization of the deleterious accelerating effects of lubricant borne debris, on the system wear rate. Improved filtration will eventually lead to the condition where significant wear debris will no longer be circulating in the system, it will be for the most part, entrapped in the filter. This condition will tend to nullify the effectiveness of both present in-line and off-line sampling approaches. Drastic modifications of present sampling techniques will have to be developed in order to obtain a representative wear debris sample from a highly filtered system.

## Q-5(d): (ii) Wear particles of spherical shape were found in a weir debris sample. What is the possible

 mode of failure for such case? Justify.Sol: Spherical particles (spheres) can act as an early warning sign of outside contamination, bearing fatigue and abnormal wear. Early identification of the presence and source of spheres helps you to:

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## ESE 2023 <br> Mains Exam Solution <br> MECHANICAL ENGINE=RING-Paper II

- Make informed decisions.
- $\quad$ Schedule actions (e.g. review monitor or repair) and
- Minimise unplanned downtime and costly repairs.

There are several possible sources that can create spheres including:

- Wear modes such as fatigue wear or sliding wear.
- Electrical discharge erosion.
- Lubricant degradation: Oil Balls
- Outside contamination including welding grinding cutting and fly ash.


## Detecting Spheres

Large diameter spherical particles are invisible to spectrometry-based oil analysis programs since ICP/ AE Spectrometers are insensitive to particles $>10 \mu \mathrm{~m}$. Other large particle techniques such as Analytical Ferrography can image and provide an indication of the size of spheres but cannot determine the elemental composition.

This image is of bearing grease from the pedestal bearing of a large mining shovel. The sample was prepared on a ferrographic slide and photographed at 200x magnification. Note the presence of spheres as well as cutting wear.

Scanning Electron Microscopy: Energy Dispersive X-ray Spectroscopy (SEM-EDS) however can determine the size, shape and composition of wear particles and contamination including spheres. Contamination spheres generated by welding/grinding/cutting debris and spheres caused by lubricant degradation can be differentiated using SEM EDS from small diameter spheres more commonly associated with bearing wear.

The following image is an SEM micrograph showing a single sphere from a sample of used engine oil. Primarily composed of $\mathrm{Fe} 26.33 \mu \mathrm{~m}$ diameter.

Since contamination spheres are abrasive, if detected by SEM-EDS analysis you should consider acting before secondary wear occurs. Actions that can be taken include:

- Inspection of filters and change if necessary.
- Drain, flush and refill the oil.
- Take a new oil sample at a shortened interval to verify the contamination has been eliminated.

With the presence of small diameter spheres, actions that may be required include:

- Inspection of the component.
- Drain, flush and refill the oil.
- Take another sample at a shortened interval to verify, if the spheres are small present.
- Perform SEM-EDS on both the oil and filter to check for signs of bearing wear and abnormal wear of other components.

Each unit of an item costs a company ₹ 40 . Annual holding costs are $18 \%$ of unit cost for interest charges, $1 \%$ for insurance, $2 \%$ allowance for obsolescence, ₹ 2 for building overheads, ₹ 1.50 for damage and loss, and ₹ 4 miscellaneous cost. Annual demand for the item is constant at 1,000 units and each order costs ₹ 100 to place.
(i) Calculate EOQ and the total costs associated with stocking the item.
(ii) If the supplier of the item will only deliver batches of 250 units, how are the stock holding costs affected?
(iii) If the supplier relaxes his order size requirements, but the company has limited warehouse space and can stock a maximum of 100 units at any time, what would be the optimal ordering policy and associated costs?
[12 MARKS]
Sol: Unit cost of item = ₹ 40

Unit holding cost per year $\left(\mathrm{C}_{\mathrm{h}}\right)$
18\% Interest
1\% Insurance
2\% Allowances for obsolescence
$21 \%$ of unit cost $\frac{21}{100} \times ₹ 40$ ]
Interest, insurance, alliance $\rightarrow$ ₹ $8.40 \rightarrow$ ₹ 8.40 per unit
Other cost $\rightarrow \quad ₹ 2.00$ for building overheads
₹ 1.50 for damages
₹ 4.00 Miscellaneous cost
Total cost $\rightarrow$
₹ 15.90 per unit
Ordering cost $\left(C_{p}\right) \rightarrow ₹ 100$ per order

$$
\begin{align*}
E O Q & =\sqrt{\frac{2 R C_{p}}{C_{h}}} \text { where, } R \text { is annual demand }=1000 \text { units }  \tag{i}\\
& =\sqrt{\frac{2 \times 1000 \times 100}{15.90}} \quad \text { No. of orders }=\frac{1000}{112}=8.92 \approx 9 \\
Q & =112.15 \text { units } \approx 112 \text { units } \\
\text { Total cost } & =\text { Holding cost }+ \text { Ordering cost } \\
& =\left[\frac{Q}{2} \times C_{h}\right]+\left[\frac{R}{Q} \times C_{p}\right] \\
& =\left(\frac{112}{2}\right) \times(15.90)+\left(\frac{1000}{112}\right) \times 100 \\
\text { Total cost } & =₹ 890.40+₹ 900=₹ 1790.40
\end{align*}
$$

(ii) In minimum order quality as per supplies is 250 units,

$$
\begin{aligned}
& \mathrm{Q}=250 \text { units } \\
& \text { then } \quad \begin{aligned}
\text { Number of orders } & =\frac{1000}{250}=4 \text { orders } \\
\text { Total cost } & =\left(\frac{R}{Q}\right) C_{p}+\frac{Q}{2} \times C_{h} \\
& =4 \times 100+\left(\frac{250}{2}\right) \times(15.90) \\
& =400+125 \times 15.90 \\
& =400+1987.50 \\
\text { Total cost } & =₹ 2387.50
\end{aligned}, \begin{aligned}
\end{aligned} \\
&
\end{aligned}
$$

