## UPSC ESE 2023

# Mains Exam Solution 

## MECHANICAL ENGINEERING

## Paper-I

## EXAM DATE: 25-06-2023 09:00 AM-12:00 PM

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Q-1(a): (i) Differentiate between rotational and irrotational flows. Can there be any possibility of having zones possessing characteristics of both rotational and irrotational flows? [6 MARKS]

Sol: When fluid particles rotate about their mass center during movement, flow is said to be rotational, otherwise irrotational. Rotation of fluid particle in flow field is caused by viscosity. Hence in viscous region, flow will be rotational. If effect of viscosity effect is negligible, flow can be considered as irrotational.

Existance of limited region of rotational and irrotational flow is possible if region of rotational flow is such that streamlines within it do not penetrate into region outside it.

Q-1(a): (ii) If the expression for the stream function is described by $\Psi=x^{3}-3 x y^{2}$, determine whether the flow is rotational or irrotational. Further find out the correct expression of the velocity potential function of the following two, considering the flow is irrotational:
(1) $\phi=y^{3}-3 x^{2} y$
(2) $\phi=-7 x^{3} y$
[6 MARKS]

Sol: $\quad \Psi=x^{3}-3 x y^{2}$
Determine whether flow is rotational or irrotational
Check for Irrotational Flow,
$\frac{\partial^{2} \psi}{\partial \mathrm{x}^{2}}+\frac{\delta^{2} \psi}{\delta \mathrm{y}^{2}}=0$
$\frac{\partial \psi}{\partial x}=3 x^{2}-3 y^{2}$
$\frac{\partial^{2} \psi}{\partial x^{2}}=6 x$
$\frac{\partial \psi}{\partial y}=-6 x y$
$\frac{\partial^{2} \psi}{\partial y^{2}}=-6 x$
check

$6 x-6 x=0$
Hence, flow is irrotational

Q-1(b): A refrigerated truck whose dimensions are $12 m \times 2.5 m \times 3 m$ is to be precooled from $30^{\circ} \mathrm{C}$ to an average temperature of $5^{\circ} \mathrm{C}$. The construction of the truck is such that a transmission heat gain occurs at the rate of $90 \mathrm{~W} /{ }^{\circ} \mathrm{C}$. If the ambient temperature is $30^{\circ} \mathrm{C}$, determine how long it will take for a system with a refrigeration capacity of 10 kW to precood this truck. The density of air may be taken as $1.2 \mathrm{~kg} / \mathrm{m}^{3}$ and its specific heat at average temperature of $17.5^{\circ} \mathrm{C}$ is $C_{p}=1.0 \mathrm{~kJ} / \mathrm{kg}{ }^{\circ} \mathrm{C}$. State the assumptions, if any.
[12 MARKS]

Sol:

$$
\begin{aligned}
\text { Volume of truck } & =12 \mathrm{~m} \times 2.5 \mathrm{~m} \times 3 \mathrm{~m} \\
\qquad V & =90 \mathrm{~m}^{3} \\
\text { Mass of air in truck } & =90 \mathrm{~m}^{3} \times 1.2 \mathrm{~kg} / \mathrm{m}^{3}=108 \mathrm{~kg} \\
\text { Heat gain inside truck } & =90^{\circ} \mathrm{W} /{ }^{\circ} \mathrm{C} \\
\text { Temperature difference } & =30^{\circ} \mathrm{C}-5^{\circ} \mathrm{C}=25^{\circ} \mathrm{C} \\
\text { Heat gain inside truck } & =90 \times 25 \\
\mathrm{Q} & =2250 \mathrm{~W}
\end{aligned}
$$

This heas is removed from truck by refrigeration system.
$Q=\dot{m} C_{p}(\Delta T)$
$10 \times 1000=\left(\frac{\text { mass }}{\text { time }}\right) \times C_{p} \times \Delta T$
$10 \times 1000=\left(\frac{108}{\Delta t}\right) \times 1000 \times 25 \quad$ (In SI Unit)
$\Delta t=\frac{108 \times 1000 \times 25}{10 \times 1000}=270$ second
Specific heat is taken at average temp $\left(\frac{30+5}{2}\right)$ i. e. $17.5^{\circ} \mathrm{C}$.

Q-1(c): An engine oil flows through a copper tube of 1 cm internal diameter and 0.02 cm wall thickness at the flow rate of $0.1 \mathrm{~kg} / \mathrm{s}$. Consider that the temperature of the oil at the entry is $30^{\circ} \mathrm{C}$. If the oil is heated to $50^{\circ} \mathrm{C}$ by stream condensing at atmospheric pressure, calculate the length of the copper tube. The properties of the oil are as follows:
$C_{p}=1964 \mathrm{~J} / \mathrm{kg}-K, \rho=876 \mathrm{~kg} / \mathrm{m}^{3}, k=0.144 \mathrm{~W} / \mathrm{m}-K$,
$\mu=0.210 \mathrm{~N}-\mathrm{s} / \mathrm{m}^{2}, \operatorname{Pr}=2870$
[12 MARKS]
Sol: Data given:
$\mathrm{C}_{\mathrm{p}}=1964 \mathrm{~J} / \mathrm{Kg}-\mathrm{K}$
$\rho=876 \mathrm{~kg} / \mathrm{m}^{3}$
$\mathrm{k}=0.144 \mathrm{w} / \mathrm{m}-\mathrm{K}$
$\mathrm{d}_{\mathrm{i}}=1 \mathrm{~cm}=0.01 \mathrm{~m}$
$\mathrm{t}=0.02 \mathrm{~cm}=0.02 \times 10^{-2} \mathrm{~m}$
$\mathrm{T}_{\mathrm{e}}=30^{\circ} \mathrm{C}$
$\mathrm{T}_{\mathrm{O}}=50^{\circ} \mathrm{C}$
$\ell=$ ?
$\mu=0.210 \mathrm{~N}-\mathrm{S} / \mathrm{m}^{2}$
$\mathrm{P}_{\mathrm{r}}=2870$
$\dot{\mathrm{m}}_{\mathrm{oil}}=0.1 \mathrm{~kg} / \mathrm{sec}$
$\mathrm{Q}=\dot{\mathrm{m}}_{\text {oil }} \times \mathrm{c}_{\mathrm{p}}\left(\mathrm{T}_{\mathrm{o}}-\mathrm{T}_{\mathrm{e}}\right)=0.1 \times 1964(50-30)=3928 \mathrm{~W}$
$R_{e}=\frac{u_{\infty} D}{V}=\frac{4 \times \dot{m}_{\text {oil }}}{\pi \times d_{i} \times \rho \times \frac{\pi}{\rho}}=\frac{4 \times 0.1}{\pi \times 0.01 \times 0.210}=60.63$
For fully developed flow in tube
$\mathrm{Nu}=3.66$
or $3.66=\frac{\mathrm{hdi}}{\mathrm{k}}=\frac{\mathrm{h} \times 0.01}{0.144}$
$\mathrm{h}=52.704 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
Heat gained by oil due to convective heat transfer $=h A_{s} \times \Delta T$

$$
\begin{align*}
& =52.704 \times \pi\left(d_{i}+2 \mathrm{t}\right) \times \ell \times(50-30) \\
& =52.704 \times \pi\left(0.01+2 \times 0.02 \times 10^{-2}\right) \times \ell \times 20 \\
& =34.43 \mathrm{eW} \quad \quad . . . . .(i i) \tag{ii}
\end{align*}
$$

By equation (i) and (ii)
$3928=34.43 \times \ell$
or $\ell=114 \mathrm{~m}$

Q-1(d): Explain the mechanism of $\mathrm{NO}_{x}$ formation and also the methods for its reduction in stationary gas turbine engines.

Sol: - Exhaust gases of an engine can have up to 2000 ppm of oxides of nitrogen. Most of this will be nitrogen oxide ( NO ), with a small amount of nitrogen dioxide $\left(\mathrm{NO}_{2}\right)$. There will also be traces of other nitrogen-oxygen combinatinos. These are all grouped together $\mathrm{NO}_{x}$, with x representing some suitable number. $\mathrm{NO}_{x}$ is very undesirable. Regulations to reduce $\mathrm{NO}_{x}$ emissions continue to become more and more stringent year by year. Released $\mathrm{NO}_{\mathrm{x}}$ reacts in the atmosphere to form ozone and is one of the major causes of photochemical smog.

- $\quad \mathrm{NO}_{x}$ is created mostly from nitrogen in the air. Nitrogen can also be found in fuel blends. Further, fuel may contain trace amounts of $\mathrm{NH}_{3}, \mathrm{NC}$, and HCN , but this would contribute only a minor degree. There are a number of possible reactions that form NO. All the restrictions are probably occuring during the combustion process and immediately after. These include but are not limited to:

$$
\begin{align*}
& \mathrm{O}+\mathrm{N}_{2} \rightarrow \mathrm{NO}+\mathrm{N}  \tag{i}\\
& \mathrm{~N}+\mathrm{O}_{2} \rightarrow \mathrm{NO}+\mathrm{O}  \tag{ii}\\
& \mathrm{~N}+\mathrm{OH} \rightarrow \mathrm{NO}+\mathrm{H} \tag{iii}
\end{align*}
$$

NO , in turn, can further react to form $\mathrm{NO}_{2}$ by various means, including

$$
\begin{align*}
\mathrm{NO}+\mathrm{H}_{2} \mathrm{O} & \rightarrow \mathrm{NO}_{2}+\mathrm{H}_{2}  \tag{iv}\\
\mathrm{NO}+\mathrm{O}_{2} & \rightarrow \mathrm{NO}_{2}+\mathrm{O} \tag{v}
\end{align*}
$$

- At low temperatures, atmospheric nitrogen exists as a stable diatomic molecule. Therefore, only very small trace amounts of oxides of nitrogen are found. However, at very high temperatures that
occur in the combustino chamber of an engine, some diatomic nitrogen $\left(\mathrm{N}_{2}\right)$ breaks down to monatomic nitrogen $(\mathrm{N})$ which is reactive

$$
\begin{equation*}
\mathrm{N}_{2}+2 \mathrm{~N} \tag{vi}
\end{equation*}
$$

- It may be noted that the chemical equilibrium constant for above equation (vi) is highly dependent on temperature. Singnificant amount of $N$ is generated in the temperature range of 2500-3000 K than can exist in an engine. Other gases that are stable at low temperatures but become reactive and contribute to the formation of NOx at high temperatures include oxygen and water vapor, which break down as follows;

$$
\begin{array}{r}
\mathrm{O}_{2} \rightarrow 2 \mathrm{O} \\
\mathrm{H}_{2} \mathrm{O} \rightarrow \mathrm{OH}+\frac{1}{2} \mathrm{H}_{2} \tag{viii}
\end{array}
$$

- If one goes a little deep into combustion chemistry it can be understood that chemical equation (vi) to (viii) to 14.9 all react much further to the right as high combustion chamber temperatures are reached. The higher the combustion reaction temperature, the more diatomic nitrogen, $\mathrm{N}_{2}$, will dissociate to monatomic nitrogen, N , and the more $\mathrm{NO}_{\mathrm{x}}$ will be formed. At low temperatures very little $\mathrm{NO}_{\mathrm{x}}$ is created.
- Although maximum flame temperature will occur at a stoichiometric airfuel ratio $(\phi=1)$, shows that maximum NOx is formed at a slightly lean equivalence ratio of about $\phi=0.95$. At this condition flame temperature is still very high, and in addition, there is an excess of oxygen that can combine with the nitrogen to form various oxides.
- In addition to temperature, the formation for $\mathrm{NO}_{\mathrm{x}}$ depends on pressure and air-fuel ratio. Combustion duration plays a significant sole in $\mathrm{NO}_{\mathrm{x}}$ formation within the cylinder. The amount of $\mathrm{NO}_{\mathrm{x}}$ generated also depends on the location of spark plug within the comsustion chamber. The highest concentration is formed around the spark plug, where the highest temperatures occur. If spark is advanced, the cylinder temperature will be increased and more $\mathrm{NO}_{\mathrm{x}}$ will be created. Because Cl engines have higher compression ratios and higher temperatures and pressures, they with divided combustion chambers and indirect injection (IDI) tend to generate higher levels of $\mathrm{NO}_{\mathrm{x} \text {. }}$


## Catlytic Converters

- Catalytic convertes are chambers mounted in the flow system through which the exhaust gases pass through. These chambers contain catalytic material, which promotes the oxidation of the emissions contained in the exhaust flow. Generally, they are called three-way converters because they are used to reduce the concentration of $\mathrm{CO}, \mathrm{HC}$, and $\mathrm{NO}_{\mathrm{x}}$ in the exhaust.


## Exhaust Gas Recirculation (EGR)

- The most effective way of reducing NOx emissions is to hold combustion chamber temperatures down. Although practical, this is a very unfortunate method in that it also reduces the thermal efficiency of the engine.
- Probably the simplest and practical method of reducing parasite gas temperature is to dilute the air-fuel mixture with a non-reacting maximum flame. This gas absorbs energy during combustion without contributing any energy input. The net result is a lower flame temperature. Any nonreacting gas would work as a diluents. Those gases with larger specific heats would absorb the most energy per unit mass and would therefore require the least amount; thus less $\mathrm{CO}_{2}$ would be required than
argon for the same maximum temperature. However, neither $\mathrm{CO}_{2}$ nor argon is readily available for use in an engine. Air is available as a diluents but is not totally nonreacting. Adding air changes the air-fuel ratio and combustion characteristics. The one nonreacting gas that is available to use in an engine is exhaust gas, and this is used in all modern automobile and other medium-size and large engines. Adding any non reacting neutral gas to inlet air-fuel mixture reduces flame temperature and $\mathrm{NO}_{\mathrm{x}}$ generation, Exhaust gas (EGR) is the one gas that is readily available for engine use.

Q-1(e): (i) Why are higher heat transfer rates experienced in dropwise condensation that in film condensation?

Sol: Dropwise condensation: The liquid condensate does not wet the condensing surface and collects in droplets. The drops from in cracks and pits on the surface, grow in size, break away from the surface, knock off other droplets and eventually run off the surface without forming a film under the influence of gravity. Typically, more than 90 percent of the surface is covered by drops which may be visible to the naked eye. In this type of condensation, a large portion of the area of solid surface is directly exposed to the vapor, hence there is no film barrier to heat flow and higher heat transfer rates are achieved. Heat transfer rates in dropwise condensation may be as much as ten times higher than in film condensation.

It is worth while to note that, irrespective of whether it is in the form of a film or droplets, the condensate offers a resistance to heat transfer less in dropwise and more in film condenstaion between the vapor and surface. There is an increase of this resistance with condensation between the vapor and surface. There is an increase of this resistance with condenstation thickness with increases in the flow direction and due to this fact, it is desirable to use short vertical surfaces or horizontal cylinders in situations involving film condensation.

Q-1(e): (ii) Distinguish between nucleate boiling and film boiling.
[6 MARKS]

## Sol: <br> Nucleate boiling

It exists in the range, $\Delta \mathrm{T}_{\mathrm{e}, \mathrm{A}} \leq \Delta \mathrm{T}_{\mathrm{e},} \leq \Delta \mathrm{T}_{\mathrm{eC}}$ where $\Delta \mathrm{t}_{\mathrm{e}, \mathrm{C}} \approx 50^{\circ} \mathrm{C}$ and comprises of second and third stage. When liquid is over heated the vapor bubbles are formed at certain favourable spots termed nucleation or active sites; these may be wall surface irregularities, air bubbles and foreign particles. The bubbles grow to a certain size influenced by pressure, temperature and surface tension at the liquid-vapor interface. Depending upon the temperature excess, the bubbles may collapse on the surface, may expand and detach from the surface to be dissipated in the body of the liquid or at sufficiently high; temperature may rise to the surface of liquid before being dissipated. This whole process results in increased value of $h$ and $q_{s}$. During local boiling (i.e. second stage), the primary mechanism of heat transfer is due to intense agitation at the heat transfer surface which creates a high heat transfer rate. In saturated or bulk boiling (i.e. third stage), the bubbles may break way from the surface because of buoyancy action and move into the body of the liquid. Therefore, it is both the agitation caused by bubbles and vapor transport of energy into the body of the liquid influence the heat transfer rate.

## Film Boiling

The comprises of fourth, fifth and sixth states of boiling. Transmition boiling, unstable film boiling or partial film boiling (i.e. fourth (stage) is found for $\Delta T_{e, C} \leq \Delta T_{e,} \leq \Delta T_{e D}$ where $\Delta t_{e, D} \approx 150^{\circ} \mathrm{C}$. In this zone, the

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bubble formation is so rapid that a vapor film or blanket begins to form on the surface. Insulating effect of vapor film (its low thermal conductivity) ignores the beneficial effect of liquid agitation and consequently the heat transfer coefficient and heat flux decrease with the increase in temperature excess $\Delta T_{e}$. At any point on the surface, conditions may oscillate between film and nucleate boiling.

Film boiling (i.e. fifth stage for $\Delta T_{e} \geq \Delta T_{e, D}$ ). In this regime, the surface is completely covered by a vapor blanket, and therefore, heat transfer from the surface to the liquid occurs by conduction and radiation throught vapor. From the minimum surface heat flux associated with point $D$, the heat flux, $q_{s}{ }^{\text {" }}$ increases with the increase of $\Delta \mathrm{T}_{\mathrm{e}}$. It is the sixth stage, radiation through vapor dominates and $\mathrm{q}_{\mathrm{s}}$ " increases rapidly.


Q-2(a): (i) Find the distance from the pipe wall at which the local velocity is equal to the average velocity for turbelent flow in pipe.
[12 MARKS]

Sol:

$$
\frac{u-u_{\text {avg }}}{u^{*}}=5.75 \log _{10}\left(\frac{y}{R}\right)+3.75
$$

For local velocity to be equal to mean velocity

$$
\begin{aligned}
u & =u_{\text {avg }} \\
0 & =5.75 \log _{10}\left(\frac{y}{R}\right)+3.75 \\
y & =0.223 R
\end{aligned}
$$

Q-2(a): (ii) Distinguish between hydrodynamically smooth and rough boundaries.
[8 MARKS]

Sol:

when $\frac{\mathrm{k}_{\mathrm{s}}}{\delta^{\prime}}<0.25$ Hydrodynamic Smooth
$k_{s} \rightarrow$ Average height of roughness projection
$\delta^{\prime} \rightarrow$ Thickness of Laminar Sub layer

$\frac{\mathrm{K}_{\mathrm{s}}}{\delta^{\prime}}>6$ boundary is rough
When thickness of laminar sublayer is large, eddies cannot penetrate up to boundary and boundary may be treated as hydrodynamic smooth.

However, when laminar sublayer thickness is small, eddies penetrate up to boundary, hence boundary is called hydrodynamically rough.

Q-2(b): (i) In a closed system, 3 kg of air at initial conditions of 400 kPa and $90^{\circ} \mathrm{C}$ adiabatically expands until its volume is 2.5 times the initial volume and temperature becomes equal to that of surroundings. If the conditions of the surroundings are 100 kPa and $25^{\circ} \mathrm{C}$, determine the following for this process:
(1) The maximum work
(2) The change in availablity
(3) The irreversibility
[15 MARKS]
Sol: Given data

$$
\begin{aligned}
& \text { mass }(\mathrm{m})=3 \mathrm{~kg} \\
& \mathrm{P}_{\mathrm{i}}=400 \mathrm{kPa}=400000 \mathrm{~Pa} \\
& \mathrm{~T}_{\mathrm{i}}=90^{\circ} \mathrm{C}=90+273=363 \mathrm{~K} \\
& \mathrm{~T}_{\mathrm{f}}=25^{\circ} \mathrm{C}=25+273=298 \mathrm{~K} \\
& \mathrm{P}_{\mathrm{f}}=100 \mathrm{kPa}=100000 \mathrm{~Pa} \\
& \mathrm{C}_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{Kg} \cdot \mathrm{~K} \\
& \mathrm{C}_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{Kg} \cdot \mathrm{~K}
\end{aligned}
$$

$R=287.5 / \mathrm{kg} . \mathrm{K}$

$$
\begin{aligned}
\Delta S=S_{2}-S_{1} & =m R \ln \left(\frac{p_{i}}{p_{f}}\right)+m C_{p} \ln \left(\frac{T_{f}}{T_{c}}\right) \\
& =3 \times 287.5 \times \ln \times\left(\frac{4}{1}\right)+3 \times 1.005 \times \ln \left(\frac{298}{363}\right) \\
& =1195.08 \mathrm{~J} / \mathrm{Kg} \cdot \mathrm{~K} \\
v_{i}= & \frac{m R T_{i}}{p_{i}}=\frac{3 \times 287.5 \times 363}{400000}=0.78 \mathrm{~m}^{3} \\
v_{2}= & 2.5 \theta_{1}=1.956 \mathrm{~m}^{3}
\end{aligned}
$$

(1) The maximim work

$$
\begin{aligned}
\mathrm{W}_{\max } & =\Delta \mathrm{V}+\Delta \mathrm{Q}+\Delta \mathrm{W} \\
& =\mathrm{mC}_{\mathrm{V}}\left(\mathrm{~T}_{\mathrm{i}}-\mathrm{T}_{\mathrm{f}}\right)+\mathrm{T}_{\mathrm{f}}\left(\mathrm{~S}_{2}-\mathrm{S}_{1}\right)+\mathrm{p}_{\mathrm{f}}\left(\mathrm{v}_{1}-\mathrm{V}_{2}\right) \\
& =3 \times 718 \times(363-298)+298 \times 1195.08+100000(0.78-1.956) \\
& =378543.84=378.543 \mathrm{~kJ}
\end{aligned}
$$

(2) The change in availability,

$$
\mathrm{W}_{\max }=378.543 \mathrm{~kJ}
$$

(3) The irreversibility $=Q-T_{f} \Delta s$

$$
\begin{aligned}
& =0-278(-1195.08) \\
& =332.23 \mathrm{~kJ}
\end{aligned}
$$

Q-2(b): (ii) Prove that for an ideal gas, the slope of an isochoric line on the $T$-s diagram is more than that of isobaric line.
[5 MARKS]

## Sol:



We have, for 1 kg of ideal gas

$$
\begin{aligned}
T d s & =d u+p d v \\
& =c_{v} d T+p d v \\
\left(\frac{\partial T}{\partial s}\right)_{v} & =\frac{T}{c_{v}}
\end{aligned}
$$

Also,

$$
\begin{aligned}
T d s & =d h-v d p \\
& =c_{p} d T-v d p \\
\left(\frac{\partial T}{\partial s}\right)_{p} & =\frac{T}{c_{p}}
\end{aligned}
$$

Since,

$$
C_{p}>C_{v}, \frac{T}{C_{v}}>\frac{T}{C_{p}}
$$

This is known in above figure. The slope of the constant volume line passing through point A is steeper than that of the constant pressure line passing through the same point.

Q-2(c): A square plate heater ( $15 \mathrm{~cm} \times 15 \mathrm{~cm}$ ) is inserted between two slabs. Slab A is $\mathbf{2 c m}$ thick ( $k=50 \mathrm{~W} / \mathrm{m}-{ }^{\circ} \mathrm{C}$ ) and slab $B$ is 1 cm thick ( $k=0.2 \mathrm{~W} / \mathrm{m}-{ }^{\circ} \mathrm{C}$ ). The outside heat transfer coefficients on side of $A$ and side of $B$ are $200 \mathrm{~W} / \mathrm{m}^{2}-^{\circ} \mathrm{C}$ and $50 \mathrm{~W} / \mathrm{m}^{2}{ }^{\circ}{ }^{\circ} \mathrm{C}$ respectively. The temperature of surrounding air is $25^{\circ} \mathrm{C}$. If the rating of heater is 1 kW , find the
(i) maximum temperature of the system;
(ii) outer surface temperature of two slabs;

Assume steady-state heat flow
[20 MARKS]

## Sol:



Let maximum temperature of system $t_{\text {max }}=t_{h}=$ at heater
Equivalent circuit is as follows :


1. Maximum temperature of system :

Data given,

$$
\begin{aligned}
& L_{A}=2 \mathrm{~cm}=2 \times 10^{-2} \mathrm{~m} \\
& \mathrm{~L}_{B}=1 \mathrm{~cm}=1 \times 10^{-2} \mathrm{~m} \\
& \mathrm{k}_{\mathrm{a}}=50 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C} \\
& \mathrm{k}_{\mathrm{b}}=0.2 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C} \\
& \mathrm{~h}_{\mathrm{i}}=200 \mathrm{~W} / \mathrm{m}^{2 \circ} \mathrm{C} \text { from slab } \mathrm{A} \text { side } \\
& \mathrm{h}_{0}=50 \mathrm{~W} / \mathrm{m}^{2 \circ} \mathrm{C} \text { from slab } B \text { side }
\end{aligned}
$$