(iii) Supplies relaxes his order size, but company has limited warehouse space for 100 units only then $Q=100$ units
then

$$
\begin{aligned}
\text { The number of orders } & =\frac{R}{Q}=\frac{1000}{100}=10 \\
\text { Total cost } & =\left(\frac{R}{Q}\right) \times C_{P}+\left(\frac{Q}{2}\right) \times C_{h} \\
& =10 \times 100+\left(\frac{100}{2}\right) \times 15.90 \\
& =1000+795 \\
\text { Total cost } & =₹ 1795.00
\end{aligned}
$$

Q-6(a): (i) The voltage length characteristic of a direct current (dc) arc is given by $V=(20+40 l)$ volts, where $l$ is length of the arc in $\mathbf{c m}$. The power source characteristic is approximated by a straight line with an open circuit voltage $=80 \mathrm{~V}$ and a short circuit current $=1000 \mathrm{amp}$.

Determine the optimum arc length and the corresponding arc power.
[12 MARKS]
Sol: Arc voltage - Arc length characteristic is given

$$
V_{a}=20+40 \ell \quad(\ell=\text { Arch length in } \mathrm{cm})
$$

Also,

$$
\begin{aligned}
& \mathrm{OCV}=80 \mathrm{~V} \\
& \mathrm{SCC}=1000 \mathrm{~A}
\end{aligned}
$$

We have power source characteristic. equation
or

$$
\begin{aligned}
& V=O C V-\left(\frac{O C V}{S C C}\right) I \\
& V=80-\left(\frac{80}{1000}\right) I \\
& V V_{P}=80-\frac{21}{25}
\end{aligned}
$$

For stable arc

$$
\begin{aligned}
V_{a} & =V_{p} \\
20+40 \ell & =80-\frac{21}{25}
\end{aligned}
$$

We get
By solving, we get

$$
I=750-500 \ell
$$

Now, arc power is given by

$$
\begin{aligned}
P & =V \times I \\
P & =(20+40 \ell)(750-500 \ell) \\
P & =15000+2000 \ell-20000 \ell^{2}
\end{aligned}
$$

For optimum arc length

$$
\begin{aligned}
& \frac{\mathrm{dp}}{\mathrm{~d} \ell}=0 \\
& \frac{\mathrm{~d}}{\mathrm{~d} \ell}\left\{15000+20000 \ell-20000 \ell^{2}\right\}=0 \\
& 20000-40000 \ell=0 \\
& 40000 \ell=20000 \\
& \text { We get arc length (optimum) } \quad \ell_{\text {opt }}=\frac{20000}{40000} \\
& \ell_{\text {opt }}=0.5 \mathrm{~cm}
\end{aligned}
$$

Arc power corresponding to $\ell_{\text {opt }}$

$$
\begin{aligned}
P & =15000+20000 \ell-20000 \ell^{2} \\
P & =15000+(20000 \times 0.5)-\left(20000 \times 0.5^{2}\right) \\
P & =15000+10000-5000 \\
P & =20000 \text { Watt or }
\end{aligned}
$$

$$
\mathrm{P}=20 \mathrm{~kW}
$$

Q-6(a): (ii) Enlist the most common defects encountered in sand mould casting. Describe the reasons for Scab and Misrun.
[8 MARKS]
Sol: Defects encountered is sand mould casting

1. Gas defects
(i) Blow holes and open blows
(ii) Air inclusions
(iii) Pin hole porosity
(iv) Shrinkage cavity
2. Moulding material defects
(i) Cuts and washes
(ii) Metal penetration
(iii) Fusion
(iv) Scab, swell
(v) Run-out
(vi) Rat tails \& buckles
3. Pouring metal defects
(i) Misrun \& cold shuts
(ii) Slag inclusions
4. Metallurgical defects
(i) Hot tear
(ii) Hot spot

SCAB: Due to improper ramming of moulding sand during mould preparation sand will not get sufficient strength and when molten metal is poured, due to pressure, the moulding sand get pressed, if this happens to the roof of the mould, it is called scab.

MISRUN: Misrun defect is caused when the molten metal is not able to fill the mould cavity completely and thus leaving unfilled cavities.

Q-6(b): (i) Compare gray, malleable, white and modular cast irons with respect to (I) composition and heat treatment, (II) microstructure and (III) mechanical properties.
[12 MARKS]

## Sol: Gray cast iron:

Composition: Iron $=93.5 \%$, Graphite $=1.75 \%$, Combined carbon $=1.75 \%$ and $\mathrm{Si}=1$ to $3 \%$.

- Gray cast iron having microstructures different type may be generated by adjustment of composition by using on appropriate heat treatment.
- Mechanically, gray iron is comparatively weak and brittle in tension due to microstructure. Strength and ductility are much higher under compressive loads.


## Nodular cast iron:

Composition: $(3.5-4 \%) \mathrm{C},(2-2.8 \%) \mathrm{Si}, 0.05 \% \mathrm{Mg}$ and $<80 \% \mathrm{Ni}$.

- The matrix phase surrounding is either pearlite or ferrite depending upon heat treatment. It is normally pearlite for an as-cast piece. However, a heat treatment for several hours at about $700^{\circ} \mathrm{C}$ will yield ferrite matrix. Casting are much more ductile than gray iron.