For steady state :

$$
Q_{H}=1000 W=Q_{A}+Q_{B}
$$

or
or

$$
1000=\frac{\left(t_{\mathrm{h}}-t_{\mathrm{a}}\right)}{\frac{L_{A}}{k A}+\frac{1}{\mathrm{~h}_{\mathrm{i}} A}}+\frac{\left(\mathrm{t}_{\mathrm{h}}-t_{\mathrm{a}}\right)}{\frac{L_{B}}{k A}+\frac{1}{h_{0} A}}
$$

$$
1000=\frac{\left(\mathrm{t}_{\mathrm{n}}-25\right)}{15 \times 15 \times 10^{-4}}\left\{\frac{1}{\frac{2 \times 10^{-2}}{50}+\frac{1}{200}}+\frac{1}{\frac{1 \times 10^{-2}}{0.2}+\frac{1}{50}}\right\}
$$

or $\mathrm{t}_{\mathrm{h}}=247.8^{\circ} \mathrm{C} \approx 248^{\circ} \mathrm{C}=\mathrm{t}_{\text {max }}$
(ii) Outer surface temperature of both slab $t_{1}$ and $t_{2}$
or

$$
\frac{248-t_{1}}{\frac{2 \times 10^{-2}}{50}}=200\left(t_{1}-25\right)
$$

or

$$
t_{1}=231.48^{\circ} \mathrm{C}
$$

Similarly,

$$
Q_{B}=\frac{\left(t_{h}-t_{2}\right)}{\frac{L_{B}}{k A}}=h_{a}\left(t_{2}-25\right)
$$

or

$$
\frac{\left(248-\mathrm{t}_{2}\right)}{\frac{1 \times 10^{-2}}{0.2}}=50\left(\mathrm{t}_{2}-25\right)
$$

or

$$
\mathrm{t}_{2}=88.71^{\circ} \mathrm{C}
$$

Q-3(a): A centifugal pump discharges 2000 litres/s of water developing a head of 20 m when running at 300 r.p.m. The impeller diameter at the outlet and outlet flow velocity are 1.5 m and $3.0 \mathrm{~m} / \mathrm{s}$ respectively. If the blades are set back at an angle of $30^{\circ}$ at the outlet, determine the
(i) manometric efficiency;
(ii) power required by the pump;
(iii) minimum speed to start the pump if the inner diameter is $\mathbf{7 5 0} \mathbf{~ m m}$.
[20 MARKS]
Sol:

$$
\begin{aligned}
\mathrm{Q} & =2000 \mathrm{l} / \mathrm{s}=2 \mathrm{~m}^{3} / \mathrm{s} \\
\mathrm{H} & =20 \mathrm{~meter} \\
\mathrm{~N} & =300 \mathrm{rpm} \\
\mathrm{D}_{2} & =1.5 \mathrm{~meter} \\
\mathrm{~V}_{\mathrm{f} 2} & =3.0 \mathrm{~m} / \mathrm{sec}
\end{aligned}
$$

Outlet vane angle $\phi=30^{\circ}$


## From outlet velocity triangle :

$$
\begin{aligned}
\tan \phi & =\frac{\mathrm{V}_{\mathrm{f} 2}}{\mathrm{u}_{2}-\mathrm{V}_{\mathrm{w} 2}} \\
\mathrm{u}_{2} & =\frac{\pi \mathrm{D}_{2} \mathrm{~N}_{2}}{60}=\frac{3.14 \times 1.5 \times 300}{60}=23.55 \mathrm{~m} / \mathrm{sec} \\
\tan 30^{\circ} & =\frac{3.0}{\left(23.55-\mathrm{V}_{\mathrm{w} 2}\right)} \\
\left(23.55-\mathrm{V}_{\mathrm{w} 2}\right) & =\frac{3.0}{\tan 30^{\circ}} \\
23.55-\mathrm{V}_{\mathrm{w} 2} & =5.196 \\
\mathrm{~V}_{\mathrm{w} 2} & =23.55-5.196=18.354 \mathrm{~m} / \mathrm{sec}
\end{aligned}
$$

(i) Manometric efficiency :

$$
\begin{aligned}
& \eta_{\text {mano }}=\frac{\mathrm{gH}_{\mathrm{w}}}{\mathrm{~V}_{\mathrm{w} 2} \mathrm{u}_{2}}=\frac{9.81 \times 20}{18.354 \times 23.55} \\
& \eta_{\text {mano }}=0.4539
\end{aligned}
$$

(ii) Power Required by :

$$
\begin{aligned}
P & =\rho Q V_{w 2} u_{2} \\
& =1000 \times 2 \times 18.354 \times 23.55=864473 \mathrm{~W} \\
P & =864.473 \mathrm{~kW}
\end{aligned}
$$

(iii) Minimum speed to start the pump :

$$
\begin{aligned}
N & =\frac{120 \times \eta_{\text {mano }} \times V_{w 2} \times D_{2}}{p\left(D_{2}^{2}-D_{1}^{2}\right)} \\
& =\frac{120 \times 0.453 \times 18.354 \times 1.5}{\pi\left(1.5^{2}-0.75^{2}\right)}=282.441 \mathrm{rpm}
\end{aligned}
$$

Q-3(b): Air flows at $12 \mathrm{~m} / \mathrm{s}$ past a smooth rectangular flat plate 0.4 m wide and 3 m long. Assuming that the transition occurs at $R e=5.5 \times 10^{5}$, calculate the total drag force when
(i) the flow is parallel to the length of the plate;
(ii) the flow is parallel to the width of the plate.

Assume
Density of air, $\rho=1.24 \mathrm{~kg} / \mathrm{m}^{3}$
Kinematic viscosity, $v=0.15$ stokes
[20 MARKS]

Sol: (i) Flow is parallel to length of plate :


First let identify Laminar/Turbulent region

$$
\begin{aligned}
\operatorname{Re} & =5.5 \times 10^{5}=\frac{\rho V x}{v}=\frac{V x}{v} \\
5.5 \times 10^{5} & =\frac{12 \times x}{0.15 \times 10^{-4}} \\
x & =0.6875 \mathrm{~m}
\end{aligned}
$$

Drag force in laminar region
$\frac{1}{2} \rho A V^{2} \times C_{D}$

$$
C_{D}=\frac{1.328}{\sqrt{\operatorname{Re}}}=\frac{1.328}{\sqrt{5.5 \times 10^{5}}}=0.00179
$$

$\frac{1}{2} \times 1.24 \times(0.687 \times 0.4) \times(12)^{2} \times 0.00179$

$$
F_{D}=0.0439 \mathrm{~N}
$$

Drage force in Turbulent Region :

$$
\mathrm{F}_{\mathrm{BC}}=\left(\mathrm{F}_{\mathrm{AC}}\right)_{\text {Turbulent }}-\left(\mathrm{F}_{\mathrm{AB}}\right)_{\text {Turbulent }}
$$

For AC

$$
\begin{aligned}
& C_{D}=\frac{0.072}{\left(\operatorname{Re}_{L}\right)^{1 / 5}}=\frac{0.07}{\left(\frac{12 \times 4}{0.15 \times 10^{-4}}\right)^{1 / 5}}=\frac{0.072}{20} \\
& C_{D}=0.0036
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{F}_{\mathrm{AC}} & =\frac{1}{2} \rho A V^{2} \times C_{D} \\
& =\frac{1}{2} \times 1.24 \times(3 \times 0.4) \times(12)^{2} \times(0.0036) \\
\mathrm{F}_{\mathrm{AC}} & =0.385 \text { Newton }
\end{aligned}
$$

For AB:
so,

$$
\begin{aligned}
C_{D} & =\frac{0.072}{\left(55 \times 10^{5}\right)^{1 / 5}}=\frac{0.72}{14.062} \\
C_{D} & =0.0051 \\
F_{A B} & =\frac{1}{2} \times 1.24 \times(0.687 \times 0.4) \times(12)^{2} \times 0.0051=0.125 \mathrm{~N} \\
F_{B C} & =0.385-0.125=0.260 \mathrm{~N}
\end{aligned}
$$

$$
\text { Total Drag Force }=\left(\mathrm{F}_{\mathrm{D}}\right)_{\text {Laminar }}+\left(\mathrm{F}_{\mathrm{D}}\right)_{\text {Turbulent }}
$$

$$
(\text { on one side })=0.0439+0.260=0.303 \mathrm{~N}
$$

(ii) Flow is parallel to width of plate :


Here, entire flow is in olaminar region.

$$
\begin{aligned}
C_{D} & =\frac{1.328}{\sqrt{R_{\mathrm{eL}}}}=\frac{1.328}{\sqrt{\frac{12 \times 0.4}{0.15 \times 10^{-4}}}=\frac{1.328}{565.685}} \\
C_{D} & =0.0023 \\
\text { Drag Force, } F_{D} & =C_{D} \times \frac{1}{2} \rho V^{2} \times A \\
& =0.0023 \times \frac{1}{2} \times 1.24 \times(12)^{2} \times(0.4 \times 3) \\
F_{D} & =0.246 \mathrm{~N}
\end{aligned}
$$

Q-3(c): Two tanks, tank A and tank B, are separated by a partition as shown in the figure. Tank A contain 3 kg of steam 1 MPa and $300^{\circ} \mathrm{C}$. Tank B contains 4 kg of saturated liquid-vapour mixture at $150^{\circ} \mathrm{C}$ with a dryness fraction of 0.5. The partition is removed and two fluids are allowed to mix until the thermal equilibrium and mechanical equilibrium are acquired. If the pressure of the final state is 300 kPa, determine -
(i) the temperature of the final state;
(ii) the quality of the steam at final state;
(iii) the amount of heat lost from the tanks.

[20 MARKS]
Sol:

| Tank A | Tank B |
| :---: | :---: |
| 3 kg | 4 kg |
| 1 MPa | $150^{\circ} \mathrm{C}$ |
| $300^{\circ} \mathrm{C}$ | $\mathrm{x}=0.5$ |

From steam table
Internal energy of tank at initial state

$$
u_{1 \mathrm{~A}}=2793.7 \mathrm{~kJ} / \mathrm{Kg}
$$

Specific volume of tank A at initial stage

$$
v_{1 \mathrm{~A}}=0.257 \mathrm{~m}^{3} / \mathrm{Kg} .
$$

For tank $B$ at $150^{\circ} \mathrm{C}$ saturated state

$$
\begin{aligned}
v_{1 B} & =v_{f}+x v_{y} \\
& =0.001091+0.5 \times(0.39248-0.001081) \\
& =0.19679 \mathrm{~m}^{3} \mathrm{Kg}
\end{aligned}
$$

Similarly, internal energy of saturated liquid vapor in tank B with the help of steam table and thermodynamic relation is

$$
\begin{aligned}
u_{1 B} & =u_{f}+x u_{f g} \\
& =631.66+0.5 \times 1927.4=1595.4 \mathrm{~kJ} / \mathrm{kg} \\
\mathrm{~V} & =M_{A} \times v_{1} A+M_{B} \times v_{1 B} \\
& =3 \times 0.25799+4 \times 0.19679=1.56113
\end{aligned}
$$

Total mass of the steam :

$$
\mathrm{m}=\mathrm{m}_{\mathrm{A}}+\mathrm{m}_{\mathrm{B}}=3+4=7 \mathrm{~kg}
$$

So, specific volume of final state,

$$
v_{2}=\frac{\mathrm{V}}{\mathrm{~m}}=\frac{1.56113}{7}=0.2230
$$

(i) Temperature of the final state is at $\mathrm{P}=300 \mathrm{kPa}$
(iii)

$$
\text { Drynetes fraction, } x_{1}=\frac{v_{2}-v_{f}}{v_{g}-v_{f}}=\frac{0.2230-0.001073}{0.6058-0.001073}=0.367
$$

So, internal energy of system correspond to final state is

$$
u_{2}=u_{f}+x_{1} u_{\mathrm{fg}}=561.11+(0.367 \times 1982.1)=1288.54 \mathrm{~kJ} / \mathrm{Kg}
$$

(iii) Head loss from the system

$$
\begin{aligned}
\mathrm{Q} & =\Delta \mathrm{U}_{\mathrm{A}}+\Delta \mathrm{U}_{\mathrm{B}}=\mathrm{m}_{\mathrm{A}} \times\left(\mathrm{u}_{2}-\mathrm{u}_{1 \mathrm{~A}}\right)+\mathrm{m}_{\mathrm{B}} \times\left(\mathrm{u}_{2}-\mathrm{u}_{1 \mathrm{~B}}\right) \\
& =3 \times(1288.54-2793.7)+4 \times(1288.54-1595.4) \\
\mathrm{Q} & =-4515.48-1227.44=-5742.32 \mathrm{KJ}
\end{aligned}
$$

Q-4(a): A truncated cone has top and bottom diameters of 10 cm and 20 cm respectively, and a height of 10 cm . Calculate the shape factor between the top surface and the side, and also the shape factor between the side and itself. Use the figure showing the radiation shape factor for radiation between two parallel coaxial disks:

[20 MARKS]

Sol:


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For surface (1),

So,

$$
\begin{align*}
\mathrm{F}_{11}+\mathrm{F}_{12}+\mathrm{F}_{13} & =1 \\
\mathrm{~F}_{11} & =0 \\
\mathrm{~F}_{12}+\mathrm{F}_{13} & =1 \tag{i}
\end{align*}
$$

Similarly for top surface (2),

$$
\begin{align*}
\mathrm{F}_{21}+\mathrm{F}_{23} & =1  \tag{ii}\\
\frac{\mathrm{~L}}{\mathrm{r}_{\mathrm{b}}} & =\frac{10}{10}=1 \text { and } \frac{r_{t}}{\mathrm{~L}}=\frac{5}{10}=0.5
\end{align*}
$$

By these two value,

$$
F_{12}=0.1
$$

By reciprocity theorem,

$$
\begin{aligned}
A_{1} F_{12} & =A_{2} F_{21} \\
\mathrm{~F}_{21} & =\frac{\mathrm{A}_{1} \mathrm{~F}_{12}}{\mathrm{~A}_{2}}=\frac{\pi \mathrm{r}_{\mathrm{b}}^{2} \times \mathrm{F}_{12}}{\pi \mathrm{r}_{\mathrm{t}}^{2}}=\frac{\pi \times 10^{2} \times 0.1}{\pi \times 5^{2}}=0.4
\end{aligned}
$$