## White and modular cast iron

- These have $\mathrm{Si}<1 \%$, most of the carbon exists as cementite instead of graphite.
- It is formed by rapid cooling rates.
- Thick section may have only surface layer of white iron that was chilled during casting process.
- Due to large amount of cementite phase it is hard and very brittle.


## Malleable cast iron

- When white cast iron is heated to a temperature of $800-900^{\circ} \mathrm{C}$ for a prolonged time period (Malleable annealing) and in neutral atmosphere (to prevent oxidation), it causes decomposition of cementite to graphite in the form of clusters or rosettes (tempered) surrounded by a ferrite or pearlite matrix. Finally strength and ductility increases.

Composition: (2.3-2.7\% C) + (1-1.5\% Si) + (<0.55\% Mn)

Q-6(b): (ii) Make a schematic plot showing the tensile engineering stress-strain behaviour for mild steel and label the salient points. State the reason of occurrence of two yield points in mild steel. Also, explain the following on the basis of the plot (I) Ductility, (II) Resilience, and (III) Toughness.
[8 MARKS]

## Sol:



Fig. Stress vs strain curve.

## 1. Proportional limit

- From diagram we observe that OP is a straight line and after point P , the curve begins to deviate from the straight line.
- Proportional limit is defined as the stress beyond which the stress-strain curve deviate from straight line. Thus point $P$ indicates the proportional limit.
- Hooke's law states that stress is directly proportional to strain (stress $\propto$ strain) is applicable upto proportional limit.


## 2. Modulus of elasticity (E)

- It is ratio of stress to strain upto point $P$.
- It is given by slope of line OP.
i.e. $E=\tan \theta=\frac{P X}{O X}=\frac{\text { stress }}{\text { strain }}\left(N / m^{2}\right)$

E for different materials are as follows: ceramics > metal > polymer

## 3. Elastic limit

- When the specimen is loaded upto point $E$, and if load is removed, material will regain its original shape and size instantaneously.
- Thus the elastic limit of the material is defined as the maximum stress without any permanent deformation


## 4. Yield Strength

- When the specimen is stressed beyond E the strain increases at a faster rate i.e. without appreciable increase in stress.
- The yield strength is defined as the maximum stress at which a marked elongation occurs without increase in the load. Thus lower yield point is considered as yield strength of material.
- For many materials $Y_{1}$ and $Y_{2}$ are very close and hence considered to be same and denoted by Y and stress corresponding to is called yield strength.
- The phenomenon involving a definite yield point in ductile materials is known as yield point phenomenon


## Resilience

It is defined as the ability of the material to absorb energy when deformed elastically and to release the energy when unloaded. This property is essential for spring materials. Resilience is measured by a quantity, called modulus of resilience or proof resilience, which is the strain energy per unit volume that is required to stress the specimen in a tension test to the elastic limit point. It is the area below stressstrain curve in a tension test upto the elastic limit.

## Toughness

It is defined as the ability of the material to absorb energy before fracture takes place. In other words, toughness is the energy for failure by fracture. This property is essential for machine components those are required to withstand impact loads. Tough materials have the ability to bend, twist or stretch before failure takes place. Toughness is measured by a quantity called modulus of toughness. Modulus of toughness is the total area under stress-strain curve in a tension test upto fracture, which also represents the work done to fracture the specimen. Toughness decreases as the temperature increases.

## Ductility

It is the ability of material to deform to a greater extent before the sign of crack, appears when it is subjected to tensile force. Ductility is measured in units of percentage elongation or percentage reduction in area in a tension test. The ductility of metal increases as the temperature increases because metals becomes soft at increasing temperature. Ductility is affected by grain size which is dependent on temperature. The energy absorbed by a ductile specimen before fracture in a tension test is more and failure takes place by yielding which is gradual.

Q-6(c): (i) (I) Derive the characteristic equation for the piezoelectric accelerometer supporting a mass (M) on a spring of stiffness ( $K$ ) and viscous damper with damping coefficient (C). Assume the input and output displacement to be $\left(x_{i}\right)$ and ( $x_{0}$ ) respectively.
(II) What is the amplitude ratio for a frequency response analysis assuming input displacement to be sinusoidal?
[10 MARKS]

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Sol: The piezoelectric accelerometer is widely accepted as the best available transducer for the absolute measurement of vibration. This is a direct result of these properties:

1. Usable over very wide frequency ranges.
2. Excellent linearity over a very wide dynamic range.
3. Acceleration signal can be electronically integrated to provide velocity and displacement data.
4. Vibration measurements are possible in a wide range of environmental conditions while still maintaining excellent accuracy.
5. Self-generating so no external power supply is required.
6. Extremely compact plus a high sensitivity to mass ratio.


The following expressions describe the forces present in the model

$$
\begin{aligned}
\mathrm{F} & =\mathrm{k}\left(\mathrm{x}_{\mathrm{s}}-\mathrm{x}_{\mathrm{b}}-\mathrm{L}\right) \text { (spring force) } \\
\mathrm{m}_{\mathrm{b}} \ddot{\mathrm{x}}_{\mathrm{b}} & =\mathrm{F}+\mathrm{F}_{\mathrm{e}} \text { (force on base) } \\
\mathrm{m}_{\mathrm{s}} \ddot{x}_{\mathrm{s}} & =-\mathrm{F} \text { (force on seismic masses) }
\end{aligned}
$$

The equation of motion for the model can be found

$$
\begin{equation*}
\ddot{x}_{s}-\ddot{x}_{b}=-\frac{F}{m_{s}}-\frac{F+F_{e}}{m_{b}}=-\frac{k}{\mu}\left(x_{s}-x_{b}-L\right)-\frac{F_{e}}{m_{b}} \tag{i}
\end{equation*}
$$

or

$$
\mu i=-k r-\frac{\mu}{m_{b}} F_{0} \sin \omega t
$$

where,

$$
\frac{1}{\mu}=\frac{1}{m_{s}}+\frac{1}{m_{b}}
$$

or

$$
\mu=\frac{m_{s} m_{b}}{m_{s}+m_{b}}
$$

$\mu$ is often referred to as the "reduced mass" and $r$ is the relative displacement of the seismic mass to the base.

$$
r=x_{s}-x_{b}-L
$$

When the accelerometer is in a free hanging position and is not being excited by external forces ( $\mathrm{F}_{\mathrm{e}}=$ 0 ) the equation of motion for its free vibration reduces to

$$
\mu \ddot{i}=-k r
$$

This simple differential equation can be solved by assuming that the displacement of $m_{s}$ relative to $m_{b}$ varies harmonically with an amplitude R. In other words

$$
\begin{aligned}
r & =R \sin \omega t \\
-\mu R \omega^{2} \sin \omega t & =-k R \sin \omega t
\end{aligned}
$$

and therefore the resonance frequency of the accelerometer, $\omega_{\mathrm{n}}$ can be written directly as

$$
\omega_{\mathrm{n}}^{2}=\frac{\mathrm{k}}{\mu}
$$

The implications of this result can be seen by rewriting this equation as follows:

$$
\begin{equation*}
\omega_{n}^{2}=k\left(\frac{1}{m_{s}}+\frac{1}{m_{b}}\right) \tag{ii}
\end{equation*}
$$

If the accelerometer is now mounted with perfect rigidity onto a structure which is heavier than the total weight of the accelerometer then $m_{b}$ becomes much larger than $m_{s}$. The resonance frequency of the accelerometer becomes lower. Taken to the limit, if the accelerometer is mounted on an infinitely heavy structure ( $\mathrm{m}_{\mathrm{b}} \rightarrow \infty$ ) then the last equation reduces to

$$
\begin{equation*}
\omega_{\mathrm{m}}^{2}=\frac{\mathrm{k}}{\mathrm{~m}_{\mathrm{s}}} \tag{iii}
\end{equation*}
$$

This is the natural frequency of the seismic mass-spring system and is defined as the mounted resonance frequency, $\omega_{\mathrm{m}}$ of the accelerometer. The mounted resonance frequency is a property of the accelerometer seismic mass spring system. Later it will be seen that this frequency is used to define the useful operating frequency range of an accelerometer.

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The applied force on the accelerometer must be included in the analysis along with the natural resonance frequency, $\omega_{\mathrm{n}}$ previously defined. The equation of motion for the model (1) now becomes

$$
\ddot{\mathrm{r}}+\omega_{\mathrm{n}}^{2} \mathrm{r}+\frac{\mathrm{F}_{0}}{\mathrm{~m}_{\mathrm{b}}} \sin \omega \mathrm{t}=0
$$

and assuming again that the displacements of the masses vary sinusoidally then

$$
-\omega^{2} R \sin \omega t+\omega_{n}^{2} R \sin \omega t+\frac{F_{0}}{m_{b}} \sin \omega t=0
$$

and therefore

$$
\begin{aligned}
R\left(\omega_{n}^{2}-\omega^{2}\right)+\frac{F_{0}}{m_{b}} & =0 \\
R & =-\frac{F_{0}}{m_{b}\left(\omega_{n}^{2}-\omega^{2}\right)}
\end{aligned}
$$

or

At frequencies well below the natural resonance frequency of the accelerometer ( $\omega \ll \omega_{n}$ ) the displacement, which is now called $R_{0}$ is expressed by

$$
R_{0}=-\frac{F_{0}}{m_{b}\left(\omega_{n}^{2}-\omega^{2}\right)}
$$

The ratio of the displacement at low frequency, $\mathrm{R}_{0}$ to the displacement at high frequency, R can be expressed as follows:

$$
\frac{R}{R_{0}}=\frac{-\frac{F_{0}}{m_{b}\left(\omega_{\mathrm{n}}^{2}-\omega^{2}\right)}}{-\frac{F_{0}}{m_{b} \omega_{n}^{2}}}
$$

and by denoting this ratio as A and rearranging the expression, then

$$
\begin{equation*}
A=\frac{1}{1-\left(\frac{\omega}{\omega_{n}}\right)^{2}} \tag{iv}
\end{equation*}
$$

This important result shows that the displacement between the base and the seismic masses increases when the forcing frequency becomes comparable to the natural resonance frequency of the accelerometer. Consequently the force on the piezoelectric elements and the electrical output from the accelerometer also increase. As the piezoelectric elements used in Bruel \& Kjaer accelerometers exhibit constant force sensitivity and increase in electrical output of an accelerometer near its resonance frequency is attributable entirely to the natural resonance of the accelerometer. The typical shape of a frequency response curve of an accelerometer and amplitude measurement errors are related to this equation.
The free hanging natural resonance frequency of the accelerometer depends heavily on the ratio of the total seismic mass to the mass of the rest of the transducer but primarily to that of the base. As a general rule the total seismic mass of an accelerometer is approximately the same as the mass of the base and this gives the relationship

$$
\frac{\text { Mounted resonance frequency }}{\text { Free hanging resonance frequency }} \approx \frac{1}{\sqrt{2}}
$$