So,

$$
F_{21}+F_{23}=1
$$

or,

$$
F_{23}=1-F_{21}=1-0.4=0.6
$$

So, the shape factor for top surface to side surface.

$$
F_{23}=0.6
$$

The shape factor for bottom surface is given by

$$
\begin{aligned}
F_{12}+F_{13} & =1 \\
F_{13} & =1-F_{12}=1-0.1=0.9
\end{aligned}
$$

By reciprocity theorem,

$$
\begin{aligned}
A_{1} F_{13} & =A_{3} F_{31} \\
F_{31} & =\frac{A_{1} \times F_{13}}{A_{3}}
\end{aligned}
$$

Area of curved surface (3) $=\pi \times\left(r_{b}+r_{t}\right) \times\left[\left(r_{b}-r_{t}\right)^{2}+L^{2}\right]^{0.5}$

$$
=\pi \times(10+5)\left[(10-5)^{2}+10^{2}\right]^{0.5}=526.86 \mathrm{~cm}^{2}
$$

So,

$$
\mathrm{F}_{31}=\frac{\mathrm{A}_{1} \mathrm{~F}_{13}}{\mathrm{~A}_{3}}=\frac{\pi r_{\mathrm{b}}^{2} \times \mathrm{F}_{13}}{\mathrm{~A}_{3}}=\frac{\pi \times 10^{2} \times 0.9}{526.86}
$$

or,

$$
F_{31}=0.54
$$

Similarly,

$$
\mathrm{F}_{32}=\frac{\mathrm{A}_{2} \times \mathrm{F}_{32}}{\mathrm{~A}_{3}}=\frac{\pi \times 5^{2} \times 0.6}{526.86}=0.09
$$

Now,

$$
\begin{aligned}
\mathrm{F}_{31}+\mathrm{F}_{32}+\mathrm{F}_{33} & =1 \\
\mathrm{~F}_{33} & =1-\mathrm{F}_{31}-\mathrm{F}_{32}=-1-0.54=0.09=0.37
\end{aligned}
$$

i.e. shape for side surface to itself is 0.37

Q-4(b): A Francis turbine supplied through an 8.0 m diameter penstock has the following particulars:
Output power = 65000 kW
Speed $=150$ r.p.m
Hydraulic efficiency $=90 \%$
Flow rate $=120 \mathrm{~m}^{3} / \mathrm{s}$
Mean diameter of turbine at entry $=5 \mathrm{~m}$
Mean blade height at entry $=1.5 \mathrm{~m}$
Entry diameter of draft tube $=4.5 \mathrm{~m}$
Velocity in tailrace $=2.5 \mathrm{~m} / \mathrm{s}$
The static pressure head in the penstock measured just before entry to the runner is 60 m . The point of measurement is $3.2 \mathbf{m}$ above the level of the tailrace. The loss in the draft tube is equivalent to $30 \%$ of the velocity head at entry to it. The exit plane of the runner is 2 m above the tailrace and the flow leaves the runner without swirl. Calculate:
(i) The overall efficiency.
(ii) THe direction of flow relative to the runner at inlet.
(iii) The pressure head at entry to draft tube.
[20 MARKS]
Sol: Data given,
Output power $=65000 \mathrm{~kW}$
$\mathrm{N}=150$ r.p.m
$\eta_{\text {nydro }}=90 \%$
$Q=120 \mathrm{~m}^{3} / \mathrm{s}$
Mean diameter, $D_{1}=5 \mathrm{~m}$
Mean blade height, $B_{1}=1.5 \mathrm{~m}$
Entry dia of draft tube, $\mathrm{d}_{1}=4.5 \mathrm{~m}$
Velocity of trailrace, $\mathrm{V}_{\text {trailrace }}=2.5 \mathrm{~m} / \mathrm{s}$
$H_{s}=60 \mathrm{~m}, \mathrm{~h}=3.2 \mathrm{~m}, \mathrm{z}=2 \mathrm{~m}$
(i) Velocity in Penstock $\mathrm{V}_{\mathrm{p}}=\frac{4 \mathrm{Q}}{\pi \mathrm{D}_{\mathrm{p}}^{2}}=\frac{4 \times 120}{\pi \times 8^{2}}=2.38 \mathrm{~m} / \mathrm{s}$

Velocity at entry of draft tube
$\mathrm{V}_{2}=\frac{4 \mathrm{Q}}{\pi \mathrm{d}_{1}^{2}}=\frac{4 \times 120}{\pi \times(4.5)^{2}}=7.54 \mathrm{~m} / \mathrm{s}$
Head before entry to runner
$H=H_{s}+\frac{V_{p}^{2}}{2 g}+h=60+\frac{2.38^{2}}{2 \times 9.81}+3.2=63.48 \mathrm{~m}$
Effective head $H_{e}=H-\frac{V_{\text {trailrace }}^{2}}{2 g}=63.48-\frac{2.5^{2}}{2 \times 9.81}=63.16 \mathrm{~m}$
Overall efficiency, $\quad \eta_{0}=\frac{P}{\rho Q g H_{e}}$
$=\frac{65 \times 10^{3} \times 10^{3}}{9.81 \times 10^{3} \times 120 \times 63.16}=87.42 \%$
(ii) $\quad \eta_{\text {hydro }}=\frac{V_{w_{1}} u_{1}}{g H e}$
$u_{1}=\frac{\pi D_{1} \mathrm{~N}}{60}=\frac{\pi \times 5 \times 150}{60}=39.26 \mathrm{~m} / \mathrm{s}$
$\mathrm{Vw}_{1}=\frac{0.92 \times 9.81 \times 63.16}{39.26} \ldots \ldots$ From (i)

$$
=14.51 \mathrm{~m} / \mathrm{s}
$$

$\tan \beta_{1}=\frac{V_{f_{1}}}{V w_{1}-U_{1}}$
$V_{f_{1}}=\frac{Q}{\rho D_{1} B_{1}}=\frac{120}{\pi \times 5 \times 1.5}=5.09 \mathrm{~m} / \mathrm{s}$
from equation (ii)

$$
\begin{aligned}
& \tan \beta_{1}=\frac{5.09}{14.51-39.26} \\
& \Rightarrow \beta_{1}=-11.62^{\circ} \text { or } 168.37^{\circ}
\end{aligned}
$$

(iii) Applying Bernoulli's equation between entry to draft tube and tailrace
$\frac{p_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+Z_{2}=0+\frac{V_{\text {tairrace }}^{2}}{2 g}+0.3 \frac{V_{2}^{2}}{2 g}$
$\frac{p_{2}}{\rho g}=\frac{V_{\text {tailrace }}^{2}}{2 g}-0.7 \frac{V_{2}^{2}}{2 g}-z_{2}$
$=\frac{2.5^{2}}{2 \times 9.81}-0.7 \times \frac{7.5^{2}}{2 \times 9.81}-2$
$=-3.688 \mathrm{~m}$

Two containers are connected with a pipe having a closed valve. One container contains a 5 kg mixture of $62.5 \% \mathrm{CO}_{2}$ and $37.5 \% \mathrm{O}_{2}$ on a mole basis $30^{\circ} \mathrm{C}$ and 125 kPa . The second container contains 10 kg of $\mathrm{N}_{2}$ at $15^{\circ} \mathrm{C}$ and 200 kPa . The valve in the pipe is opened and gases are allowed to mix. During the mixing process, 100 kJ of heat energy is supplied to the combined tank. Determine the volume of the mixture and write an energy balance equation.
[20 MARKS]
Sol: Under specified condition $\mathrm{CO}, \mathrm{N}_{2}$ and $\mathrm{O}_{2}$ can be treated as gases and the mixture as an ideal gas mixture.


The molar mass of $\mathrm{CO}_{2}, \mathrm{NO}_{2}$ and $\mathrm{O}_{2}$ are $44.0,28.0$ and $32.0 \mathrm{~kg} / \mathrm{kmol}$ respectively.
The gas constants of $\mathrm{CO}_{2}, \mathrm{~N}_{2}$ and $\mathrm{O}_{2}$ are $0.1889,0.2968,0.2598 \mathrm{KJ} / \mathrm{KgK}$ respectively.
Analysis :
The molar mass of the mixture in tank 1 are
$\mathrm{y}_{\mathrm{CO}_{2}} \mathrm{M}_{\mathrm{CO}_{2}}+\mathrm{y}_{\mathrm{O}_{2}} \mathrm{M}_{\mathrm{O}_{2}}=0.625 \times 44+0.375 \times 32=39.5 \mathrm{~kg} / \mathrm{kmol}$
The gas constant in tank 1 is

$$
\mathrm{R}_{1}=\frac{\mathrm{R}_{\mathrm{u}}}{\mathrm{M}_{\mathrm{m}}}=\frac{8.314 \mathrm{~kJ} / \mathrm{kmol} . \mathrm{K}}{39.5 \mathrm{~kJ} / \mathrm{kmol}}=0.2104 \mathrm{~kJ} / \mathrm{kgK}
$$

The volume of the tanks and the total volume are :

$$
\begin{align*}
V_{1} & =\frac{M_{1} R_{1} T_{1}}{P_{1}} \\
& =\frac{(5 \mathrm{~kg})(0.2104 \mathrm{~kJ} / \mathrm{kgK})(30+273) \mathrm{K}}{(125 \mathrm{kPa})}=2.551 \mathrm{~m}^{3} \\
V_{2} & =\frac{M_{2} R_{2} T_{2}}{P_{2}}=\frac{(10 \mathrm{~kg})(0.2968 \mathrm{~kJ} / \mathrm{kgK})(15+273) \mathrm{K}}{(200 \mathrm{kPa})} \\
V_{\text {Total }} & =V_{1}+V_{2}=2.551 \mathrm{~m}^{3}+4.276 \mathrm{~m}^{3} \\
V_{\text {Total }} & =6.828 \mathrm{~m}^{3}  \tag{i}\\
M_{\mathrm{FCO}_{2}} 1 & =\frac{y_{\mathrm{CO}_{2}} \mathrm{M}_{\mathrm{CO}_{2}}}{M_{m}}=0.625 \times \frac{44 \mathrm{~kg} / \mathrm{kmol}}{39.5 \mathrm{~kg} / \mathrm{kmol}}=0.6963 \\
M_{\mathrm{FO}_{2}} 1 & =\frac{\mathrm{y}_{\mathrm{O}_{2}} M_{\mathrm{O}_{2}}}{M_{m}}=0.375 \times \frac{32 \mathrm{~kg} / \mathrm{kmol}}{39.5 \mathrm{~kg} / \mathrm{kmol}}=0.3037
\end{align*}
$$

The masses in tank 1 and the total mass after mixing are :

$$
\begin{aligned}
\mathrm{M}_{\mathrm{CO}_{2}} 1 & =\mathrm{M}_{\mathrm{fCO}_{2}} 1 \cdot \mathrm{~m}_{1}=0.6963 \times 5 \mathrm{~kg}=3.481 \mathrm{~kg} \\
\mathrm{M}_{\mathrm{O}_{2}} 1 & =\mathrm{M}_{\mathrm{fO}_{2}} 1 \cdot \mathrm{~m}_{1}=0.3037 \times 5 \mathrm{~kg}=1.519 \mathrm{~kg} \\
\mathrm{M}_{\text {total }} & =\mathrm{m}_{1}+\mathrm{m}_{2}=5+10=15 \mathrm{~kg}
\end{aligned}
$$

The mass fraction of the combined mixture are :

$$
\begin{aligned}
\mathrm{M}_{\mathrm{FCO}_{2}} 2 & =\frac{\mathrm{M}_{\mathrm{CO}_{2}} 1}{\mathrm{M}_{\text {total }}}=\frac{3.481}{15}=0.2321 \\
\mathrm{M}_{\mathrm{FO}_{2}} 2 & =\frac{\mathrm{M}_{\mathrm{O}_{2}} 1}{\mathrm{M}_{\text {total }}}=\frac{1.519}{15}=0.1012 \\
\mathrm{M}_{\mathrm{FN}_{2}} 2 & =\frac{\mathrm{m}_{1}}{\mathrm{~m}_{\text {total }}}=\frac{10}{15}=0.6667
\end{aligned}
$$

It is given that temperature of mixture is $39.4^{\circ} \mathrm{C}$ i.e. $\mathrm{T}_{\text {mix }}=39.4^{\circ} \mathrm{C}=312.4 \mathrm{~K}$.
The gas constant of the final mixture 3 .
$R_{\text {mix }}=M_{\mathrm{FCO}_{2}} \cdot \mathrm{R}_{\mathrm{CO}_{2}}+\mathrm{M}_{\mathrm{FO}_{2}} \cdot \mathrm{R}_{\mathrm{O}_{2}}+\mathrm{M}_{\mathrm{FN}_{2}} \cdot \mathrm{R}_{\mathrm{N}_{2}}$
$R_{\text {mix }}=(0.231)(0.1889)+(0.1012(0.2598)+(0.6667)(0.2968)$

$$
\mathrm{R}_{\text {mix }}=0.2680 \mathrm{~kJ} / \mathrm{kgK}
$$

The final pressure is determine from the gas relation to be

$$
\begin{aligned}
\mathrm{P}_{\text {mix }} & =\frac{\mathrm{M}_{\text {tota }} \mathrm{R}_{\text {mix }} \mathrm{T}_{\text {mix }}}{\mathrm{V}_{\text {total }}}=\frac{(15 \mathrm{~kg})(0.2680 \mathrm{~kJ} / \mathrm{kgK})}{\left(6.828 \mathrm{~m}^{3}\right)}(312.4 \mathrm{~K}) \\
\mathrm{P}_{\text {mix }} & =184 \mathrm{kPa} \\
& \text { SECTION -B }
\end{aligned}
$$