Q-6(c): (ii) An accelerometer is designed with a seismic mass of 0.05 kg , a spring constant of $5000 \mathrm{~N} /$ $m$, and a damping constant of $30 \mathrm{NS} / \mathrm{m}$. If the accelerometer is mounted to an object experiencing displacement $x_{i}=5 \sin$ (100t) mm, find and expression for the steady state relative displacement of seismic mass relative to housing as $s$ function of time $x_{r}(t)$.
[10 MARKS]
Sol: Data given:

$$
\begin{aligned}
& \text { Spring constant, } \mathrm{k}=5000 \mathrm{~N} / \mathrm{m} \\
& \text { Seismic mass, } m=0.05 \mathrm{Kg} \\
& \text { Damping constant, b }=30 \mathrm{NS} / \mathrm{m} \\
& \omega_{\mathrm{n}}=\sqrt{\frac{\mathrm{k}}{\mathrm{~m}}}=\sqrt{\frac{5000}{0.05}}=316.2 \mathrm{rad} / \mathrm{sec} \\
& \frac{\omega}{\omega_{n}}=\frac{100}{316.2}=0.316 \\
& \Rightarrow \\
& \begin{aligned}
\left(\frac{\omega}{\omega_{n}}\right)^{2} & =0.1 \\
\zeta & =\frac{b}{2 \sqrt{k m}}=\frac{30}{2 \sqrt{5000 \times 0.05}}=0.949
\end{aligned} \\
& \left|\overline{\mathrm{x}}_{\text {in }}\right|_{\text {actual }}=\mathrm{x}_{\text {in }} \omega^{2}=5 \times(100)^{2} \\
& =5 \times 10^{4} \mathrm{~mm} / \mathrm{s}^{2} \\
& =50 \mathrm{~m} / \mathrm{s}^{2} \\
& H_{a}(\omega)=\frac{1}{\left[1-\left(\frac{\omega}{\omega_{n}}\right)^{2}\right]^{2}+4 \zeta\left(\frac{\omega}{\omega_{n}}\right)^{2}}=1.08 \\
& X_{t}=\frac{1}{\omega_{n}^{2}} H_{a}(\omega)\left(X_{\text {in }} \omega^{2}\right)=\frac{1}{(316.2)^{2}} \times 1.08 \times 50 \\
& X_{t}=5.4 \times 10^{-4} \mathrm{~m}=0.54 \mathrm{~mm} \\
& \phi=-\tan ^{-1}\left(\frac{2 \zeta \frac{\omega}{\omega_{n}}}{1-\left(\frac{\omega}{\omega_{n}}\right)^{2}}\right)=-\tan ^{-1}\left(\frac{2 \times 0.949 \times 0.316}{1-0.1}\right) \\
& =-33.7^{\circ}=-0.588 \mathrm{rad} \\
& \mathrm{X}_{\mathrm{r}}(\mathrm{t})=\mathrm{X}_{\mathrm{t}} \sin (\omega \mathrm{t}+\phi) \\
& =0.54 \sin (100 \mathrm{t}-0.588) \mathrm{mm}
\end{aligned}
$$

Q-7(a): (i) An engine is to be designed to have a minimum reliabiity of 0.8 and minimum availability of 0.98 over a period of $2 \times 10^{3}$ hours. Determine MTTR and frequency of failures of engine.
[8 MARKS]
Sol: Given,

$$
\begin{aligned}
\text { Reliability, } R & =0.8 \\
\text { Availability, } \mathrm{A} & =0.98 \\
\text { Time, } \mathrm{t} & =2000 \mathrm{hrs}
\end{aligned}
$$

We have,
or
Freq. of failures, $\lambda=11.15 \times 10^{-5}$ per hour
Now, we can calculate mean time between failure

We have

$$
\begin{aligned}
\mathrm{MTBF} & =\frac{1}{\lambda}=\frac{1}{0.00011155}=8965 \mathrm{hrs} \\
\mathrm{~A} & =\frac{\mathrm{MTBF}}{\mathrm{MTBF}+\mathrm{MTTR}} \\
0.98 & =\frac{8965}{8965+\text { MTTR }} \\
8965+\mathrm{MTTR} & =9147.96 \\
\text { MTTR } & =182.95 \text { hours }
\end{aligned}
$$

Q-7(a): (ii) Explain the mechanism of chip formation. What are the conditions that result in the formation of
(I) Continuous chips without built up edge.
(II) Continuous chips with built up edge.
(III) Discontinuous chips
[12 MARKS]
Sol: The basic mechanism of chip formation, therefore, consists of a deformation of metal lying just ahead of the cutting edge of tool, by process of shear, in a narrow zone (called shear zone or primary deformation zone) extending from the cutting edge of the tool obliquely up to the uncut surface of workpiece in front of the tool. During metal cutting, the metal in the area in front of the cutting edge of the tool is severely compressed causing high temperature shear stress in the metal, the shear stress being maximum along a narrow zone or plane called the shear plane. When the shear stress in the workpiece metal just ahead of the cutting edge of tool reaches a value exceeding the ultimate strength of the metal, particles of the metal start shearing away (or rupture) and flow plastically along the shear plane, forming "segments of chip" which flow upwards along the face of the tool. In this way, more and more new chip segments are formed and the cycle of
compression, plastic flow and rupture is repeated resulting into the birth of a continuously flowing chip. Since the width of shear zone is small, chip formation is often described as a process of successive shears of thin layers of workpiece metal along particular surfaces. Chips are highly compressed body and have burnished and deformed underside (due to deformation at secondary shear zone on account of friction between chip and tool face). The primary shear zone deformations are required for the formation of chip, whereas deformations in secondary shear zone are secondary deformations which, in fact, are disturbances and are not required.

(i) Continuous chip without built-up edge has its elements bonded together and is formed by continuous deformation of metal without fracture ahead of the cutting edge of tool and followed by smooth flow of chip up the tool face. Upper side of a continuous chip has small notches and the lower side is smooth and shiny as the chip slides over the tool. This type of chip is formed in machining at high speed soft ductile metals such as mild steel and copper and is considered the most desirable type of chip.


Fig. Formation of a continuous chip.
(ii) Continuous chip with built-up edge is very much similar to the continuous type chip except that a built-up edge is found adhering on the nose of the tool.


Fig. Formation of a continuous chip with a built-up edge.
Such a chip is formed while machining ductile metal and existence of high friction at the chip-tool interface. The upward flowing chip exerts pressure on the tool face which is very high being
maximum at the cutting edge or nose of the tool. As a result of this, excessively high temperature is generated because of which the compressed metal adjacent to tool nose gets welded to it. This extra metal welded to the nose or point of the tool is called built-up edge. The built-up edge is highly strain-hardened and brittle because of which when the chip flows up the tool, a part of the built-up edge is broken and carried away with the chip while the rest of it keeps adhering with the workpiece surface, making it rough. The presence of built-up edge at the nose of the tool alters the rake angle of the tool and consequently the cutting forces are changed. Factors responsible for formation of built-up edge are low cutting speed, excessive feed, smaller rake angle and poor lubrication or cooling of tool during cutting. Besides giving rough machined surface and fluctuating cutting force and tool vibration, built-up edge also carries away some material from the tool leading to the formation of a crater which results in tool wear. Formation of a built-up edge can be avoided by (i) reducing friction at chip tool interface by means of polishing the tool face and use of adequate supply of lubricant, (ii) keeping larger rake angle and (iii) maintaining low feeds and higher cutting speed as the latter generates high temperature which reduces weld strength and reduces possibility of formation of built-up edge through welding.

Besides the above types of chips, homogeneous strain chips are also there which are produced in machining metals like titanium alloys and others suffering a marked decrease in yield strength with temperature and poor thermal conductivity. Such chips are banded with regions of large and small strains.
(iii) Discontinuous chip consists of elements separated into short segments. This type of chip is obtained in machining hard and brittle metals such as cast iron and bronze. When workpiece metal is brittle, it has little capacity for deformation before fracture and the chip separates along the shear plane. Chips may be in the form of completely individual segments or loose chips formed by adhering of segments. These loose chips fracture easily. It may be noted that in machining hard and brittle metals, as the tool advances ahead, the shear plane angle gradually reduces until the value of compressive stress working on the shear plane becomes too low to prevent rupture. It is at this stage that any further advancement of the tool results in the fracture of the metal ahead of it, thus producing a segment of chip, repetition of which results in discontinuous chips. Machining of ductile metals at very slow speed may also give discontinuous chips. In case of brittle metal, the presence of these chips affords fine finish, increased tool life and low power consumption. Discontinuous chips in machining ductile metal result in poor finish and excessive tool wear.


Fig. Formation of a discontinuous or segmental chip.

Q-7(b): Explain with the working principle a suitable Non-Destructive Testing (NDT) technique to be used for detecting surface as well as fully embedded defects for a wide range of materials including polymers. Also, list the other NDT techniques with reasoning that are not suitable for inspection of above described requirements.
[20 MARKS]

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Sol:
One suitable non-destructive testing (NDT) technique that can be used for detecting surface defects as well as fully embedded defects in a wide range of materials, including polymers, is Ultrasonic Testing (UT).

Ultrasonic Testing utilizes high-frequency sound waves to inspect the internal structure of a material. It involves the transmission of ultrasonic waves into the material and the analysis of the reflected waves to identify any defects or irregularities.

Here's how Ultrasonic Testing can be used to detect surface and embedded defects in different materials, including polymers:

## 1. Surface Defect Detection:

- Contact Method: In this method, a couplant (gel or oil) is applied between the transducer and the material's surface to ensure efficient sound wave transmission. The transducer emits ultrasonic waves that travel into the material and reflect back from the surface. By analyzing the time taken for the waves to return and their amplitude, surface defects such as cracks, delaminations, or corrosion can be detected.
- Immersion Method: This method involves immersing the material in a water tank or coupling medium. The transducer is placed in the water, and ultrasonic waves are transmitted through the water into the material. The reflected waves are analyzed to identify surface defects.


## 2. Embedded Defect Detection:

- Pulse-Echo Method: In this method, a single transducer acts as both a transmitter and a receiver. The transducer emits ultrasonic pulses into the material, and the reflected waves from the back surface or internal defects are detected. This technique is suitable for detecting embedded defects, such as voids, inclusions, or disbands.
- Through-Transmission Method: This method involves the use of two transducers, one acting as a transmitter and the other as a receiver. The transmitter transmits ultrasonic waves into the material, and the receiver detects the waves that pass through the material. By comparing the transmitted and received signals, internal defects can be identified.

Polymers, being a wide range of materials, can be effectively inspected using Ultrasonic Testing due to its versatility and adaptability. The choice of transducer frequency, coupling medium, and technique variation can be optimized based on the specific polymer material and defect characteristics to achieve accurate and reliable results.

It is important to note that the selection of the appropriate Ultrasonic Testing technique and parameters should consider the material's properties, thickness, defect size, and the required level of inspection sensitivity. Qualified NDT professionals with expertise in Ultrasonic Testing can provide further guidance and ensure proper inspection procedures are followed for effective defect detection in polymers and other materials.