Q-5(a): A six-cylinder SI engine operates on a four-stroke cycle. The bore of each cylinder is 75 mm and the stroke is 100 mm . The clearnace volume per cylinder is 60 cc, At a speed of 4000 r.p.m, the fuel consumption is $18 \mathrm{~kg} / \mathrm{h}$ and the torque developed is $140 \mathrm{~N}-\mathrm{m}$. Calculate the
(i) brake thermal efficiency
(ii) relative efficiency on the basis of brake power

The calorific value of the fuel can be taken as $45000 \mathrm{~kJ} / \mathrm{kg}$.
[12 MARKS]
Sol: Given data,
bore, $(\mathrm{d})=75 \mathrm{~mm}=0.075 \mathrm{~m}$
Stroke $(\mathrm{L})=100 \mathrm{~mm}=0.1 \mathrm{~m}$
$V_{c}=60 \mathrm{cc}=60 \times 10^{-6} \mathrm{~m}^{3}$
Speed, $N=4000$ r.p.m
Fuel consumption $=18 \mathrm{~kg} / \mathrm{h}=0.005 \mathrm{~kg} / \mathrm{s}$
$\mathrm{T}=140 \mathrm{~N} . \mathrm{m}$

No. of cylinder, $k=6$
$r=2$ for four stroke
Stroke power, B.P $=\frac{2 \pi \mathrm{NT}}{60}=\frac{2 \pi \times 84000 \times 140}{60}=58643=58.64 \mathrm{~kW}$
(i) Brake thermal efficiency,

$$
\begin{aligned}
\eta_{b t} & =\frac{B . P}{\text { Power of fuel burnt }} \\
& =\frac{B . P}{m_{f} \times C_{v}} \\
& =\frac{58643}{0.005 \times 45000 \times 10^{3}} \\
& =26.06 \%
\end{aligned}
$$

(ii) Relative efficiency, $\eta_{r}=\frac{\eta_{b t}}{\eta_{\text {airstandard }}}$
air standard efficiency, $\eta_{\text {airstd }}=1-\frac{1}{\left(r_{c}\right)^{\gamma-1}}$
Compression ratio, $r_{c}=\frac{\frac{\pi}{4} \times d^{2} \times L+V_{c}}{V_{c}}=\frac{V_{s}+V_{c}}{V_{c}}$

$$
=\frac{\frac{\pi}{4} \times(0.075)^{2} \times 0.1+\left(60 \times 10^{-6}\right)}{\left(60 \times 10^{-6}\right)}=8.36
$$

so, $\eta_{\text {airstd }}=1-\frac{1}{(8.36)^{(1.4-1)}}=57.23 \%$
So, relative efficiency, $\eta_{r}=\frac{26.06}{57.23}=45.53 \%$

Q-5(b): Draw the T-s and h-s diagrams for steam jet refrigeration system and write the expressions for the following:
(i) Nozzle efficiency
(ii) Entrainment efficiency
(iii) Compression efficiency
[12 MARKS]
Sol: The steam jet refrigeration system (also known as ejector refrigerator system) is one of the oldest methods of producing refrigerating effect. The basic component of this system are an evaporator, a compression device, a condenser and a refrigerant control device. This system employes a steam ejector or booster (instead of mechanical compressor) to compress the refrigerant to the required condenser pressure level. In this system, water is used as the refrigerant. SInce the freezing point of water of $0^{\circ} \mathrm{C}$, therefore , it cannot be used for applications below $0^{\circ} \mathrm{C}$. The steam jet refrigeration system is widely used

## ESE 2023 Mains Exam Solution MECHANIGAL ENGINEERING - Paper I

in food processing plants for precooling of vegetables and concentrating fruit juices, gas plants, paper mills, breweries etc.

## Analysis of steam jet refrigeration system

The temperature-entropy ( $\mathrm{T}-\mathrm{s}$ ) and enthalpy-entropy ( h -s) diagrams for a steam jet refrigeration system are shown in the figures (a) and (b) respectively.


The point A represents the initial condition of the motive steam before passing through the nozzle and the point $B$ is the final condition of the stream, assuming isentropoic expansion. The point $C$ represents the initial condition of the water vapour in the flash chamber or evaporator and the point E is the condition of the mixture of high velocity steam from the nozzle and the entrained water vapour before compression. Assuming isentropic compression, the final condition of the mixture discharged to the condenser is represented by point F . The condition of motive steam just before mixing with the water vapour is shown at point $D$. The make-up water is supplied at point $G$ whose temperature is slightly lower than the condenser temperature and is throttled to poing H in the flash chamber.

## Efficiencies used in steam jet refrigeration system

The various efficiencies used in stream jet refrigeration system are discussed below:

1. Nozzle efficiency: It is defined as the ratio of actual enthalpy drop to the isentropic enthalpy drop of the motive steam passing through the nozzle. Mathematically, nozzle efficiency,

$$
\eta_{N}=\frac{\text { Actual enthalpy drop }}{\text { Isentropic enthalpy drop }}=\frac{A P}{A B}=\frac{h_{A}-h_{B^{\prime}}}{h_{A}-h_{B}}
$$

The nozzle efficiency may vary from 85 to 90 percent
2. Entrainment efficiency. The water vapours formed in the flash chamber or evaporator comes out with a very low velocity as compared to the velocity of the stream $(\mathrm{V})$ coming out of the nozzle which is given by

$$
V=\sqrt{2000\left(h_{A}-h_{B}^{\prime}{ }^{\prime}\right)}=44.72 \sqrt{h_{A}-h_{B}^{\prime}}
$$

The expression $\left(h_{A}-h_{B}{ }^{\prime}\right)$ represents the kinetic energy of the motive steam. This kinetic energy gives the required momentum to the water vapours coming out of the flash chamber or evaporator. The process of giving the momemtum to the water vapour formed in the flash chamber by the high velocity stream is called entrainment of vapour. During the entrainment of water vapour from the flash chamber, the motive steam loses some of its kinetic energy. This process of entrainment is inefficient and part of the original motive force available for compression is reduced. This is taken into consideration by a factor known as entrainment efficiency. Mathematically, entrainment efficiency.

$$
\eta_{E}=\frac{h_{A}-h_{D}}{h_{A}-h_{B}{ }^{\prime}}
$$

The entrainmment efficiency may be taken as 65 percent.
3. Compression efficiency: It is detailed as the ratio of the isentropic enthalpy increase to the actual enthalpy increase required for the compression of the mixture of motive steam and the water vapours, in the diffuser. Mathematically, compression efficiency.

$$
\eta_{c}=\frac{\text { Isentropic enthalphy increase }}{\text { Actual enthalpy increase }}=\frac{h_{F}-h_{E}}{h_{F}-h_{E}}
$$

The compression efficiency may be taken as 75 to 80 percent.
Q-5(c): Briefly describe a natural draught cooling tower. Explain why it is hyperbolic in shape.
[12 MARKS]
Sol: Natural Draught Cooling Towers: There is no fan used by natural draught colling tower. They depend for air flow upon the natural driving pressure caused by the difference in density between the cool outside air and the hot, humid air inside.

Air + water vapour


The driving pressure differential is expressed as $\Delta p_{d}=\left(\rho_{0}-\rho_{i}\right) g H$
Where $\quad H=$ height of tower above the fill (m)
$\rho_{0}$ and $\rho_{i}=$ density of outside and inside air respectively $\left(\mathrm{m}^{3} / \mathrm{kg}\right)$
Since $\left(\rho_{0}-\rho_{i}\right)$ is relatively small, so $H$ must be large to cause desired $\Delta p_{d}$ and as a result natural draught cooling tower are therefore very tall. It is to be noted that the tower body, above the water distribution system and the fill, is an empty shell of circular cross-section but with vertical hyperbolic
profile and due to this natural draught towers are called hyperbolic towers. The advantage of natural draught hyperbolic colling tower include its superior strength and greatest resistance to outside wind loading compared to other shape. The natural draught may be counter-flow or cross-flow type. In counterflow the fill is inside whereas in cross-flow, the fill sits in a ring outside the tower outside the stilts

## Q-5(d): Distinguish among the following:

## (i) Renewable energy

(ii) Green energy
(iii) Clean energy

Also, mention the relative environmental effects of the above.
[12 MARKS]

## Sol: Renewable energy

We define renewable energy as electricity produced from natural sources that either replenish themselves over short periods - or never run out at all. When most of us hear the term renewable energy, we often think of solar and wind - but the name also encompasses many other forms of generation such as hydro (water) geothermal (heat), and biomass (plant and waste material).

While renewable energy systems use natural resources as fuel, they can still affect the environment. Large scale hydropower, can have an environmental impact due to its land use, effects on marine life and deforestation. Despite these challenges, hydropower still produces energy from a renewable source and creates no carbon emissions, making it for better for the atmosphere than traditional oil and gas. When we buy renewable energy - such as through a power retailer or purchasing renewable energy certificates - we are having a positive impact on the environment.

## Green Energy -

Green energy is a subset of renewable energy and represents generation sources with the smallest environmental footprint - such as sunlight, wind, heat and water. The name can also include low-impact hydroelectric sources and specific forms of biomass.
In addition to reversing climate change, many green energy sources are also less expensive than those created using fossil fuels. For example, large scale solar and wind 0 - which have substantially lowered in cost over the last ten years - are now cheaper than coal fired power stations in more than $50 \%$ of areas around the world.

Green energy creates no carbon emissions and has the lowest impact on the environment. Relative to fossil fuel energy sources, it vastly improves the quality of the air we breathe, the water we drink and conserves many of your precious natural resources.

## Clean Energy -

Clean energy is electricity that does not create any greenhouse gases during its production - although it is not necessarily renewable.
And so, while all forms of green and renewable energy are also "clean energy", so is nuclear power as it does not create any carbon emissions or pollutants during generation. May forms of bio-gas made from organic matter, household waste, and manure are also regarded as clean energy, although they may not always be completely renewable.
All clean energy sources are a significant improvement over fossil-fuels and create no greenhouse gasses. And in cases such as biofuel and landfill gas capture, clean energy production can even solve multiple problems at the same time.

While there are subtle differences between the three terms, they each represent the future of our energy supply. As we accelerate towards a carbon-free world, no single form of sustainable electricity will meet all of our needs; we require a combination of different clean, green and renewable energy sources.
For individuals as well as companies, there are many ways we can make a difference. We can invest in renewable energy, purchase carbon credits, work to improve energy efficiency and reduce waste.

Ultimately creating zero emission societies will require a combination of green energy solutions that each have a positive environmental impact. By reduction our consumption, using cleaner forms of energy and rethinking the way we live work and move - we can create a truly sustainable future.

Clean energy is produced through means that do not release greenhouse gases or any other pollutants to the environment. Clean energy can be produced from renewable sources but the term is not synonymous with renewable energy i.e. not all renewable energy sources are clean. For example, some of the ways by which geothermal energy, which his renewable is processed have negative impacts on the environment. On the other hand, green energy refers to energy from renewable sources i.e. for an energy source to be green, it must be renewable.

Q-5(e): Describe the emission norms for indian vehicles if they have to comply with Bharat State (BS) Emission Standards-VI. Mention the devices and technology introduced to meet the BS-Vi norms.
[12 MARKS]
Sol: While the BS4 emission norms allow motor vehicle to emit not more than $80 \mathrm{mg} / \mathrm{km}$ of $\mathrm{NO}_{2}$ (nitrogen oxides). The BS6 emission norms have reduced it to $60 \mathrm{mg} / \mathrm{km}$. The particulate Matter (PM) for patrol vehicles have been restricted to $4.5 \mathrm{mg} / \mathrm{km}$ in the BS6 emission regime.

Automakers generally adopt a two-prolonged approach which involves on the engine and the after treatment, in terms of the cylinder measures, enhanced combustion chamber and fuel injector design will refine the combustion process and result in finer atomization of fuel. And, on the other hand, release of pollutants like PM and NOx is arrested using exhaust treatment system.

Direct injection patrol and diesel engines rely on particulate filters to reduce the emission of PM or soot into the atmosphere. A higher level of NOx - a challenge especially with diesel engines - is tackled either by employing a Lean NOx Trap (LNT) or a Selective Catalytic Reduction (SCR) system.

As the name suggest, LNT works by filtering out NOx content from the exhaust gases SCR on the other hand, injects a water based urea solution (AdBlue) into the exhaust flow to reduce the nitrogen oxide emission into inert nitrogen and water. Though more effective than LNT, SCR's complex and expensive setup limits its applications. Jeep and Mercedes are two automakers that have introduced SCR based diesel vehicles in the Indian market.

For 2 wheelers, the carburetor, an intrinsic part of a vehicle will now be replaced by an electronic fuel injection. AS compared to BS4 (the emission norms being followed before BS6), Nitrogen Oxides (NOx) emission will not reduce by $25 \%$ for patrol vehicles.

Q-6(a): A gasoline engine has a stroke volume of $0.002 m^{3}$ and a compression ratio of 6 . At the end of the compression stroke, the pressure is 10 bar and the temperature is $400^{\circ} \mathrm{C}$. Ignition is set so that the pressure rises along a straight line during combustion and attains its highest value of 30 bar after the piston has travelled (1/40) of the stroke. The charge consists of a gasoline-air mixture in proportion of 1:18 by mass. Calculate the heat lost per kg of charge during combustion. Take $R=287 \mathrm{~J} / \mathrm{kg}-\mathrm{k}$, calorific value of the fuel $=45 \mathrm{MJ} / \mathrm{kg}, C p=1 \mathrm{~kJ} / \mathrm{kg}$.
[20 MARKS]

## Sol:


$V_{1}-V_{2}=0.002 \times 10^{6}=2000 \mathrm{CC}$
$V_{2}=\frac{2000}{6-1}=400 \mathrm{CC}$
$V_{3}=\frac{2000}{40}+400=450 \mathrm{CC}$
$\mathrm{T}_{2}=400+273=673 \mathrm{~K}$
$\mathrm{T}_{3}=\mathrm{T}_{2} \frac{\mathrm{p}_{3} \mathrm{~V}_{3}}{\mathrm{p}_{2} \mathrm{v}_{2}}=673 \times \frac{30}{10} \times \frac{450}{400}=2271.37 \mathrm{k}$
In order to estimate the head added to the mixture during the process $2 \rightarrow 3$ it is required to calculate the work done and the increase in internal energy between 2 and 3 .
$\mathrm{W}_{2-3}=\left(\frac{30+10}{2}\right) \times 10^{5} \times(450-400) \times 10^{-6}$
Area of the trapezoid $2^{\prime} 23^{\prime} 3=100 \mathrm{~J}$
mixture mass, $m=\frac{p v}{R T}$

$$
\begin{aligned}
& =\frac{10 \times 10^{5} \times 400 \times 10^{-6}}{287 \times 673} \\
& =2.07 \times 10^{-3} \mathrm{~kg}
\end{aligned}
$$

$\Delta \mathrm{E}=\mathrm{E}_{3}-\mathrm{E}_{2}$
$=2.07 \times 10^{-3} \times(1-0.287)(2271.37-673)=2.36 \mathrm{~kJ}$
$\mathrm{Q}=0.1+2.36=2.46 \mathrm{~kJ}$
This is the quantity of heat actually given to the mixture in one cycle. But the heat liberated in one cycle must have been
$\frac{1}{19} \times 2.07 \times 10^{-3} \times 45000=4.902 \mathrm{~kJ}$

Heat lost during explosion $=4.902-2.46$

$$
=2.442 \mathrm{~kJ}
$$

Heat lost in $\mathrm{kJ} / \mathrm{kg}=\frac{2.442}{2.07 \times 10^{-3}}$

$$
=1180 \mathrm{~kJ} / \mathrm{kg} \text { of charge }
$$

Q-6(c): The angles at inlet and discharge of the blading of a $50 \%$ reaction turbine are $35^{\circ}$ and $20^{\circ}$ respectively. The speed of rotation is 1500 r.p.m and at a particular stage, the mean ring diameter is $0.67 \mathbf{m}$ and the steam condition is at $\mathbf{1 . 5}$ bar, 0.96 dry. Determine:
(i) the required height of blading to pass $3.6 \mathrm{~kg} / \mathrm{s}$ of steam;
(ii) the power developed by the ring.
[20 MARKS]
Sol: Given;

$$
\beta_{4}=35^{\circ}=\delta, \beta_{2}=\alpha=20^{\circ}
$$



By since law,

$$
\frac{V_{1}}{\sin 145^{\circ}}=\frac{V_{b}}{\sin 15^{\circ}}=\frac{V_{r 1}}{\sin 20^{\circ}}
$$

$$
V_{1}=52.62 \times \frac{0.5736}{0.2588}=116.63 \mathrm{~m} / \mathrm{s}=V_{r_{2}}
$$

$$
V_{r 1}=52.62 \times \frac{0.342}{0.2588}=69.54 \mathrm{~m} / \mathrm{s}=\mathrm{V}_{\mathrm{r} 2}
$$

$$
\Delta \mathrm{V}_{\mathrm{w}}=\mathrm{V}_{1} \cos \alpha+\mathrm{V}_{2} \cos \beta_{1}=116.63 \cos 20^{\circ}+69.54 \cos 35^{\circ}
$$

$$
=109.6+56.964=165.567 \mathrm{~m} / \mathrm{s}
$$

$$
v_{1}=0.001052+0.96 \times 1.15937=1.114 \mathrm{~m}^{3} / \mathrm{kg}
$$

$$
\omega_{1}=3.6 \mathrm{~kg} / \mathrm{s}
$$

$$
=\frac{\pi \mathrm{D}_{\mathrm{m}} \mathrm{~h}_{\mathrm{b}} \mathrm{~V}_{1} \sin \alpha}{\mathrm{v}_{1}}=\frac{\pi \times 0.67 \times \mathrm{h}_{\mathrm{b}} \times 116.63 \sin 20^{\circ}}{1.114}
$$

$$
h_{b}=0.0478 \mathrm{~m}=47.8 \mathrm{~mm}
$$

$$
=\omega_{s} \Delta V_{w} V_{b}=3.6 \times 166.564 \times 52.62 \times 10^{-3}
$$

$$
=31.552 \mathrm{~kW}
$$

Q-7(a): The following data refer to a boiler unit consisting of an economizer, a boiler and a superheater:
Mass of water evaporated per hour $=5940 \mathrm{~kg}$
Mass of coal burnt per hour $=\mathbf{6 7 5} \mathbf{~ k g}$
Lower calorific value of coal $=31600 \mathrm{~kJ} / \mathrm{kg}$
Pressure of steam at boiler stop valve = 14 bar
Temperature of feedwater entering economizer $=32^{\circ} \mathrm{C}$
Temperature of feedwater leaving economizer $=115^{\circ} \mathrm{C}$
Dryness fraction of steam leaving boiler and entering superheater $=0.96$
Temperature of steam leaving sueprheater $=260^{\circ} \mathrm{C}$
Specific heat of sueprheater steam $=2.3 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$
Determine the following:
(i) Percentage of heat in coal utilized in economizer, boiler and superheater
(ii) Overall efficiency of the boiler unit

Assume specific heat of water $=4.187 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$
[20 MARKS]
Sol: Data given, Mass of water evaporated per hour $=5940 \mathrm{~kg}$
Mass of coal burnt $=675 \mathrm{~kg}$
Lower calorific value of coal $=31600 \mathrm{~kJ} / \mathrm{kg}$
Pressure of steam at boiler stop valve, $p_{1}=14$ bar
Temperature of feedwater entering economizer, $\mathrm{t}_{\mathrm{e} 1}=32^{\circ} \mathrm{C}$
Temperature of feedwater leaving economizer, $\mathrm{t}_{\mathrm{e} 1}=115^{\circ} \mathrm{C}$
Dryness fraction of steam leaving boiler and entering superheater $=0.96$
Temperature of steam leaving sueprheater, $\mathrm{t}_{\text {sup }}=260^{\circ} \mathrm{C}$
Specific heat of sueprheater steam $=2.3 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$
Head utilised by 1 kg of feedwater in economiser,

$$
\begin{aligned}
\mathrm{h}_{f_{1}} & =1 \times 4.18 \times\left(\mathrm{t}_{\mathrm{e}_{2}}-\mathrm{t}_{\mathrm{e} 1}\right) \\
& =1 \times 4.18 \times(115-32) \\
& =346.94 \mathrm{~kJ} / \mathrm{KG}
\end{aligned}
$$

Heat utilized in boiler per kg of feed water

$$
h_{\text {boiler }}=\left(h_{f}+x h_{f g}\right)-h_{f 1}
$$

At 14 boil temperature, from stream table

$$
\begin{aligned}
\mathrm{t}_{\mathrm{s}} & =195^{\circ} \mathrm{C} \\
\mathrm{~h}_{\mathrm{f}} & =830.1 \mathrm{~kJ} / \mathrm{kg} \\
\mathrm{~h}_{\mathrm{fg}} & =1957.7 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

$h_{\text {boiler }}=(830.1+0.96 \times 1957.7)-346.94$

$$
=2362.55 \mathrm{~kJ} / \mathrm{kg}
$$

heat utilised in super heater by 1 kg of feed water

$$
\begin{aligned}
\mathrm{h}_{\text {superheater }} & =(1-\mathrm{x}) \mathrm{h}_{\mathrm{fg}}+\mathrm{C}_{\mathrm{ps}}\left(\mathrm{t}_{\text {sup }}-\mathrm{t}_{\mathrm{s}}\right) \\
& =(1-0.96) \times 1957.7+2.3 \times(260-195) \\
& =227.80 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

Also, mass of water evaporated / hour/ kg of coal burnt $=\frac{5940}{675}=8.8 \mathrm{~kg}$
(i) \% of heat in coal utilised in economiser
$=\frac{346.9}{31600} \times 8.8 \times 100=9.66 \%$
\% of heat in coal utilized in boiler
$=\frac{2362.55}{31600} \times 8.8 \times 100$
= 65.79\%
$\%$ of heat in coal utilized in superheater
$=\frac{227.80}{31600} \times 8.8 \times 100$
$=6.34 \%$
(ii) Overall efficiency of boiler unit

Total heat absorbed in kg of water
$=h f_{1}+h_{\text {boiler }}+h_{\text {superheater }}$
$=346.94+2362.55+227.80$
$=2937.24 \mathrm{~kJ} / \mathrm{kg}$
$\eta_{\text {overall }}=8.8 \times \frac{2937.24}{31600}=81.72 \%$

Q-7(b): (i) Explain the various factors affecting anaerobic digestion process. Why do anaerobic microbes normally grow at a much lower rate than aerobic bacteria?
[10 MARKS]
Sol: Anaerobic digestion is the breakdown of organic matter by the microbes under the absence of oxygen. it comprises four stages: Hydrolysis, Acidogenesis, Acetogenesis, and methanogenesis. The syntropic nature of this process makes each statge uniquely important. For example, acidogeneis is a pre-requisite for methanogenesis, but if high amounts of acids are formed during the acidogenesis stage, it will affect the methanogens and hinder digestion. During the process, biogas is produced as a by-product, with
methane and carbon dioxide as major constituents. Due to the high energy potential of methane, anaerobic digestion of solid wastes is widely studied by researcheers all over the world. Various factors affecting anaerobic digestion were indentified over the years during the studies. The main factors included in this chapter are $\mathrm{C} / \mathrm{N}$ ratio, $\mathrm{F} / \mathrm{M}$ ratio. pH . temperature, Organic loading rate, hydraulic loading rate, presence of toxins (inherent component, or a by-product of the metabolism). Each of these nevertheless depends heavilly on the type of substrate and inoculum employed. In the field conditions, it is seldom practical to maintain all these factors, few of these being uncontrollable in nature. Hence, it is often the trade-off between the factors that could be brought withing the desirable range. C/N ratio, OLR, and Detention time belong to this group. Mixing of the feedstock is yet another aspect that affects the efficacy of the digestion process. Anaerobes tend to grow slower than obligate aerobes because their energy yield from oxidizing organic molecules is smaller than that of aerobes.