List other NDT technique with reasoning that are not suitable for inspection of above described required.
While Ultrasonic Testing (UT) is a suitable NDT technique for inspecting a wide range of materials, including polymers, let's discuss some other NDT technique that may not be as suitable for the specific requirement of detecting both surface and fully embedded defects in various materials:

1. Visual Inspection: Visual inspection involves the direct visual examination of the material's surface for any visible defects. While it can be effective in detecting surface defects, it may not be suitable for identifying fully embedded defects as they are not visible to the naked eye. Visual inspection can still be used as a preliminary screening method, but it may not provide comprehensive results for embedded defects.

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2. Liquid Penetrant Testing (PT): Liquid Penetrant Testing is primarily used for detecting surfacebreaking defects in materials. It involves applying a liquid penetrant to the material's surface, which seeps into any surface cracks of discontinuities. After a specified time, excess penetrant is removed, and a developer is applied to make the penetrant indications visible. While PT is excellent for surface defect detection, it is not as effective for identifying fully embedded defects.
3. Magnetic Particle Testing (MT): Magnetic Particle Testing is a technique used to defect surface and near-surface defects in ferromagnetic materials. It involves magnitizing the material and applying magnetic particles on the surface. The particles will gather at areas with magnetic flux leakage caused by defects, making them visible under appropriate lighting conditions. However, MT is not suitable for inspecting non-ferromagnetic materials like polymers, limiting its applicability.
4. Eddy Current Testing (ECT): Eddy Current Testing is primarily used for detecting surface and nearsurface defects in conductive materials. It relies on including electrical currents in the material and measuring changes in the current flow caused by defects. While ECT can be effective for surface defect detection, it may not be as suitable for fully embedded defect detection, especially in nonconductive materials like polymers.
5. Radiographic Testing (RT): Radiographic Testing involves using x-rays or gamma rays to inspect materials for defects. It can detect both surface and internal defects. However, for inspecting polymers and other non-metallic materials, RT may not be the most suitable technique. Polymers tend to have low X-ray or gamma-ray absorption, resulting in poor image contrast and limited defect visibility.
It's important to note that the suitability of NDT technique depends on the specific inspection requirements, material properties, and defect characteristics. NDT professionals should carefully evaluate these factors to select the most appropriate technique or combination of techniques for accurate and reliable defect detection.

Q-7(c): (i) A 12-bit Analog-to-Digital Converter operating at a sampling rate of 5 kHz is used with a sensor. What is the size of computer memory (in bytes) required to store 20 seconds of sensor data? What will be the memory size in case a 8-bit Analog to Digital Converter is used? Why is it not possible to connect sensors such as accelerometers, strain gauges and thermocouple directly to a microprocessor or computer?
[12 MARKS]
Sol: Given:

$$
\text { Sampling rate }=5 \mathrm{kHz}
$$

Case-I: To find memory size in case a 12 -bit ADC:
Total number of samples $(\mathrm{N})=20 \times 5 \times 10^{3}=100 \times 10^{3}$
Now, Total number of bits $=12 \times \mathrm{N}=12 \times 100 \times 10^{3}=1200 \times 10^{3}$ bit

$$
=\frac{1200 \times 10^{3}}{8} \text { bytes }=150 \times 10^{3} \text { byte }
$$

Case-II: To find memory size in case a 8 -bit ADC:

$$
\begin{aligned}
\text { Total number of bits } & =8 \times \mathrm{N}=8 \times 100 \times 10^{3} \text { bit } \\
& =\frac{800 \times 10^{3}}{8} \text { byte }=100 \times 10^{3} \text { byte }
\end{aligned}
$$

Q-7(c):

Sol: We have, for optical encoder

$$
\text { Number of pulse } n_{p}=\frac{x \cdot n_{s}}{p}
$$

where,

$$
\begin{aligned}
x & =\text { Distance moved by table }=100 \mathrm{~mm} \\
n_{s} & =\text { Pulse generated by optical encoder }=200 / \mathrm{rev} . \\
p & =\text { pitch }=5 \mathrm{~mm} \\
n_{p} & =\frac{100 \times 200}{5}=4000 \text { pulse }
\end{aligned}
$$

Q-8(b): (ii) With the help of a schematic diagram, explain he working principle of a resolver. How does the output for resolver differ from that of an encoder?
[10 MARKS]

## Sol: Resolvers :

- The function of a resolver is to resolve a vector into its sine and consine components.
- The output of the resolver is in the form of two signals, one proportional to the sine of the angle and the other proportional to cosine of the angle.
- Hence, resolvers are used for the conversion of angular position of a shaft into Cartesian coordinates.
- A resolver is an analog electromagnetic transducer that can be used in a wide variety of position and velocity feedback applications ranging from semiconductor manufacturing to oil and gas drilling.
- Resolver is an analog device and the electrical outputs are continuous through one complete mechanical revolution and hence, it offers infinite theoretical resolution.
- Basic principle of working:
(i) The device consists of a rotor attached to a shaft that moves with the load, and a stator that remains stationary.
(ii) Stator windings are supplied with an alternating voltage that produces an alternating magnetic flux which induces voltages in the two rotor windings.
(iii) The output voltage of the rotor windings is proportional to the stator voltage and the coupling between stator and rotor windings.
(iv) The way in which the windings are placed, the rotor output voltages are proportional to the sine and cosine of the rotor angel.