Q-7(b): (ii) A family biogas plant is required to be designed to utilize the cow dung of five cows. The hydraulic retention time is 30 days. The temperature of the digester is to be maintained at $30^{\circ} \mathrm{C}$. The dry matter consumption per day is 2 kg . The biogas yield is $0.25 \mathrm{~m}^{3} / \mathrm{kg}$. The effciency of the burner is $60 \%$. The heat of combustion of methane is $26 \mathrm{MJ} / \mathrm{m}^{3}$. The methane proportion is $70 \%$. The density of feedstock material may be taken as $50 \mathrm{~kg} / \mathrm{m}^{3}$. Find (1) the volume of biogas digester and (2) its thermal power.
[10 MARKS]

## Sol:

Date given:
No. of cows $=5$
HRT $=30$ days
$C V=26 \mathrm{MJ} / \mathrm{m}^{3}$
Dry matter consumption $=2 \mathrm{~kg} /$ day
Biogas yield $=0.25 \mathrm{~m}^{3} / \mathrm{kg}$
$\eta_{\mathrm{b}}=0.6$
Proportion of city in biogas $=0.7$
Density $P=50 \mathrm{~kg} / \mathrm{m}^{3}$
(i) Volume of biogas digestion:

Mass of dry input per day $=2 \times 5=10 \mathrm{~kg} /$ day
Volume of fluid in digester,

$$
\begin{aligned}
V_{f} & =\frac{m(\mathrm{~kg} / \text { day })}{\text { density }} \\
& =\frac{10}{50}=0.2 \mathrm{~m}^{3} / \text { day }
\end{aligned}
$$

Digestion volume $=V_{f} \times H R T=0.2 \times 30=6 \mathrm{~m}^{3}$
Volume of biogas $\mathrm{V}_{\mathrm{b}}$ = Biogas yield per kg of dry input $\times$ mass of dry input
$=0.25 \times 10=2.5 \mathrm{~m}^{3} /$ day
(ii) Thermal power:

$$
\begin{aligned}
E & =\eta_{b} \times C_{v} \times \text { volume of biogas } \times \text { factor of methane } \\
& =0.6 \times\left(2.26 \times 10^{3} \mathrm{~kJ} / \mathrm{m}^{3}\right) \times 2.5 \times 0.7 \\
& =27.3 \times 10^{3} \mathrm{~kJ} / \text { day } \\
& =\frac{27.3 \times 10^{3}}{24 \times 3600} \\
& =0.3159 \mathrm{~kW}
\end{aligned}
$$

Q-7(c): (i) A refrigeration system with R-22 as refrigerant operates with an evaporating temperature of $-10^{\circ} \mathrm{C}$ and a condensing temperature of $35^{\circ} \mathrm{C}$. If the vapour leaves the evaporator saturated and is compressed isentropically, what is the COP of the cycle -(1) if saturated liquid enters the expansion device and (2) if the refrigerant entering the expansion device is with $10 \%$ vapour?
[10 MARKS]

## Sol:



Entry to expansion device (3) is saturated liquid.
From chart (approximate)
$\mathrm{h}_{1}=400 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{h}_{2}=4.0 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{h}_{3}=240 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{h}_{4}=240 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{COP}=\frac{\mathrm{h}_{1}-\mathrm{h}_{4}}{\mathrm{~h}_{2}-\mathrm{h}_{1}}=\frac{400-240}{430-400}=5.3$
Entry to expansion devices is $10 \%$ vapor $90 \%$ liquid.

## From chart :

$$
\begin{aligned}
\mathrm{h} & \longrightarrow \\
h_{1} & =400 \mathrm{~kJ} / \mathrm{kg} \\
h_{2} & =430 \mathrm{~kJ} / \mathrm{kg} \\
h_{3} & =h_{f}+0.1\left(h_{g}-h_{f}\right) \\
h_{3} & =240+0.1(415-240)=257.5=h_{4} \\
C O P & =\frac{h_{1}-h_{4}}{h_{2}-h_{1}}=\frac{400-257.5}{430-400} \\
C O P & =4.75
\end{aligned}
$$

Q-7(c): (ii) What is a liquid-to-suction heat exchanger in refrigeration and air conditioning? Illustrate the benefits of liquid-to-suction heat exchanger.
[10 MARKS]
Sol: Liquid Suction Heat Exchanger :
An LSHX is a counter flow heat exchanger installed between the outlet of condenser and inlet of compressor. In LSHX the hot refrigerant at the outlet of the condenser exchanges heats with cooled refrigerant at the outlet of the evaporator.



## A liquid-to-suction heat exchanger is beneficial in the following ways:

1. It provides subcooling to the liquid refrigerant prior to it entering the expansion valve. This eliminates the possibiilty of 'flash gas' forming in the liquid line and enables the expansion valve to operate with greater stability.

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2. The added subcooling which is provided to the liquid regrigerant increases the efficiency of the system.
3. The heat which is transferred from the liquid to the suction increases the superheat of the suction gas thereby reducing the possibility of liquid 'floodback' to the compressor. Liquid floodback isconsidered one of the major result in improved compressor reliability.

## Q-8(a): (i) Describe the working principle of hydrogen fuel cell. Also, comment on the reversible energy conversion efficiency of fuel cells. <br> [10 MARKS]

## Sol: Operation Principle of Fuel Cells:

A fuel cell is an electro-chemical device in which the chemical energy of fuel is continuously converted into electric energy. This conversion of energy takes place at constant pressure and temperature.

The main components of a hydrogen $\left(\mathrm{H}_{2}\right)$ - oxygen $\left(\mathrm{O}_{2}\right)$ fuel cell are:
(1) A fuel electrode (anode)
(2) An oxidant electrode (cathode)
(3) An electrolyte (a solution of $\mathrm{H}_{2} \mathrm{SO}_{4}$ for acidic fuel cell and KOH for alkaline fuel cells)
(4) Additional components are container, separators, sealings, fuel and oxidant supply etc.

The basic feature of a fuel cell is that the fuel and the oxidant are combined in the form of ions than in form of neutral molecules.

Fig. represents the schematic diagram of a fuel cell using hydrogen as fuel and oxygen as oxidant and alkaline solution of KOH as electrolyte.


A hydrogen-oxygen $\left(\mathrm{H}_{2}-\mathrm{O}_{2}\right)$ fuel cell
It is called alkaline fuel cell (AFC). It consists of two permeable Nickel electrodes immersed in an electrolyte of good conductivity. An electrolyte may be an alkaline solution of KOH as shown or acidic solution of $\mathrm{H}_{2} \mathrm{SO}_{4}$ called respectively as alkali fuel cells and acidic fuel cells.

## Consistency in results since over a decade



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The porous fuel electrode is anode (negative pole) and the other porous oxidant electrode is cathode (positive pole). These electrodes are separated by a porous gas barrier called separator (not shown in Fig.)
The anode is supplied $\mathrm{H}_{2}$ gas as fuel at a certain pressure and the cathode is supplied $\mathrm{O}_{2}$ as oxidant at a pressure. These gases pass through the respective electrodes and bubble around through the electrolyte solution. The pores provide an opportunity to gases, electrodes and electrolyte to come in contact for their electrochemical reactions. The electrodes are connected through an external circuit as shown.

The electro-chemical reactions are generally slow and a catalyst is required in the electrodes to accelerate the reaction. Platinum is the best catalyst but costly. In general, less expensive catalysts like nickel and silver are used according to application and design.

## Chemical reactions with alkaline $\mathrm{H}_{2}-\mathrm{O}_{2}$ fuel cell:

The hydrogen gas is ionized at anode and it produces a free electron and $\mathrm{H}_{+}$ions. Every hydrogen molecules brought to electrode surface is dissociated into two H atoms by catalytic property of electrode. These enter into electrolyte solution as hydrogen ions leaving behind two electrons which pass through the external circuit to the cathode (positive electrode) The reaction at anode is as follows:

Anode:

$$
\begin{equation*}
\mathrm{H}_{2} \rightarrow 2 \mathrm{H}^{+}+2 \mathrm{e} \tag{i}
\end{equation*}
$$

The oxygen supplied to cathode (positive electrode) reacts with water of electrolyte and the electrons transmitted to it to produce hydroxyl $\left(\mathrm{OH}^{-}\right)$ions. Thus,

Cathode :

$$
\begin{equation*}
\frac{1}{2} \mathrm{O}_{2}+\mathrm{H}_{2} \mathrm{O}+2 \mathrm{e} \rightarrow 2 \mathrm{OH}^{-} \tag{ii}
\end{equation*}
$$

These hydroxyl ions migrate from cathode to anode through electrolyte. The hydrogen and hydroxyl ions then combine in the electrolyte to produce water i.e.

$$
\begin{equation*}
2 \mathrm{H}^{+}+2 \mathrm{OH}^{-} \rightarrow 2 \mathrm{H}_{2} \mathrm{O} \tag{iii}
\end{equation*}
$$

Above equation shows that OH ions produced at one electrode (cathode) are involved in a reaction at the other electrode (anode). By adding the above three equations, the overall process is chemical reaction of $\mathrm{H}_{2}$ and $\mathrm{O}_{2}$ gases to form water i.e.

$$
\begin{equation*}
\mathrm{H}_{2}+\frac{1}{2} \mathrm{O}_{2} \rightarrow \mathrm{H}_{2} \mathrm{O} \tag{iv}
\end{equation*}
$$

Thus the net reaction of a fuel cell in which hydrogen and oxygen supplied is the produce water, electrical energy and heat.

Chemical reaction with acidic electrolyte $\left(\mathrm{H}_{2} \mathrm{SO}_{4}\right)$ :
Anode:

$$
\mathrm{H}_{2} \rightarrow 2 \mathrm{H}^{+}+2 \mathrm{e} \ldots(\mathrm{v})
$$

Cathode :

$$
\begin{align*}
\frac{1}{2} \mathrm{O}_{2}+2 \mathrm{H}^{+}+2 \mathrm{e} & \rightarrow \mathrm{H}_{2} \mathrm{O}  \tag{vi}\\
\mathrm{H}_{2}+\frac{1}{2} \mathrm{O}_{2} & \rightarrow \mathrm{H}_{2} \mathrm{O} \tag{vii}
\end{align*}
$$

Overall reaction :

## Conversion Efficiency of Fuel Cells

The electrical energy generated by a fuel cell depends on what is called the "free" energy rather than on the heat energy, of the overall cell reaction. The free energy of formation of 1 mole (18 grams) of liquid
water from hydrogen and oxygen gases at atmospheric pressure is 56.67 kcal or 237 kJ at $25^{\circ} \mathrm{C}$. The heat energy (or enthalpy) of the reaction under the same conditions is $68.26 \mathrm{kcal}(286 \mathrm{~kJ})$. The theoretical efficiency of th conversion of heat energy into electrical energy in a hydrogen-oxygen fuel cell is thus (56.67/68.26) $\times 100=83$ percent.

Efficiencies as high as 70 percent have been observed, but the practical cells using pure hydrogen and oxygen generally have conversion efficiencies in the range of 50 to 60 percent. The efficiencies are some what lower when air is the source of oxygen. The overall thermal to electrical conversion efficiencies are also lower when the hydrogen is derived from hydrocarbon sources. Nevertheless, they should be higher than those obtainable from the same fuels is most steam-electric plants.

The theoretical emf (or voltage) of a fuel cell can be calculated from the reaction free energy. For the hydrogen-oxygen cell at $25^{\circ} \mathrm{C}$, with the gases at atmospheric pressure, the ideal emf. is 1.23 volts; at $200^{\circ} \mathrm{C}$, it is about 1.15 volts. The discharge voltages observed in actual cell are always below the theoretical value, the difference increasing with increasing strength of the current drawn from the cell (Refer to the figure)


For the moderate currents at which fuel cells normally operate the emf is 0.7 to 0.8 volts. This deviation from the theoretical emf accounts for the conversion efficiency of a fuel cell being below the ideal maximum value.
The departure of a fuel cell from ideal behaviour arises from several factors one is the inherent slowness of the electrode reactions: this is dominant at low current drains. It can be reduced by an effective electrochemical catalyst and by increasing the operating temperature. At larger currents there is an additional contribution from the electrical resistance of teh electrolyte (multiplied by the current strength). A low resistance (i.e. high conductivity) electrolyte is therefore desirable.
Even in an ideal hydrogen-oxygen cell, $100-83=17$ percent of the chemical reaction energy (enthalpy) would be liberated as heat. the proportion is incresed in an actual cell because the conversion efficiency is less than the maximum of 83 percent. In order to avoid an excessive pemperature rise, heat is removed from the fuel cell during operation. Possible ways of doing this are by the flow of excess air past the positive electrode or by circulating the electrolyte through an external cooler.
In some fuel cell designs it is proposed to utilized the heat to provide speace heating of a building and to supply hot water. The heat released in high temperature cells might be used for industrial pusposes (i.e. process heat) or to generate stream.