(iv) When one of the stator windings $S_{1} S_{3}$ is excited by an a.c. source then $S_{2} S_{4}$ gets short circuited. The output voltage is given by

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{R} 1-3}=\mathrm{E}_{\mathrm{S} 1-3} \cos \theta \\
& \mathrm{E}_{\mathrm{R} 2-4}=\mathrm{E}_{\mathrm{S} 1-3} \sin \theta
\end{aligned}
$$

(v) When the two stator windings are excited, the output is given by

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{R} 1-3}=\mathrm{E}_{\mathrm{S} 1-3} \cos \theta+\mathrm{E}_{\mathrm{S} 2-4} \sin \theta \\
& \mathrm{E}_{\mathrm{R} 2-4}=\mathrm{E}_{\mathrm{S} 2-4} \cos \theta-\mathrm{E}_{\mathrm{S} 1-3} \sin \theta
\end{aligned}
$$

(vi) When the two rotor windings are excited, we get the following outputs from the stator windings.

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{S} 1-3}=\mathrm{E}_{\mathrm{R} 1-3} \cos -\mathrm{E}_{\mathrm{R} 2-4} \sin \theta \\
& \mathrm{E}_{\mathrm{S} 2-4}=\mathrm{E}_{\mathrm{R} 2-4} \cos +\mathrm{E}_{\mathrm{R} 1-3} \sin \theta
\end{aligned}
$$

Q-8(c): What are the fundamental arm architecture of a basic robot arm on the basis of geometric work envelope? How can these fundamental arm architecture be derived from one another? What arm configurations do Gantry and SCARA robots correspond to ? Also, show the geometric work envelope and arm configuration of Gantry and SCARA robots with a suitable figure.
[20 MARKS]
Sol: A work envelope is generally defined as how far the robot arm's end-effecter mounting plate can reach vertically, horizontally and backward. The dimensions do not include the additional reach guaranteed by tools attached to the robot wrist. Any unreachable area beyond the work envelope is referred as a dead zone. There are six major types of robot configurations: Cartesian, cylindrical, spherical, Selective Compliance Articulated Robot Arm (SCARA), spine configuration and pendulum configuration.

Some work envelopes are flat, confined almost entirely to one horizontal plane.
A robot gantry is an industrial robot with a robotic arm mounted on an overhead rail system or frame. The gantry structure comprises a series of beams or struts that provide the robot with stability and precision to move along the $\mathrm{X}, \mathrm{Y}$ and Z axes. Robot gantries are commonly used in applications where heavy payloads must be lifted and moved over large working area. The robots can be configured to operate in various environments, including clean rooms, hazardous environments, and other specialized applications.

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## Gantry Structure

The gantry structure is the framework that supports the robot arm and the end effector module. It consists of two or more parallel beams that move along the X and Y axes and a vertical column that moves along the Z-axis. The structure must be rigid and stable to ensure accuracy and repeatability in the robot's movements.

The gantry structure can be made of different materials, including aluminum, steel, and carbon fiber, depending on the application arm to perform a specific task. The end effector can be a gripper, a suction cup, a welding gun, a cutting tool, or any other device that can manipulate the part or material being processed.

The material used must be strong enough to support the weight of the robot arm and the end effector while also being lightweight to reduce the overall weight of the gantry.

## Robot Arm

The robot arm is part of the gantry that holds the end effector and moves along the $\mathrm{X}, \mathrm{Y}$ and Z axes. The robot arm's design and specifications depend on the application requirements, such as the reach, payload capacity and speed.

The robot arm can be equipped with different types of end effectors, such as grippers suction cups and welding guns to perform specific tasks.

## Cartesian Gantry

Cartesian gantries, also known as linear robot gantries, are the most commonly used type of robot gantry. They consist of two or more linear axes that move in a straight line along the $\mathrm{X}, \mathrm{Y}$ and Z axes. The robot arm is mounted on a carriage that moves along the gantry structure, allowing it to reach different points within the work envelope.
Cartesian gantries are known for their high accuracy and repeatability, making them ideal for applications that require precise positioning. They are often used in Computer Numerical Control (CNC) machining, 3D printing and pick and place.

## Articulated Gantry

Articulated gantries consist of multiple segments that are connected by joints or links. Each joint can rotate around its axis, giving the robot a greater degree of freedom and flexibility than a Cartesian robot gantry. Articulated gantries are commonly used in applications that require a high level of dexterity, such as pick and place applications, welding and painting.

Articulated gantries can reach any point within their workspace by bending their joints, and they are often used in applications that require complex movements. They are also use robots that can work safely alongside human operators.

## Parallel gantry

Parallel gantries, also known as parallel manipulators or delta robot consist of a series of parallel links that are connected to a fixed base and a moving platform. The robot arm is mounted on the moving platform, and the links are driven by a series of actuators that control the movement of the platform.

Parallel gantries are known for their high payload capacity and stiffness, making them suitable for heavy duty applications such as material handling and assembly. They are often used in the food and beverage industry for packaging and pelletizing operations, where speed and accuracy are critical.

## Hybrid Gantry

Hybrid gantries combine the features of two or more gantry system configurations to create a customized solution for specific applications. For example, a hybrid gantry may combine the high accuracy of a Cartesian gantry with the flexibility of an articulated gantry, or the high payload capacity of a parallel gantry with the precision of a Cartesian gantry.

Hybrid gantries are often used in applications that require a unique combination of features that cannot be achieved with a single gantry type.

SCARA is an acronym for Selective Compliance Assembly Robot arm. One Scara configuration is shown in figure. In Scara, the robot arm has following movements:

1. Linear movement that allows the arm to extend and retract be cause of one orthogonal joint.
2. Rotary movement at the top of the column about the shoulder joint (along vertical axis) because of one revolving joint.
3. Rotary movement at the output arm about the elbow joint (along vertical axis) because of one rotational joint.