Q-8(a): (ii) A flat plate solar collector measuring $2 \mathrm{~m} \times 1.2 \mathrm{~m}$ has a loss resistance of $0.13 \mathrm{~m}^{2} \mathrm{~K} / \mathrm{W}$ and a plate transfer efficiency of 0.85 . The glass cover has transmittance of 0.9 and the absorptance of the plate is also 0.9 . Water enters at a temperature of $35^{\circ} \mathrm{C}$. The ambient temperature is $20^{\circ} \mathrm{C}$ and the irradiance in the plane of the collector is $750 \mathrm{~W} / \mathrm{m}^{2}$. Calculate the flow rate needed to produce a temperature rise of $10^{\circ} \mathrm{C}$. The density of water and its specific heat at mean film temperature may be taken as $1000 \mathrm{~kg} / \mathrm{m}^{3}$ and $4.2 \mathrm{~J} / \mathrm{g}-{ }^{\circ} \mathrm{C}$ respectively.
[10 MARKS]

Sol: Data given:
Plate area $=A_{p}=2 \times 1.2=2.4 \mathrm{~m}^{2}$
Loss resistence $=0.13 \mathrm{~m}^{2} \mathrm{k} / \mathrm{w}=\mathrm{R}_{l}$
Transmittance of plate $=\tau=0.9$
Plate transfer efficiency $=0.85=\eta_{\text {pf }}$
absorptance $=0.9=\alpha$
Inlet temp. of water $=35^{\circ} \mathrm{C}=\mathrm{T}_{\text {s }}$
Ambient temp. $=20^{\circ} \mathrm{C}=\mathrm{T}_{\mathrm{a}}$
$\mathrm{G}=750 \mathrm{w} / \mathrm{m}^{2}$
$\Delta T=10^{\circ} \mathrm{C}$
Radiant heat flux striking the plate
$Q_{r}=T \times A_{p} \times G=0.9 \times 2.4 \times 750$
$Q_{\mathrm{r}}=1620$
Net heat flow into the plate:
$\mathrm{Q}_{\mathrm{n}}=\mathrm{Q}_{\mathrm{r}} \times \alpha-\left[\frac{\mathrm{T}_{\mathrm{s}}-\mathrm{Ta}}{\mathrm{R}_{\ell}} \times \mathrm{A}_{\mathrm{p}}\right]$
or, $Q_{n}=1620 \times 0.9-\left[\frac{35-20}{0.13} \times 2.4\right]$

$$
=1181.076 \mathrm{~W}
$$

Useful power from the collector
$\mathrm{Q}_{\mathrm{u}}=\eta_{\mathrm{pf}} \times \mathrm{Q}_{\mathrm{n}}$
$L$ or $Q_{n}=0.85 \times 1181.076=1003.91 \mathrm{~W}$
Now $Q_{u}=m_{c p} \Delta T=\dot{m} \times C_{p} \times\left(T_{2}-T_{1}\right)$
or $1003.91=\dot{\mathrm{m}} \times 4.2 \times 10^{3} \times 10$
or $\dot{m}=0.0239 \mathrm{~kg} / \mathrm{sec}$
flow rate $=\frac{\dot{m}}{\rho}=2.39 \times 10^{-5} \mathrm{~m}^{3} / \mathrm{hr}$

$$
=0.086 \mathrm{~m}^{3} / \mathrm{hr}
$$

Q-8(b): A two-pass surface condenser is required to handle the exhaust from a turbine developing 15 MW with specific steam consumption of $5 \mathrm{~kg} / \mathrm{kWh}$. The condenser vacuum is $\mathbf{6 6 0 \mathrm { mm } \text { of mercury when }}$ the barometer reads 760 mm of mercury. The mean velocity of water is $3 \mathrm{~m} / \mathrm{s}$ and the water inlet tempeature is $24^{\circ} \mathrm{C}$. The condensate is saturated water and the outlet quality of exhaust steam is 0.9 dry . The overall heat transfer coefficient based on outer area of tubes is $4000 \mathrm{~W} / \mathrm{m}^{2-}{ }^{\circ} \mathrm{C}$. The

(i) Mass of cooling water circulated in $\mathrm{kg} / \mathrm{min}$
(ii) Condenser surface area
(iii) Number of tubes required per pass
(iv) Tube length

Assume atmospheric pressure to be 760 mm of mercury or 1.01325 bar and specific heat of water $=4.187 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$.
[20 MARKS]
Sol: Two tube pass surface
Power (P) = $15 \mathrm{MW}=15 \times 10^{3} \mathrm{~kW}$
Specific steam consumption $\dot{\mathrm{m}}_{\mathrm{s}}=5 \mathrm{~kg} / \mathrm{kWh}$
Barometer reads $\mathrm{p}_{\mathrm{atm}}=760 \mathrm{~mm}$ of Hg
Condenser vaccum $P_{g}=-660 \mathrm{~mm}$ of Hg
Water inlet temperature $=\mathrm{T}_{\mathrm{ci}}=24^{\circ} \mathrm{C}$
Condenstate is saturated water and outlet temperature of cooling water is $4^{\circ} \mathrm{C}$ less than the condenstate temperature.
$T_{c a}=T_{h o}-4^{\circ} \mathrm{C}$
Quality of exhaust steam $=x=0.9$
overall heat transfer coefficient, $\mathrm{u}_{0}=4000 \mathrm{w} / \mathrm{m}^{2} \mathrm{C}$
Outer diameter of water tube, $d_{0}=38.4 \mathrm{~mm}$
Inner diameter of water tube, $d_{i}=29.6 \mathrm{~mm}$
The absolute pressure

$$
\begin{aligned}
P_{\text {abs }} & =P_{a t m}+P_{g} \\
& =760-660 \mathrm{~mm} \text { of } \mathrm{Hg} \\
& =100 \mathrm{~mm} \text { of } \mathrm{Hg}
\end{aligned}
$$

$$
\therefore \mathrm{h}=100
$$

$$
\therefore P_{\mathrm{abs}}=\rho g h
$$

$$
=13.6 \times 10^{-3} \times 4.31 \times \frac{100}{100}
$$

$$
=13341.6 \mathrm{~Pa}
$$

$$
\Rightarrow P_{\text {abs }}=0.133 \text { bar (or) }
$$

$$
\mathrm{P}_{\mathrm{abs}}=13.34 \mathrm{kPa}
$$

$\mathrm{P}=0.133 \mathrm{bar}$
$T_{h}=T_{\text {sat }}=51^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{fg}}=2592 \mathrm{~kJ} / \mathrm{kg}$
$\therefore \mathrm{T}_{\mathrm{co}}=51-4=47^{\circ} \mathrm{C}$
mass of steam condensed per minute

$$
\begin{aligned}
\mathrm{m}_{\mathrm{n}} & =\mathrm{p} \times \mathrm{m}_{\mathrm{s}} \\
& =15 \times 10^{3} \times 5 \\
& =75000 \mathrm{~kg} / \mathrm{h} \\
& =\frac{75000}{60} \\
& =1250 \mathrm{~kg} / \mathrm{min}
\end{aligned}
$$

Heat rejected by steam per minute

$$
\begin{aligned}
Q & =m_{h} \times(\text { xhfg }) \\
& =20.83 \times(0.9 \times 2592) \\
& =48600 \mathrm{~kW}
\end{aligned}
$$

$\therefore$ mass of water circulated $\left(\mathrm{m}_{\mathrm{c}}\right)$
By energy balance
$Q=m_{c} c_{p c}\left(T_{c o}-T_{c i}\right)$
where $\mathrm{c}_{\mathrm{pc}}=4.187 \mathrm{~kJ} / \mathrm{kg} \mathrm{m}$
$\therefore 48600=\mathrm{m}_{\mathrm{c}} \times 4.187 \times(47-24)$
$\mathrm{m}_{\mathrm{c}}=\frac{48600}{4.187 \times(47-24)}$
$\mathrm{m}_{\mathrm{c}}=504.67 \mathrm{~kg} / \mathrm{s}$
$\mathrm{m}_{\mathrm{c}} \simeq 30280 \mathrm{~kg} / \mathrm{min}$
(ii) Condenser surface area.

It operates in counter flow, so
$\Delta \mathrm{T}_{1}=\mathrm{T}_{\mathrm{n}}-\mathrm{T}_{\mathrm{co}}=51-47=4^{\circ} \mathrm{C}$
$\Delta T_{2}=T_{n}-T_{c}=51-24=27^{\circ} \mathrm{C}$
Logarithmic mean temperature difference (MTD)
$=\Delta \mathrm{T}_{\mathrm{lm}}=\frac{\Delta \mathrm{T}_{1}-\Delta \mathrm{T}_{2}}{\ln \left(\frac{\Delta \mathrm{~T}_{1}}{\Delta \mathrm{~T}_{2}}\right)}$
$=\frac{4-27}{\ln \left(\frac{4}{27}\right)}$
$=12.04^{\circ} \mathrm{C}$
We know
$Q=u_{0} A_{0} \Delta T_{\text {im }}$
$48600 \times 10^{3}=4000 \times \mathrm{A}_{0} \times 12.04$
$\mathrm{A}_{0}=\frac{48600 \times 10^{3}}{4000 \times 12.04}$
$\mathrm{A}_{0}=1009.136 \mathrm{~m}^{2}$
iii) No. of tubes required per pass by continuity equation
$m_{c}=\left(\frac{\pi}{4}\right) d_{1}^{2} \times \rho \times v_{m} \times N$
Where $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$
$\mathrm{N}=\mathrm{No}$ of pass of tubes.
$\therefore 504.67=\frac{\pi}{4} \times(0.0296)^{2} \times 1000 \times 3 \times N$
$N=\frac{504.67}{2.064}$
$N=244.46$
$\mathrm{N} \sim 244$ tubes
(iv) length of tube per pass
$\mathrm{A}_{0}=\pi \mathrm{d}_{0} \mathrm{~L} \times \mathrm{N} \times 2$ passes
because it is 2 tube pass surface
$\mathrm{A}_{0}=\pi \mathrm{d}_{0} \mathrm{~L} \times \mathrm{N} \times 2$
Because it is 2 tube pass surface
$\mathrm{A}_{0}=\pi \mathrm{d}_{0} \mathrm{~L} \times \mathrm{N} \times 2$
$\Rightarrow L=\frac{A_{0}}{\pi d_{0} \times N \times 2}$
$=\frac{1009.136}{\pi \times 0.0384 \times 244 \times 2}$
$\mathrm{L}=17.14 \mathrm{~m}$

Q-8(c): The total pressure maintained in an Electrolux refrigerator is 15 bar. The temperaure obtained in the evaporator is $-15^{\circ} \mathrm{C}$. The quantities of heat supplied to the generator are (i) 420 kJ to dissociate one kg of vapour and (ii) $1460 \mathrm{~kJ} / \mathrm{kg}$ for increasing the total enthalpy of $\mathrm{NH}_{3}$. The enthalpy of $\mathrm{NH}_{3}$ entering the evaporator is $330 \mathrm{~kJ} / \mathrm{kg}$. Take the following properties of $\mathrm{NH}_{3}$ at $-15^{\circ} \mathrm{C}$ :
Pressure = 2.45 bar
Enthalpy of vapour $=1666 \mathrm{~kJ} / \mathrm{kg}$
Specific volume $=0.5 \mathrm{~m}^{3} / \mathrm{kg}$
The hydrogen enters the evaproator at $25^{\circ} \mathrm{C}$
Gas constant for $\mathrm{H}_{2}=4.218 \mathrm{~kJ} / \mathrm{kg}-{ }^{\circ} \mathrm{C}$
$C_{p}\left(\right.$ for $\left.\mathrm{H}_{2}\right)=12.77 \mathrm{~kJ} / \mathrm{kg}^{\circ}{ }^{\circ} \mathrm{C}$
Find the COP of the system assuming $\mathrm{NH}_{3}$ leaves the evaporator in saturated condition.
[20 MARKS]
Sol: Data given, Total pressure of electrolux refrigerator $\left(p_{T}\right)=15$ bar
Evaporator temperature $\left(T_{e}\right)=-15^{\circ} \mathrm{C}=258 \mathrm{~K}$


The diagram of an electrolux refrigerator is shown above. Consider the flow of $1 \mathrm{Kg}^{\text {of } \mathrm{NH}_{3} \text {, heat given }}$ to the generator.

$$
Q_{G}=Q_{G 1}+Q_{G 2}=40+1460=1880 \mathrm{KJ} / \mathrm{Kg}
$$

Since, in the evaporator $\mathrm{p}_{\mathrm{T}}=15$ bar and $\mathrm{p}_{2}=2.45$ bar $=\mathrm{p}_{\mathrm{NH} 3}$, therefore pressure of hydrogen $\left(\mathrm{H}_{2}\right)$.

$$
\mathrm{p}_{\mathrm{H}_{2}}=\mathrm{p}_{\mathrm{T}}-\mathrm{p}_{2}=15-2.45=12.55 \mathrm{bar}=1255 \mathrm{kN} / \mathrm{m}^{2}
$$

Let m Kg of $\mathrm{H}_{2}$ flow through the evaporation $/ \mathrm{Kg}$ of $\mathrm{NH}_{3}$. The ammonia $\left(\mathrm{NH}_{3}\right)$ occupies $v=0.5 \mathrm{~m}^{3} / \mathrm{Kg}$. The some is the volume of $\mathrm{H}_{2}$ at $\mathrm{T}=-15^{\circ} \mathrm{C}=258 \mathrm{~K}$.

$$
\mathrm{m}=\frac{\mathrm{p}_{\mathrm{H}_{2}} \mathrm{~V}}{125}=\frac{1255 \times 0.5}{4.218 \times 258}=0.5766 \mathrm{~kg}
$$

Considering the energy balance for the evaporation.

$$
\begin{aligned}
1 \mathrm{Kg}\left(\mathrm{~h}_{2}-\mathrm{h}_{1}\right) & =\mathrm{mC}_{\mathrm{p}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)+\mathrm{Q}_{\mathrm{E}} \\
1(1666-330) & =0.5766 \times 12.77(298-258)+\mathrm{Q}_{\mathrm{r} 2} \\
1336 & =294.52+\mathrm{Q}_{\mathrm{E}} \\
\Rightarrow \quad \mathrm{Q}_{\mathrm{E}} & =1041.47 \mathrm{KJ} / \mathrm{Kg} \text { of } \mathrm{NH}_{3} \\
\text { So, } \quad \mathrm{COP} & =\frac{\mathrm{Q}_{\mathrm{E}}}{\mathrm{Q}_{\mathrm{G}}}=\frac{1041.47}{1880}=0.5539
\end{aligned}
$$

